A SEMI-ANALYTIC F.E. MODEL FOR

INERTIAL INTERACTION ANALYSIS

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SUMMARY

A semi-analytic finite element model for the seismic inertial interaction analysis of structures with pile foundation is presented here.

The properties of the elements are established through the elastic Mindlin's formula. The results of the analysis of a typical vent stack of a nuclear power plant, for the simplified Veletsos' accelerogram, is also shown.

THE MATHEMATICAL MODEL

The soil profile is divided into finite elementes which are infinite in the x direction, Fig. 1a. A typical finite element is indicated in Fig. 1b. This element is defined by two line nodal points i and j, Fig. 1b. The displacement field for each element is defined by:

$$u_x = \emptyset_i u_i + \emptyset_j u_j \qquad (1)$$

in which

$$\emptyset_i = (1 - \frac{z}{L})\, e^{-k[x]}; \quad \emptyset_j = \frac{z}{L}\, e^{-k[x]} \qquad (2)$$

The definitions in Eqs. 1 and 2 express the semi-analytical formulation of the soil model. In Eq.2 the displacement field variation in the z coordinate is linear and in the x direction is derived according to Mindlin's theory.

The displacement field $u_x(x,y,z)$ for a horizontal distributed load with height dn is given by the Mindlin's formula (Ref.1).

In Fig. 6 from Ref.1 it is observed that the displacement field is pratically constant along the height H and zero below this height (Fig.2). In Mindlin's expression the Poisson's ration is taken as ν=0,5 and E=3G. The displacement u_x in a generical point with coordinates x, y, and z is expressed as

$$u_x = \frac{p}{G}\; u\left(\frac{x}{H}, \frac{y}{H}, \frac{z}{H}\right) \qquad (3)$$

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in which H is the height of the soil profile and u is an adimensional parameter.

The displacement u_{0x} in a generical point in the plane y=0 is given by

$$u_{0x} = \frac{p}{G} u_0 \left(\frac{x}{H}, \frac{z}{H} \right) \quad (4)$$

The adimensional parameter

$$u_T = \frac{1}{H} \int_0^\infty u\left(\frac{x}{H}, \frac{y}{H}, \frac{z}{H} \right) dy \quad (5)$$

represents a mean displacement in a width equal to the height H. The integral in Eq.5 is calculated numerically taking 10H instead of ∞.

The parameters U_T and U_0 express the mean of the parameters u_T and u_0 along the height H:

$$U_T = \frac{1}{H} \int_0^H u_T dz; \; U_0 = \frac{1}{H} \int_0^H u_0 dz \quad (6)$$

The parameters u_0 and u_T are tabulated as functions of x/H and z/H and U_0 and U_T as function of z/H. The meaning of U_T and U_0 is indicated in Fig.3a. U_0 is the adimensional parameter for y=0 and U_T is defined in order to have $U_T H = \int_0^\infty U dy$.

The half width of the model is chosen in such a way that in the point of application of the load (x=0) the quocient U_T/U_0 approaches B/H. This is equivalent to assume a mean displacement along the width equal to the displacement for y=0 satisfying Eq.5.

For x=0 U_0, has a singularity. Therefore the value of B is found by extrapolation of the curve B/H versus X/H (Fig.3b). The value B=0.3H is adopted (2B = 0.6H).

As B=0.3H, U is defined as $U_T/0.3$. The mean displacement u for the model with half-width equal to B is

$$u = \frac{p}{G} U \left(\frac{x}{H} \right) \quad (7)$$

Introducing

$$\frac{p}{u} = K\left(\frac{x}{H}\right) = \frac{G}{U(x/H)} \quad (8)$$

a plot of G/K (x/H) = U(x/H) as function of X/H is given in Fig.4. The expression adopted for this curve is

$$\frac{G}{K} = 0.477 \; (0.7^{x/H}) \text{ for } u > 0 \quad (9)$$

Taking logarithim of both sides of Eq.9 and the anti-logarithim of the resulting expression one obtains

$$\frac{G}{K} = 0.477 \; e^{0.36 \, x/H} \quad (10)$$

In terms of displacements

$$U = U_0\ 0.477\ e^{-0.36\ x/H} \qquad (11)$$

The stiffness and consistent mass matrices are obtained from the standard expressions of the F.E. Method, with the interpolation functions given by Eq.2:

$$K_i = E'Kdh \begin{bmatrix} 1/3 & 1/6 \\ 1/6 & 1/3 \end{bmatrix} \ (12a) \quad \text{and} \quad M_i = \frac{d\rho h}{K} \begin{bmatrix} 1/3 & 1/6 \\ 1/6 & 1/3 \end{bmatrix} \ (12b)$$

where E' is the stiffness coefficient ρ the specific mass, and $K = 0.36/H$ (Eq.10).

An equivalent rectangular finite element with nodes i and j free and i' and j' built in is now defined, Fig.5. The parameters L', E', ρ' are obtained from the given parameters ρ, H and G.

The equality of the terms in the stiffness and mass matrices of the actual and the equivalent element gives

$$L' = \frac{1}{K} = 2.8H \quad (13a) \qquad \text{and} \qquad \rho' = 3\rho \quad (13b)$$

E' is obtained from the equality of the static stiffness of the equivalent element, in a unit area, and the stiffness given by the Mindlin's theory:

$$K_0 = \frac{E'd}{L'} = \frac{E'd}{2.8H} \quad (14a) \quad \text{and} \quad K_M = 2.1G \ \therefore\ E' = 9.8G \quad (14b)$$

NUMERICAL ANALYSIS

As example of application of the finite element presented above, the seismic analysis of a 150m height chimney was performed. The foundation of this structure consists of 12 concrete piles (diameter-130cm). The considered soil profile is heterogeneous, with two layers (sand-18m thick, + clay -, 12m), upon rigid rock. A linear behavior of the soil is assumed, with constant values for stiffness and damping

The seismic input (considered in the top of the rock) is the simplified accelerogram presented by Veletsos in Ref.2. This accelerogram (Fig.6) gives a response spectra similar to a spectra of a real earthquake. In this analysis it is assumed $\ddot{y}_g = 1m/seg^2$ and $t_1 = 0.35$ seg.

The analysis is performed in two steps (kinematic and inertial analysis), according to the superposition theorem (see Ref.3) presented by Kausel.

As the analysed structure is very flexible with a negligible embedded region, and supported by long piles, the kinematic analysis is performed neglecting the modification in the seismic displacements of the soil, due to the presence of the structure. The model for the kinematic analysis is the conventional shear-beam model with arbitrary area. Since it is admited that the piles are long (according to Flores, Ref.4), the forces in a pile are obtained coupling its horizontal displacements with the soil shear-beam.

The basic input for the inertial analysis is the resulting accelerogram in the soil surface. In Fig.8 the response spectra if this accelerogram is compared with the spectra in the rock surface (Veletsos accelerogram).

The pile group factor was evaluated through the theory developed by Poulos (Ref.5). The stiffness of the pile group was obtained by inversion of the flexibility matrix (assembled with the interaction coefficients between two piles, given in the graphics of the Ref.5).

The model for the inertial analysis consists of the soil finite element model developed previously, the condensed pile group (taking into account the pile group effect according to Poulos and the rocking stiffness) and the superstructure modeled through plane frame elements (Fig.9).

In Fig.10 the final results of the analysis i.e. the bending moments and shear forces along the pile lenght, are shown. The final forces in the piles are the sum of the forces obtained in both steps of the anlysis (in Fig. 10 there are shown the kinematic and total forces along the piles).

The analysis was performed by the direct integration (step - by - step) method with help of the computer program Lorane - Dina (Ref.6).

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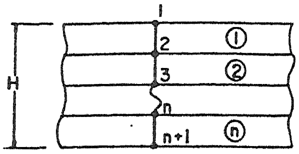


FIG. 1a

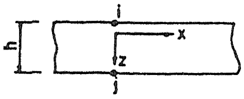


FIG. 1b

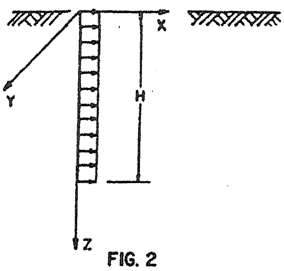


FIG. 2

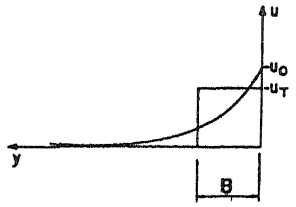


FIG. 3a

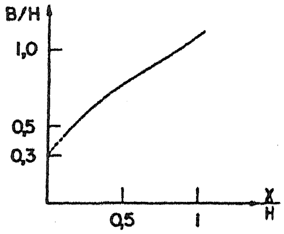


FIG. 3b

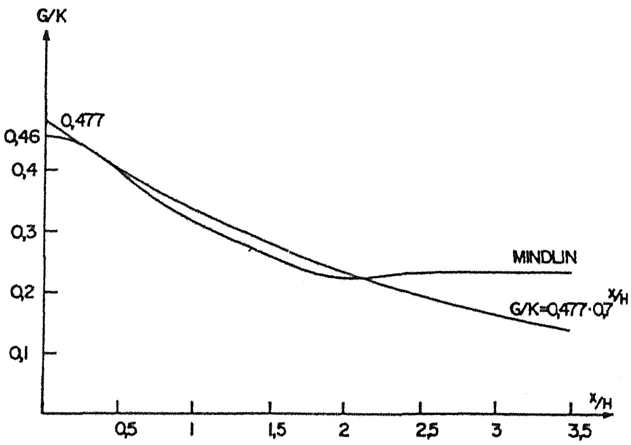


FIG. 4

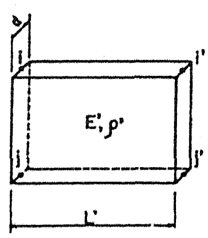
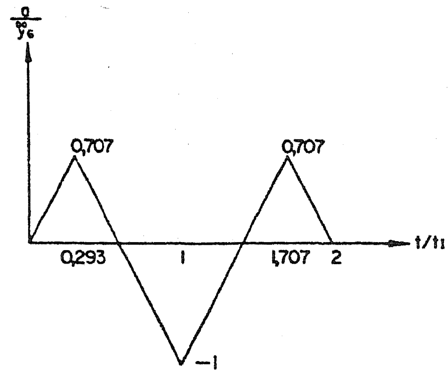


FIG. 5



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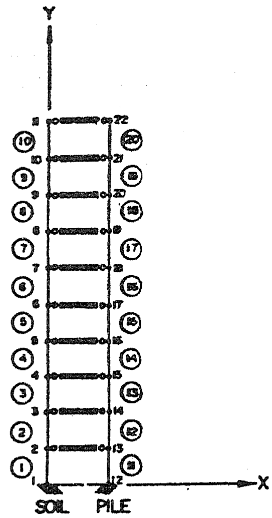

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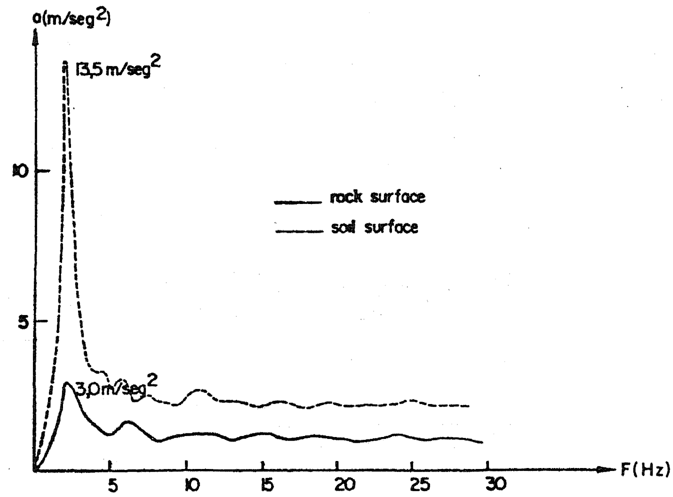

FIG. 8

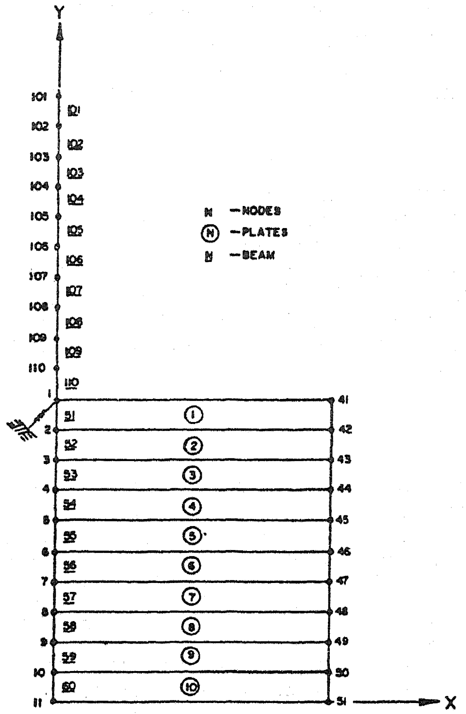

FIG. 9

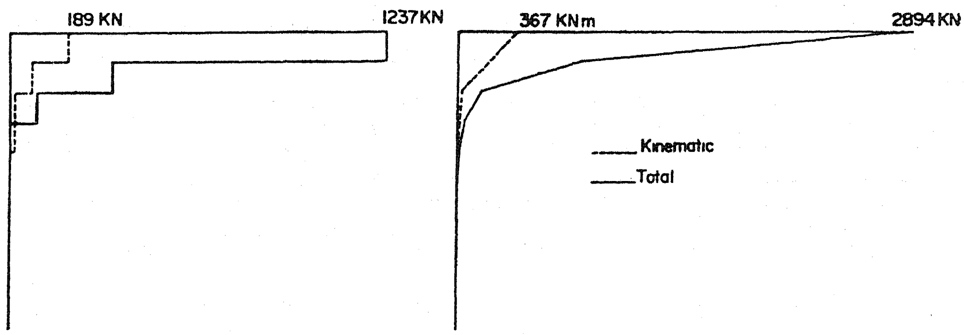

FIG. 10a

FIG. 10b