SUBSTRUCTURE METHOD FOR EARTHQUAKE RESPONSE OF HIGH-ELEVATED MULTI-SPAN CONTINUOUS BRIDGE

Hirokazu TAKEMIYA[I] and Kenji KAWANO[II]

SYNOPSIS

Seismic response analysis is made for a multi-span continuous high pier bridge using random vibration approach. Particular interest is placed on the out-of-phase difference effect of input motions on structural response. Effective inclusion of the soil-structure interaction is made through substructure method: formulating the soil-foundation system and pier-girder system separately and then integrating them at their interface. Travelling SH wave is modeled based on one dimensional propagation theory. Conclusion is drawn that the in-phase input gives a possible maximum response for the present soil-structure system.

INTRODUCTION

This paper aims at an earthquake response analysis of a high-elevated double deck three-span continuous bridge (Fig. 1) in the direction perpendicular to the bridge axis. The long span and high piers make the total structure relatively flexible. For such a structure an isolated pier with girder inertia effect may fail to predict the correct behaviour during earthquake motions [1]. Furthermore, the bridge axis variation of girder response is of great importance.

Deliberate consideration is first taken for the soil-structure interaction since it becomes significant in a certain situation, changing vibrational characteristics of the superstructure from when rigidly supported. The so-called dynamic substructure method is applied herein, which deals with the soil-foundation system and superstructure separately and then integrates them with use of continuity requirements at their interface. Caisson foundations are used in the present structure. The soil medium is modeled such that a uniform elastic layer overlies a firm half-space substratum. Its reaction is reproduced by the spring-dashpot system representation [2]. The superstructure with rigid constraint is formulated from the finite element method.

Another important aspect relevant to the above structure is the fact that it has multiple input points for earthquake motion. When the girder has a long span, the input motions become different at foundations due to the travelling nature of waves. This out of phase effect is examined using a simple shear wave propagation which has an oblique incident angle at the substratum.

Since earthquake motions are random phenomena in nature, random vibration theory is fully applied throughout this investigation.

[I] Associate Professor, Department of Civil Engineering, Okayama University, Okayama, 700, JAPAN

[II] Research Associate, Department of Civil Engineering, Kyoto University, Kyoto, 606, JAPAN

FORMULATION

Wave Propagation- This is investigated for a uniform surface layer on a firm half-space substratum (Fig. 2). The one dimensional SH wave propagation, when an oblique incident wave arrives at the surface layer bottom with an angle α_2 from the vertical axis, is of interest. The soil particle motion is solved from the corresponding shear beam equation with appropriate boundary conditions. Phase difference and attenuation if damping exists are observed even at equal level locations due to the oblique incident angle. For instance, at two points l' and m' in Fig. 2 which are apart in horizontal distance D(m), the particle motion at m' is delayed from that at l' by the time

$$\tau_o = \frac{D \sin \alpha_2}{V_{s2}} \tag{1}$$

in which V_{s2} is the shear wave velocity in the substratum.

Now assume the following stationary random motion (acceleration) at the surface layer bottom which is characterized by the power spectral density function of

$$G_b(\omega) = \frac{1 + (2h_g\omega/\omega_g)^2}{\{1-(\omega/\omega_g)^2\}^2 + (2h_g\omega/\omega_g)^2} S_o \tag{2}$$

in which ω_g and h_g are parameters which control the shape, refrecting the substratum characteristics and S_o is a constant related to the intensity. After some manipulation one can get the stationary random motion (acceleration) at free surface, which has the power spectral density function of

$$G_s(\omega) = \frac{G_b(\omega)}{\cos^2\lambda_1 + q^2\sin^2\lambda_1} \tag{3}$$

in which $q=(\rho_1 G_1/\rho_2 G_2)^{1/2}\cdot(\cos\alpha_1/\cos\alpha_2)$, $\lambda_1=\omega\cos\alpha_1 H/Vs_1$, ρ_i=mass density, G_i=shear rigidity and other notations are clear from Fig. 2. For the succeeding analysis the corresponding auto-correlation function is preferred. The Wiener-Khintchine relationship gives

$$R(\tau) = A\exp(-h_g\omega_g\tau)\,(e_1\cos\omega_d\tau - e_2\sin\omega_d\tau) \tag{4}$$

in which

$$A = S_0\pi\omega_g/(2h_g\sqrt{1-h_g^2}\,(x_1^2+y_1^2))$$

$$x_1 = (1-q^2)c^2\cos 2\gamma + q^2\ ,\quad y_1 = c^2(1-q^2)\sin 2\gamma$$

$$c^2 = \cos^2 x + \sinh^2 y\ ,\quad \tan\gamma = -\tan x\tanh y$$

$$x = (\omega_g\sqrt{1-h_g^2}\,H\cos\alpha_1)/V_{s1}\ ,\quad y = (\omega_g h_g H)/V_{s1}$$

$$a_1 = (1+4h_g^2)\sqrt{1-h_g^2}\ ,\quad b_1 = h_g(4h_g^2-1)$$

$$e_1 = a_1x_1+b_1y_1\ ,\quad e_2 = b_1x_1-a_1y_1\ ,\quad \omega_d = \omega_g\sqrt{1-h_g^2}$$

Substructure- The soil-caisson foundation system is formulated based on the assumption that a rigid body is partially embedded in the surface layer and has two degree-of-freedom of horizontal translation and rocking about its gravity center. Modeling of the soil reaction is made by a combination of spring and dashpot whose quatitative determination is referred to the Veletsos and Verbic work [3] for the half-space substratum and the Beredugo and Novak work [4] for the side soil medium. This means that one deals with the respective stratum independently. The details are described in Ref.[2]. The equation of motion is thus expressed, when an input motion is considered at the gravity center, as

$$[M]_{sub}\{\ddot{x}\}_{sub} + [C]_{sub}\{\dot{x}\}_{sub} + [K]_{sub}\{x\}_{sub} = \{F\}_{sub} + [\beta]_c^T\{R\}_{sup} \qquad (5)$$

in which $\{x\}_{sub}$ denotes the relative displacement; $[M]_{sub}$, $[C]_{sub}$, and $[K]_{sub}$ are the mass, damping and stiffness matrices, respectively; $\{F\}_{sub}$ is the driving force; $\{R\}_{sup}$ is the internal forces acting at the caisson top; $[\beta]_c$ stands for the displacement influence matrix from caisson's gravity center to its top.

Superstructure- Common finite element formulation gives the governing equation for the released boundary condition; namely, the pier base is subject to have degrees of freedom in connection with the substructure motion. When the boundary nodal displacement $\{x_j\}$ is separated from others $\{x_i\}$, and the corresponding partitioning is made for the mass, damping and stiffness matrices, then

$$\begin{bmatrix}[M_{jj}] & [M_{ji}] \\ [M_{ij}] & [M_{ii}]\end{bmatrix}\begin{Bmatrix}\{\ddot{x}_j\} \\ \{\ddot{x}_i\}\end{Bmatrix}_{sup} + \begin{bmatrix}[C_{jj}] & [C_{ji}] \\ [C_{ij}] & [C_{ii}]\end{bmatrix}\begin{Bmatrix}\{\dot{x}_j\} \\ \{\dot{x}_i\}\end{Bmatrix}_{sup} + \begin{bmatrix}[K_{jj}] & [K_{ji}] \\ [K_{ij}] & [K_{ii}]\end{bmatrix}\begin{Bmatrix}\{x_j\} \\ \{x_i\}\end{Bmatrix}_{sup} = \begin{Bmatrix}\{F_j\} \\ \{0\}\end{Bmatrix}_{sup} \qquad (6)$$

in which $\{F_j\}$ denotes the internal forces at interface with the substructure. Expressing the free nodal displacement as the sum of quasi-static component $\{x_i^u\}$ due to the movement of caisson tops, and the dynamic component $\{x_i^c\}$ for the constraint boundary,

$$\{x_i\}_{sup} = [\beta]_{sup}\{x_j\}_{sup} + \{x_i^c\}_{sup} \qquad (7)$$

in which $[\beta]_{sup}$ stands for the displacement influence matricx, that is found for the present statically indeterminate system, by

$$[\beta]_{sup} = - [K_{ii}]^{-1}[K_{ij}] \qquad (8)$$

Substituting Eq.7 into Eq.6, one can establish the superstructure equation in the constraint displacement and boundary displacement.

Integrated Structure- In order to combine the above respective subsystems the displacement compatibility and force equilibrium are taken into account. These are respectively

$$\{x_j\}_{sup} = \{[\tilde{\alpha}](\{\tilde{x}\}_{sub}+\{\tilde{x}_g\})\} \qquad (9) \qquad \text{and} \qquad \{F_j\}_{sup} + \{\tilde{R}\}_{sup} = \{0\} \qquad (10)$$

in which $\{\tilde{x}_g\}$ is the input displacement at the respective caisson gravity center and the motion defined by Eq.4 is substituted for this.

The simultaneous equations are then obtained for the complete system in terms of the constraint superstructural displacement and substructural displacement.

$$\left[M_{compl}\right]\begin{Bmatrix}\{\ddot{x}_i^c\}\\ \{\ddot{x}\}_{sub}\end{Bmatrix} + \left[C_{compl}\right]\begin{Bmatrix}\{\dot{x}_i^c\}\\ \{\dot{x}\}_{sub}\end{Bmatrix} + \left[K_{compl}\right]\begin{Bmatrix}\{x_i^c\}\\ \{x\}_{sub}\end{Bmatrix}$$

$$= \begin{Bmatrix}[F_1]\{\ddot{x}_g\}\\ [F_2]\{\ddot{x}_g\}\end{Bmatrix} + \begin{Bmatrix}\{0\}\\ [F_3]\{x_g\}\end{Bmatrix} \qquad (11)$$

In order to decrease the degrees of freedom without loss of accuracy, the classical normal modes are presumed for the $\{x_i^c\}$. Generally, in seismic response analysis the higher modes less contribute. The orthogonality conditions are given for the modal matrix $[\Phi_i^c]$ by

$$[\Phi_i^c]^T[M_{ii}][\Phi_i^c] = [I]\,;\ [\Phi_i^c]^T[C_{ii}][\Phi_i^c] = \lceil 2\xi_l\omega_l \rfloor;\ [\Phi_i^c]^T[K_{ii}][\Phi_i^c] = \lceil \omega_l^2 \rfloor \qquad (12)$$

Random Response Analysis- The response covariance matrix plays most important role. For the superstructural sections this is evaluated by

$$E[\{x_i\}_{sup}\{x_i\}_{sup}^T] = [\beta]_{sup}E[\{x_j\}\{x_j\}^T][\beta]_{sup}^T + [\beta]_{sup}E[\{x_j\}\{q_i^c\}^T][\Phi_i^c]^T$$

$$+ [\Phi_i^c]E[\{q_i^c\}\{x_j\}^T][\beta]_{sup}^T + [\Phi_i^c]E[\{q_i^c\}\{q_i^c\}^T][\Phi_i^c]^T \qquad (13)$$

As an approximation, one may impose classical normal modes decomposition further on the complete system equation Eq.11. This is not an improper assumption for the soil-structure interaction problem [2]. Hence,

$$E[\{x\}\{x\}^T] \simeq [\Phi]E[\{q\}\{q\}^T][\Phi]^T \qquad (14)$$

in which $E[\{q\}\{q\}^T]$ stands for the modal response covariances.

In view of the possible use of the well-known response spectrum, which is based on the in-phase input, even for a multiple input system, the respective modal response is investigated in connection with the out-of-phase effect from

$$E[q^2] = \sum_{j=1}^{N}\gamma_j^2\sigma_j^2 + 2\sum_{i,j=1}^{N}\gamma_i\gamma_j\sigma_{ij}^2 \qquad (15)$$

in which γ_i denotes the modal participation factor and σ_j is the rms response of a unit mass single degree-of-freedom system for a sole input and σ_{ij} is for two different inputs; N is the numbers of input points.

NUMERICAL RESULTS AND DISCUSSION

A numerical computation was carried out for a modified structural model, as shown in Fig. 1, of the high elevated girder bridge to be constructed in the Bannosu area as a part of the Honshu-Shikoku Strait Bridges. For finite element discretization, five segments are taken for each pier part and four segments for the upper and lower girder decks in each span. A caisson structure (height=20m, equivalent radius on equal area basis=17m), embedded 15m down to the firm substratum, composes the foundations. Soil properties for strata are indicated in Fig. 2. The parameters characterizing the substratum power spectral density function are chosen as ω_g=6π and h_g=0.6. Others not specified are used as variables in this investigation.

The dynamic characteristics of the soil-structure interaction system together with the rigidly supported system are shown in Fig. 3 for the girder modes and indicated in Table 1 for the natural frequencies (periods). Vibration mode shapes are normalized with respect to the mass matrix. Changes in vibration modes are recognized due to the soil-structure interaction from when rigidly supported.

Since the present structure has a multiple foundations in which earthquake motions are input and in view of their propagation in the soil medium, emphasis is placed on the input phase difference effect on the superstructure response. Herein, in the forcing function in Eq. 11 the predominant acceleration is only taken into account since the displacement input is judged to be minor for practical situations, from experience. In addition to the phase difference τ_0 defined Eq. 1, the oblique incident wave front itself has other phase difference times due to the path through which it propagates. The gross effect is discussed below, denoting it by τ_0.

The rms displacement responses computed by the classical normal modes method for the complete structure (approximate solution) are shown in Fig. 4 for two representative nodes as indicated in Fig. 1 by A and B. The values are normalized by a specific excitation level that nondimensionalizes the fundamental mode response of rigidly supported structure to a white noise of intensity S_0. Oscillatory variation of response is observed with the phase difference τ_0/T_1 where T_1 is the fundamental period of superstructure. In these figures the soil-structure interaction system is compared with the rigidly supported one for some soil profiles. Note that the surface layer filtering effect on the input motion is less significant among the examples investigated, but the soil-structure interaction effec is strong. In-phase input gives a greater response value for the interaction system than when rigidly supported at node A, while a smaller response value at node B. The rigid supporting indicates that the maximum response is attained at the in-phase input situation for node B while at a specific time lag input for node A. However, for the interaction system the in-phase input yields a possible maximum response value at node B and almost equal response value even at node A by assuming in-phase input situation. The similar trend can be noted for the rms bending moment variation in Figs. 5. Figs. 6 depict rms displacement variation of the complete structure for a certain soil profile.

In order to explain the above phenomenon. the respective modal response is investigated against time lag τ_0 between adjacent piers. Fig. 7 shows the rigidly supported case to exclude the soil-structure interaction effect. The ordinate p_f is the response ratio devided by the value when each input is assumed independent. Note that the first mode indicates a decaying tendency with oscillation from τ_0=0 which corresponds to the in-phase input situation, but the second and third modes give peak values at specific values of τ_0 within a fundamental period of time lag. This trend means that higher modes have a possibility to be magnified for a multiple input system. The use of response spectrum, therefore, may underestimate response where these modes contributions are significant, while in case of the first being predominant it gives a possible maximum value.

CONCLUSIONS

From these investigations, one may conclude that:

The out-of-phase input effect on structural response is closely related to the vibration modes in such a way that the first mode contribution is the greatest at no phase difference input, while the higher modes grow at specific time lag input situation.

For the present structure, due to the soil-structure interaction, the fundamental mode contribution dominates the response, which means that a possible maximum response value is obtained for almost every structural section by assuming in-phase input situation.

ASKNOWLEDGMENT

The authors thank Professor Yoshikazu Yamada of Kyoto University for his support throughout this research, and Mr.Akira Hirano for his help in computations.

REFERENCES

1. Yamada,Y.,Takemiya,H. and Noda,S. (1980), "Layered Soil-Pile-Structure Dynamic Interaction", Proc. 7th Wld. Conf. Earthq. Engng., Istanbul, Turkey
2. Yamada,Y.,Takemiya,H. and Kawano,K. (1979), "Random Response Analysis of A Nonlinear Soil-Suspension Bridge Pier", Int.J. Earthq. Engng. Struct. Dyn., Vol.7, pp.31-47
3 Veletsos,A.S. and Verbic,B. (1974), "Basic Response Function for Elastic Foundations", J. Engng. Mech. Div., ASCE, Vol.100, No.EM4, pp.189-201
4. Beredugo,Y.O. and Novak,M. (1972), "Coupled Horizontal and Rocking Vibration of Embedded Footing", Canad. Geotech.J.,Vol.9,pp.477-497

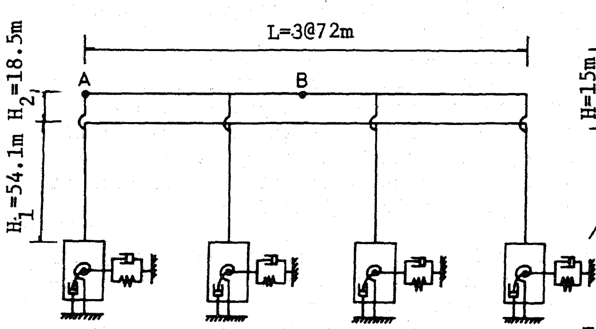


Fig.1 Soil-Foundation-Structure

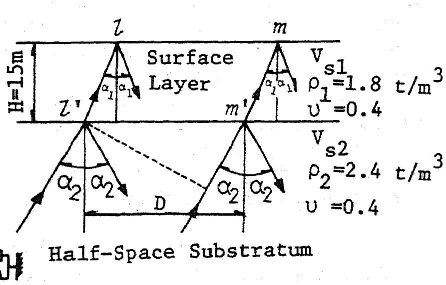


Fig.2 Wave Propagation Model

Table 1 Natural Frequencies

Modes	Rigidly Supported System(ω_i) (rad/s (s))	Interaction System($\tilde{\omega}_i$) (rad/s) V_{s2}=500m/s	
		V_{s1}=150m/s	V_{s2}=250m/s
1st	10.06(0.624)	7.39	7.75
2nd	11.52(0.545)	7.93	8.39
3nd	14.50(0.433)	8.53	9.13
4nd	19.37(0.324)	8.81	9.44
5nd	19.54(0.321)	12.75	13.03

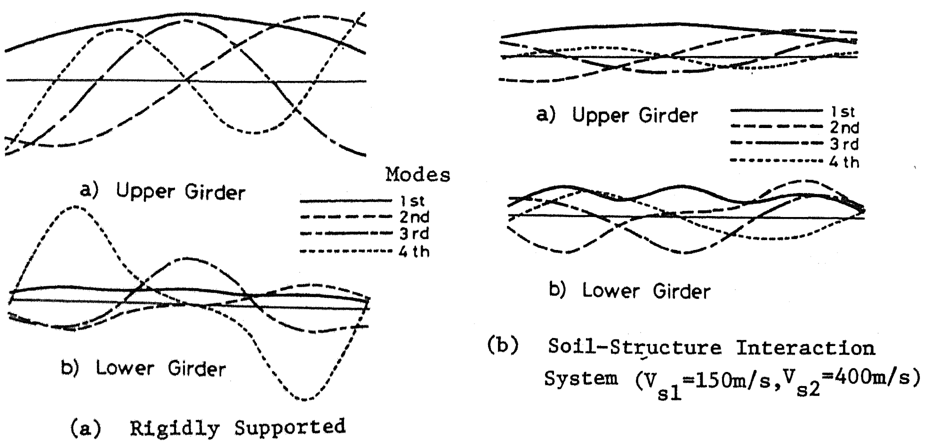


Fig.3 Girder Vibration Modes

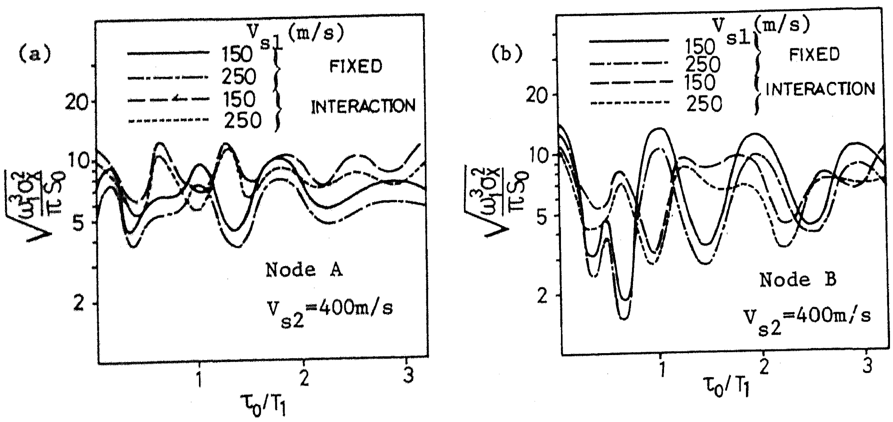


Fig.4 RMS Response vs Phase Lag

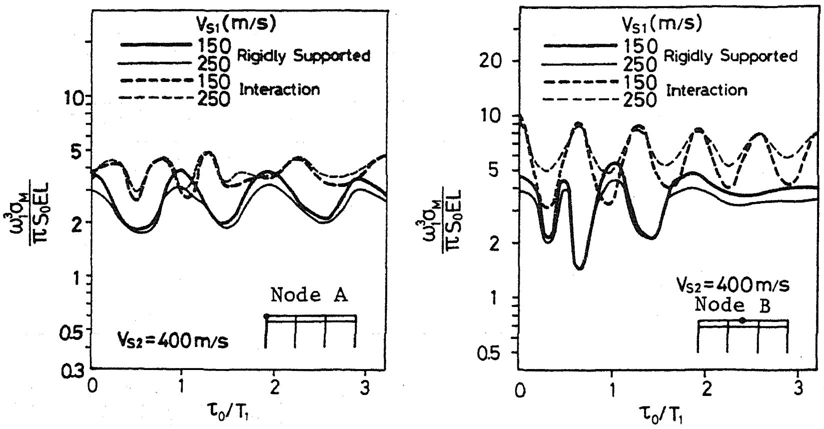
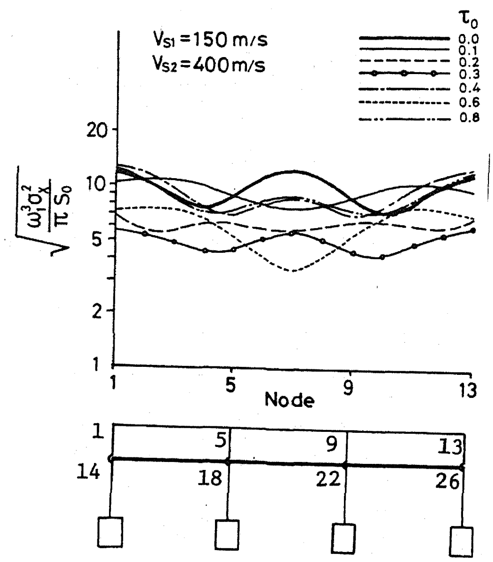


Fig.5 Bending Moment vs Phase Lag



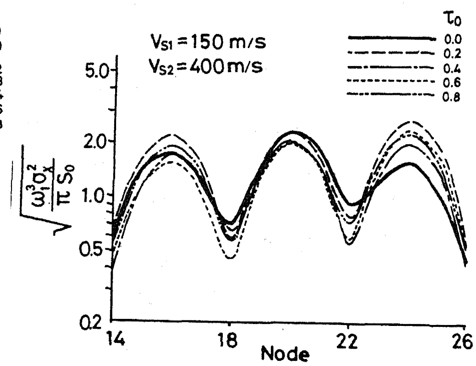


Fig.6 RMS Response of Interaction System

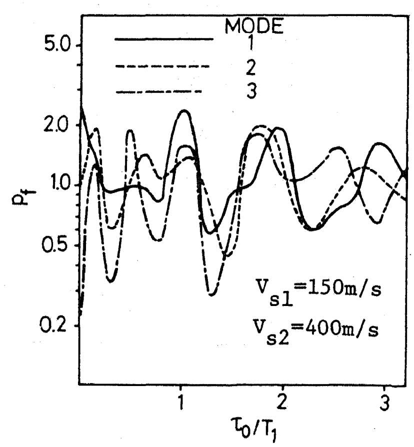


Fig.7 Phase Lag Effect on Participation Factor