MODAL ANALYSIS OF NONLINEAR SYSTEMS

Tauqir M. Sheikh,[I] Jose M. Roesset,[II] and C.P. Johnson[III]

Nonlinear dynamic analyses are performed using the elastic modes as generalized coordinates. In this way the integration can be carried out with an explicit formula without the stringe limitation on the time step imposed by a direct solution of the equations of motion. Results are presented for a 10 story frame and for a 15 story building with coupled shear walls. The solutions are compared to those obtained with the program DRAIN 2D which performs a direct integration using an implicit scheme. The effect of the number of modes considered on the accuracy of the solution is discussed.

INTRODUCTION

Since the early work by Clough and Benuska in 1966 (3) a considerable amount of work has been done on the dynamic behavior and response of structures subject to seismic excitation using dual or single component models (2, 4) to reproduce nonlinear behavior of the members, and explicit or implicit integration schemes to solve numerically the equations of motion. The purpose of this work was to investigate the possibility of using modal analysis, with the elastic modes of the structure, to predict the nonlinear dynamic response. An explicit integration scheme can then be used for each mode, the time step of integration being controlled by the number of modes considered. Modal damping can be defined directly without the need to compute an artificial damping matrix and any frequency variation can be accomodated. Finally inspection of the modes and their contribution provides valuable insight into the behavior. The question of accuracy, related to the time step and hence the number of modes, exists. From the studies performed it appears that three to five modes will be sufficient for most typical structures.

FORMULATION

The equations of motion are written in the form

$$M\ddot{U} + C\dot{U} + F = -MI\,\ddot{u}_g \tag{1}$$

where M is the mass matrix, C the damping matrix assumed to satisfy the orthogonality condition, F are the internal forces in the structure (KU in the elastic range), K is the stiffness matrix, U, $\dot{U}$ and $\ddot{U}$ represent the relative displacements velocities and accelerations, I is a unit vector (all components unity) and $\ddot{u}_g$ is the ground acceleration of the earthquake.

If Φ_i are the mode shapes for the undamped system in the elastic range, normalized so that

$$\Phi_i^T M \Phi_i = 1 \tag{2}$$

$$\Phi_i^T K \Phi_i = w_i^2 \tag{3}$$

I. Graduate Student, The University of Texas at Austin.
II. Professor of Civil Engineering, The University of Texas at Austin.
III. Assoc. Prof. of Civil Engineering, The University of Texas at Austin.

$$\Phi_i^T C \Phi_i = 2D_i w_i \quad (4)$$

$$\Phi_i^T MI = \Gamma_i \quad (5)$$

where w_i is the ith natural circular frequency, D_i is the damping ratio at the frequency w_i (for the ith mode) and Γ_i is the static participation factor of the ith mode, expressing the solution in the form

$$U = \Sigma a_i(t)\Phi_i \quad (6)$$

represents simply a change of coordinates.

In the elastic range the vector of forces F can also be expressed as

$$F = \Sigma a_i(t)K\Phi_i \quad (7)$$

and the modal equations of motion become

$$\ddot{a}_i(t) + 2\beta_i w_i \dot{a}_i(t) + w_i^2 a_i(t) = -\Gamma_i \ddot{u}_g \quad (8)$$

For a nonlinear system the expansion of the displacements in terms of the modes is still valid but expression (7) for the forces cannot be applied. One can write, however at a time $t_{n+1} = (n+1)\Delta t$

$$F_{n+1} = F_n + \Delta F_n = F_n + K_{tn}\Delta U_n \quad (9)$$

where K_t is the tangent stiffness matrix and

$$\Delta U_n = U_{n+1} - U_n \quad (10)$$

The modal equations are then

$$\ddot{a}_i(t) + 2\beta_i w_i \dot{a}_i(t) + \Phi_i^T F = -\Gamma_i \ddot{u}_g \quad (11)$$

with

$$\Delta F = K_t \Delta U \quad (12)$$

Using the central difference formula to solve equations (11) and (12)

$$(1+\beta_i w_i \Delta t)a_{i,n+1} = 2a_{i,n} - (1-\beta_i w_i \Delta t)a_{i,n-1} - \Delta t^2(\Gamma_i \ddot{u}_{gn} - \Phi_i^T F_n) \quad (13)$$

and

$$F_{n+1} = F_n + K_{tn}\Sigma(a_{i,n+1} - a_{i,n})\Phi_i \quad (14)$$

or

$$\Phi_i^T F_{n+1} = \Phi_i^T F_n + \Sigma(a_{i,n+1} - a_{i,n})(\Phi_i^T K_{tn}\Phi_i) \quad (15)$$

Notice that in this form although the modal responses are in fact coupled, the coupling is provided by equations (14) or (15). In the solution algorithm, if forces and displacements are known at time n one can form the tangent stiffness matrix K_{tn} and solve for the values of $a_{i,n+1}$ for all desired modes. Knowing $a_{i,n+1}, K_{tn}$ one can then solve equation (15) computing $\Phi_i^T F_{n+1}$ and the solution of (13) proceeds to the next time step.

If all the modes of the structure are considered the solution performed this way will coincide with a direct solution of the equations of motion (1). The main interest of this approach is the possibility of obtaining accurate solutions by considering only a limited number of modes. It is interesting to notice that if m modes are used the procedure provides a convenient way to select an equivalent m degree of freedom system to reproduce the structure. In particular for only one mode in equivalent single degree of freedom system is provided (7) .

TEN STORY FRAME

To check the validity of the proposed procedure a ten story, one bay frame, originally designed and studied by Anerson (1), was considered. While this frame is in itself of little interest, and rather untypical of building structures, it has been extensively studied by various authors (1, 2, 7) and it provided therefore a good subject for comparison. The frame is shown in Fig. 1. Its first natural period, including gravity forces and P-Δ effects is 2.175 seconds and its smallest (tenth) period is 0.096 seconds. This implies that a time step of integration of 0.01 seconds, which would be normally used as a minimum to reproduce properly the variation of the earthquake acceleration, or even 0.02 seconds, are sufficient to guarantee stability. Under these conditions the savings performed by selecting less than 10 modes for the numerical solution are rather small. Study of this frame provided still information on the number of modes needed to obtain a good accuracy.

Fig. 2 shows the story displacements obtained from the present analysis using a time step of 0.01 seconds and all ten modes, those reported by Aziz (2) and the results obtained using the program DRAIN2D (5) (with the same time step). In all three cases damping was assumed to be proportional to the mass matrix (i.e. decreasing with frequency) with a value of 0.05 for the first mode, and the structure was subjected to the N component of the 1940 El Centro earthquake. Results from the present method and DRAIN2D are almost identical and very similar to those of Aziz, with differences smaller than 5% at the 7th and 9th floor (it is not clear whether Aziz used exactly the same accelerogram). Fig. 3 shows the values of the column moments from the three analyses and Fig. 4 the corresponding results for the girders. Differences are still very small, of the order of 10% as a maximum for the column moments and 5% or less for the girder moments, the results of the modal analysis being typically intermediate between the other two. It should be noticed however that these minor differences in moments can result in much larger variations in moment ductilities if yielding occurs. This is due to the somewhat arbitrary definition of ductility ratios (1, 2, 6). Fig. 3 would seem to indicate that the columns have not yielded. Although the column moments are less than the plastic capacities, when the axial forces and the interaction diagram are taken into account there is in fact a small amount of yielding in some of them. Figs. 5 and 6 show the same results assuming damping proportional to the stiffness matrix, the mass matrix or constant. In all cases the damping is 0.05 in the first mode. The variations in the results are of the same order as those observed for the different solution algorithms. Since the actual amount of damping and its frequency dependence are not well known these results indicate the difficulty in predicting reliable estimates of the ductility ratios when they are close to 1.

Figs. 7, 8 and 9 show the effect of the number of modes considered on the floor displacements, column moments and girder moments respectively. Results are presented for 1, 2, 3 and 10 modes. The results for 5 modes coincided within the accuracy of the drawings with those for 10 modes. It can be seen that the solution with 3 modes is already very close to what could be called the exact solution with all the modes. Even the results with 2 modes would be sufficiently accurate for practical purposes considering all the other uncertainties, and the use of only 1 mode while producing larger variations accounts for most of the solution.

SHEAR WALL BUILDING

A more interesting test for the method is provided by a 15 story frame made of two coupled shear walls as shown in Fig. 10. The fundamental period in this case is of 0.8 seconds with a smallest period of the order of 0.0133 seconds. A direct solution with the central difference formula would require a time step of integration of 0.0042 seconds.

Figs. 11, 12 and 13 show a comparison of results from the modal analysis with all the modes and DRAIN2D using in both cases a time step of 0.0015 seconds. Displacements are again almost identical as are column moments and girder moments except in the lower floors (particularly the bottom story) where the modal solution gives values about 3% larger than DRAIN2D.

Figs. 14, 15 and 16 show the effect of the number of modes for 1, 2, 3 and all 15 modes. Again in this case the results for 5 modes could not be distinguished from those with all modes within the accuracy of the drawing. Results for 3 modes are already very good, the maximum discrepancies occurring in the upper stories for column moments (variations of the order of 30 to 50% where the moments are very small and of little consequence for design purposes) and in the bottom stories for girder moments (with differences of the order of 2%). It is interesting to notice that the shape of the curve for the girder moments provided by 3 modes is almost identical to that obtained from DRAIN2D. As more modes are considered the moments in the girders of the bottom story tend to increase while those in the third, fourth and fifth story decrease. The variations are, however, very small. Even the solution for 2 modes would probably be adequate for design purposes.

REFERENCES

1. Anderson, J. C. and Bertero, V. V. "Seismic Behavior of Mulistory Frames Designed by Different Philosophies," Report No. EERC-69-11. Un. of California, Berkeley. October 1969.
2. Aziz, T. S. A. "Inelastic Dynamic Analysis of Building Frames," Report R76-37, M.I.T.
3. Clough, R. W. And Benuska, K. L. "FHA Study of Seismic Design Criteria for High Rise Buildings," HUD TS-3, Aug. 1966.
4. Giberson, M.F. "Two Nonlinear Beams with Definitions of Ductility," Journal of the Structures Division. ASCE-STR. Feb. 1969, pp. 137-157.
5. Kanaan, A. E. and Powell, G. M. "General Purpose Computer Program for Dynamic Analysis of Inelastic Plane Structures." Report EERC 73-22 Berkeley. April 1973.
6. Mahin, S. A . and Bertero, V. V. "An Evaluation of Some Methods for Predicting Seismic Behavior of Reinforced Concrete Buildings," Report EERC 75-5. Berkeley. Feb. 1975.
7. Pogue, J. R. "On the Use of Simple Models in Nonlinear Dynamic Analysis" Report R76-43. M.I.T.

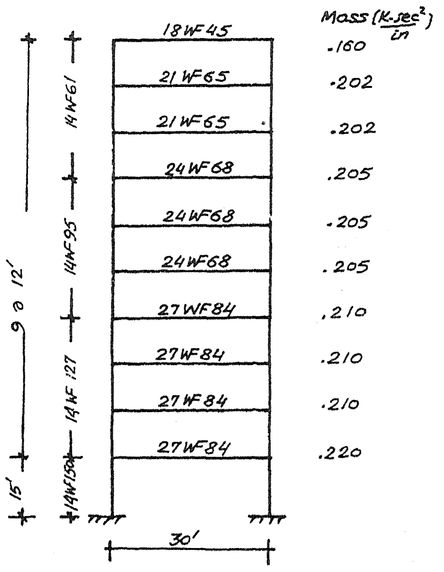

FIG 1 - ANDERSON FRAME

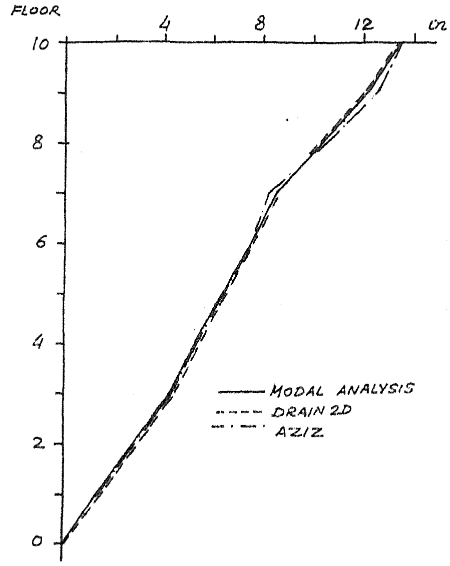

FIG 2 FLOOR DISPLACEMENTS

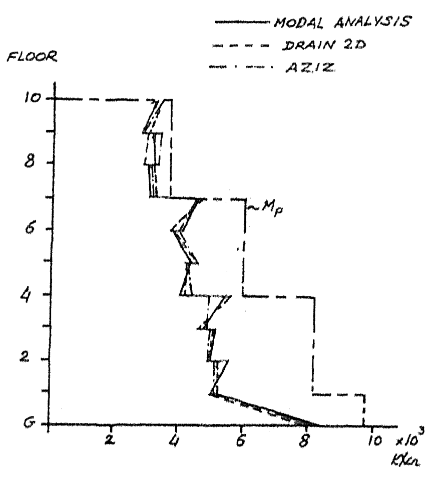

FIG. 3 COLUMN MOMENTS

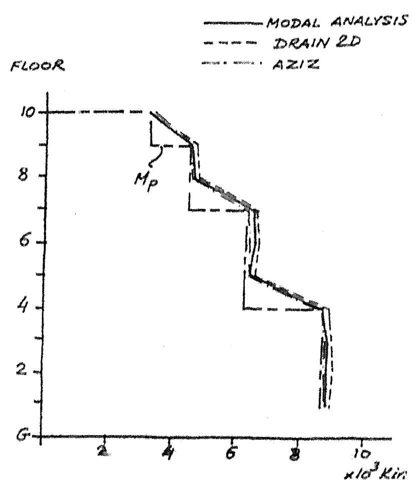

FIG. 4 GIRDER MOMENTS

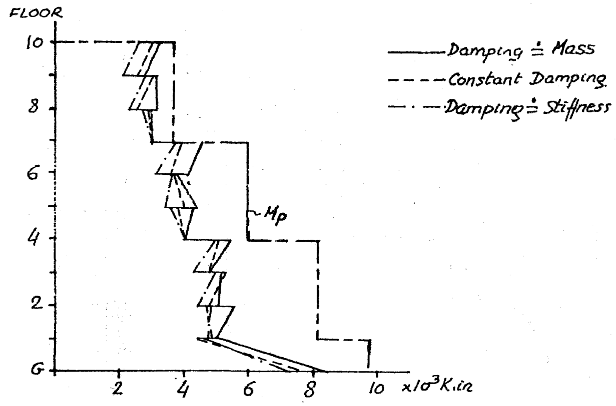

FIG.5 EFFECT OF DAMPING NATURE ON COLUMN MOMENTS

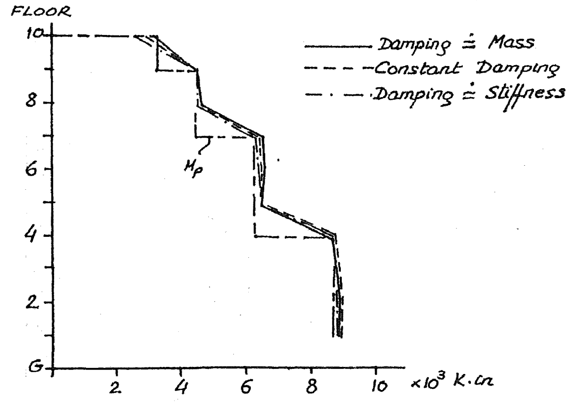

FIG.6 EFFECT OF DAMPING NATURE ON GIRDER MOMENTS

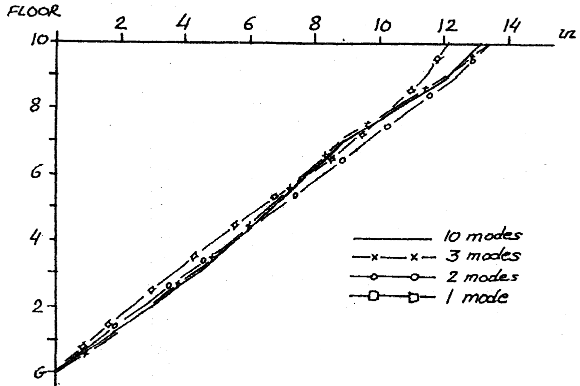

FIG 7 EFFECT OF NUMBER OF MODES ON DISPLACEMENTS

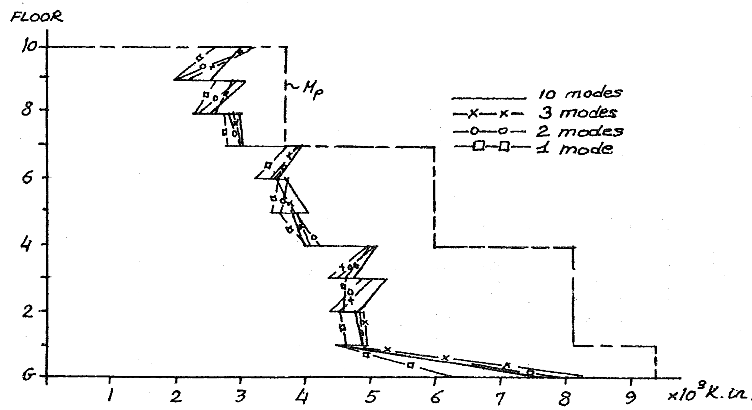

FIG 8 EFFECT OF NUMBER OF MODES ON COLUMN MOMENTS

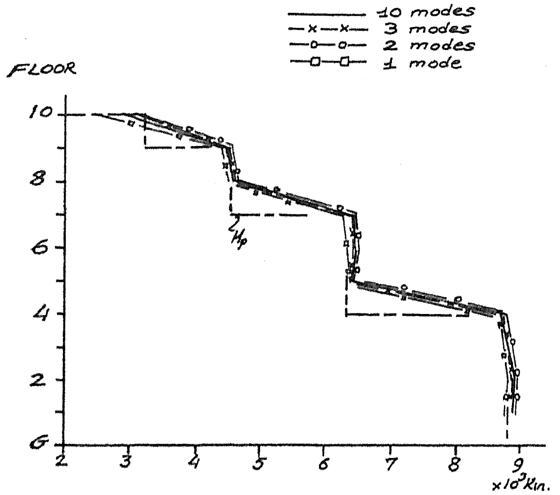

FIG 9 EFFECT OF NUMBER OF MODES ON GIRDER MOMENTS -

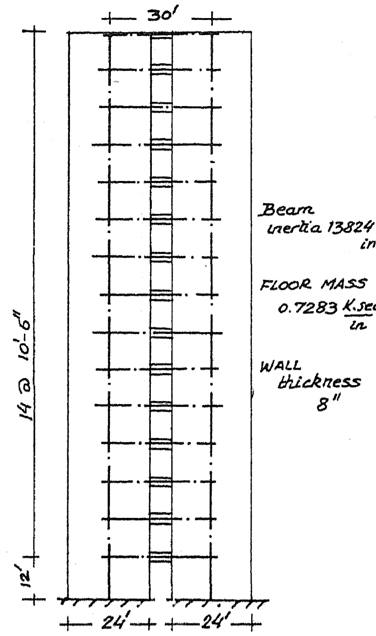

FIG. 10 COUPLED SHEAR WALLS

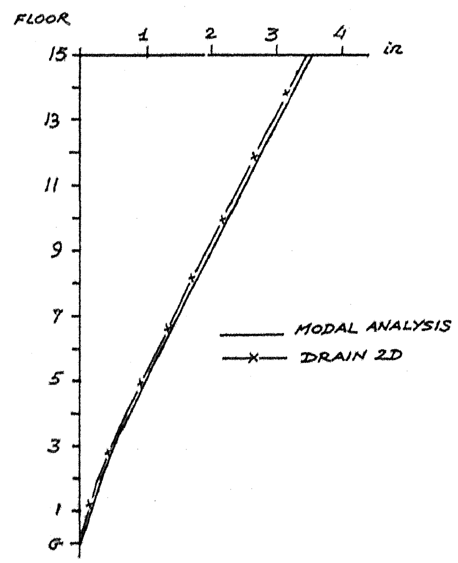

FIG. 11 FLOOR DISPLACEMENTS

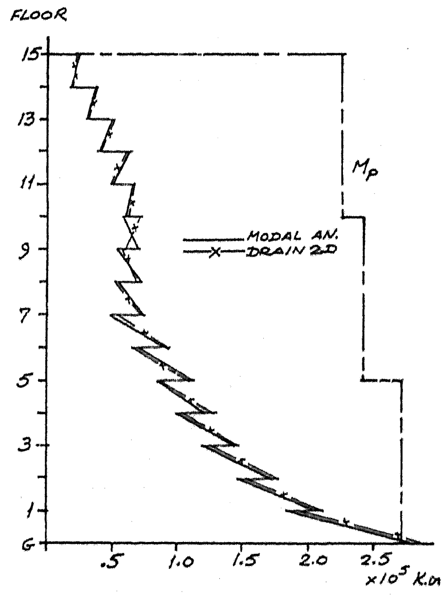

FIG.12 COLUMN (WALL) MOMENTS

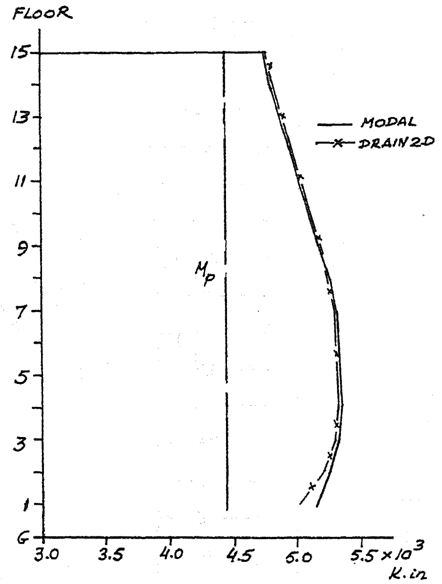

FIG. 13 GIRDER MOMENTS

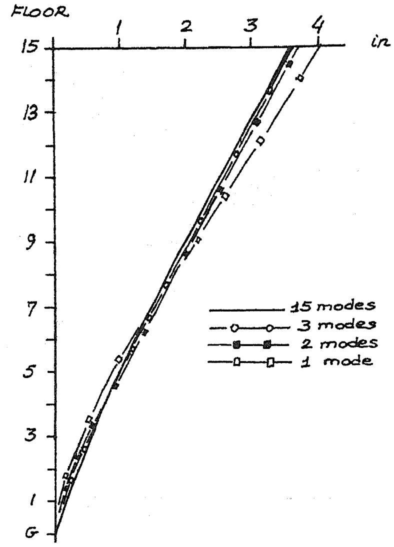

FIG. 14 EFFECT OF NUMBER OF MODES ON FLOOR DISPLACEMENTS

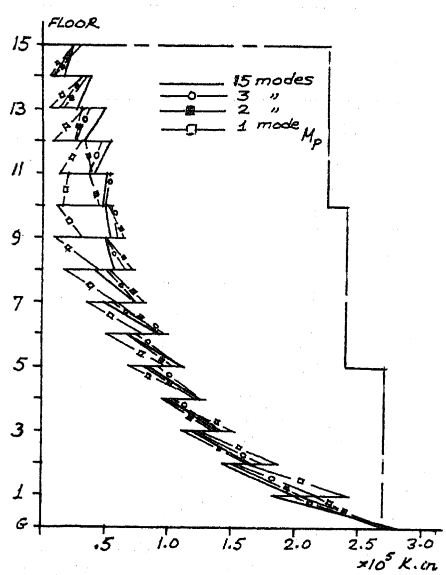

FIG 15 EFFECT OF NUMBER OF MODES ON COLUMN (WALL) MOMENTS.

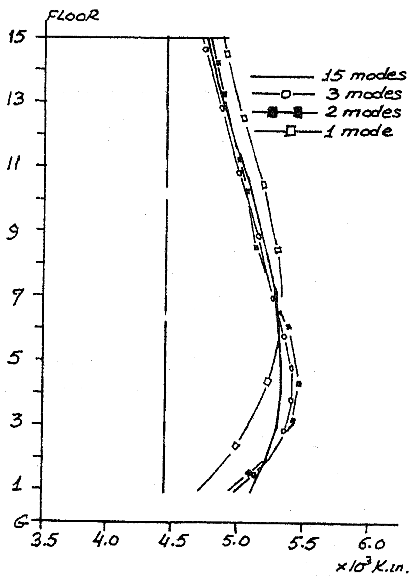

FIG. 16 EFFECT OF NUMBER OF MODES ON GIRDER MOMENTS.