DYNAMIC ANALYSIS FOR RECTANGULAR WATER TANKS

by

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SUMMARY

The dynamic analysis procedure for a flexural rectangular water tank, including the sloshing, the bulging and the base frame deformation, simultaneously, is presented by the use of the Rayleigh-Ritz method. The theoretical results are compared with the data measured in the actual size test, using the large-scale shaking table.

INTRODUCTION

With the constructions of large buildings, the water tanks have increased the capacities. The GRP(Grass Reinforced Plastics) has been employed as the materials of water tanks for the maintenances of good water qualities. The employment of GRP and the large capacities shuold mean the decrease of tank rigidities. Thus, there is the anxiety about the neglect of tank flexibilities. In the recent earthquakes; Near Izu-Oshima Earthquake on Jan. 14th, 1978 and two earthquakes taken place off Miyagi Pref. on Feb. 20th and Jun. 12th, 1978, the many water tanks installed in buildings and on ground were damaged. There were many damage examples caused by lacks of wall stiffnesses, shown in Fig. 1.

The dynamic characteristics of rectangular water tanks considering the elastic deformations of walls have been studied by Dr. Fumiki Kito. He investigated the natural frequencies of elastic walls of partially-filled water tanks, by the energy method, assuming the various edge conditions. In the analysis presented herein, the shear deformations of side walls parallel to the excitational direction and the bending deformations of pressure walls perpendicular to its direction are considered, and the velocity potentials of water are assumed to consist of the base frame motions, the wall motions and the sloshings. The purpose of Kito's method was to obtain the virtual mass of water in tanks. However, in this paper, the total vibrational equations and the seismic responses of rectangular water tanks are obtained by the Rayleigh-Ritz method.

ANALYSIS OF A FLEXURAL RECTANGULAR WATER TANK

In the present analysis, the walls and the base frame are considered to make elastic deformations. The analytical model is shown in Fig. 4. The side wall bending deformations perpendicular to the excitational direction are neglected. The vibrations only in the excitational direction, namely in x-axis, are investigated. The x-axis displacement u at any point of the pressured walls will be expressed as follow.

$$u=\sum_{m=1,3}\sum_{n=1,2}\zeta_{mn}\sin\frac{m\pi y}{b}\sin\frac{n\pi z}{d}+\sum_{k=1,3}\eta_k\sin\frac{k\pi z}{2d}+(u_0+u_g) \qquad (1)$$

where the first term is the bending displacement of the pressure wall, the second one is the shear displacement of the side wall, u_0 is the relative displacement of the base frame, and u_g is the ground displacement.

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The velocity potential of water in a tank will be defined in accordance with the results of sloshing analyses and Kito's method, by the following expression.

$$\phi=-(\dot{u}_0+\dot{u}_g)\left(x-\frac{a}{2}\right)+\sum_{s=1,3}\dot{\beta}_s\cos\frac{s\pi x}{a}\cosh\frac{s\pi z}{a}+\sum_{i=0,2}\sum_{j=1,3}\delta_{ij}\sinh\pi\sqrt{\left(\frac{i}{b}\right)^2+\left(\frac{j}{2h}\right)^2}\left(x-\frac{a}{2}\right)\cos\frac{i\pi y}{b}\cos\frac{j\pi z}{2h} \quad (2)$$

where the third term has been indicated by Dr. Kito, previously. Eq.(2) satisfies the Laplace's equation. The water is assumed as the ideal fluid of incompression and nonviscosity. Assuming the velocity continuous condition of the water and pressure walls in x-axis, the third term δ_{ij} of Eq.(2) may be represented by the wall displacements ζ_{mn} and η_k, using Fourier developments.

The vibrational equations governing the generalized coordinates ζ_{mn}, η_k, β_s, u_0 will be obtained by the Rayleigh-Ritz method. The potential energies of this system may be given by the bending deformation of pressure walls like the simply supported plates, the shear deformation of side walls, the elastic deformation of base frame, and the free surface motion of water in z-axis. The potential energy of the free surface motion may be evaluated by following expressions.

$$U_W=\frac{\rho_w g}{2}\int_0^b\int_0^a w_s^2\,dx\,dy \quad (3) \qquad w_s=\int_{-\infty}^{t}\dot{w}_s\,dt=-\int_{-\infty}^{t}\left.\frac{\partial\phi}{\partial z}\right|_{z=h}dt \quad (4)$$

In which, ρ_w is the unit volume mass of water, g is the acceleration of gravity, w_s is the z-axis displacement of the free surface. The kinetic energies of this system may be given by the mass of bottom plate and base frame, the mass of roof, the distributed mass of pressure walls, the distributed mass of side walls, and the water in a tank. The kinetic energy of the water in a tank may be evaluated from the following volume integral.

$$T_W=\frac{\rho_w}{2}\int_0^a\int_0^b\int_0^h\left\{\left(\frac{\partial\phi}{\partial x}\right)^2+\left(\frac{\partial\phi}{\partial y}\right)^2+\left(\frac{\partial\phi}{\partial z}\right)^2\right\}dz\,dy\,dx \quad (5)$$

The vibrational equations are obtained by substituting the Lagragian into the Eular's equation of the calculus of variations, and expressed in the following matrix form.

$$[M]\{\ddot{V}\}+[K]\{V\}=-\{m_0\}\ddot{u}_g \quad (6)$$

where

$$\{V\}^T=(\zeta_{11},\zeta_{12},\cdots,\zeta_{mn},\eta_1,\eta_3,\cdots,\eta_k,\beta_1,\beta_3,\cdots,\beta_s,u_0) \quad (7)$$

$$\{m_0\}^T=(\{m_{\zeta 0}\}^T,\{m_{\eta 0}\}^T,\{m_{\beta 0}\}^T,m_{00}) \quad (8)$$

$$[M]=\begin{bmatrix}[M_{\zeta\zeta}] & [M_{\zeta\eta}] & 0 & \{m_{\zeta 0}\}\\ & [M_{\eta\eta}] & 0 & \{m_{\eta 0}\}\\ \text{SYM.} & & [M_{\beta\beta}] & \{m_{\beta 0}\}\\ & & & m_{00}\end{bmatrix} \quad (9) \qquad [K]=\begin{bmatrix}[K_{\zeta\zeta}] & [K_{\zeta\eta}] & [K_{\zeta\beta}] & 0\\ & [K_{\eta\eta}] & [K_{\eta\beta}] & 0\\ \text{SYM.} & & [K_{\beta\beta}] & 0\\ & & & k_{00}\end{bmatrix} \quad (10)$$

In Eq.(9), the submatrixes concerning the sloshing motions and the wall motions should be vanished. Letting the right hand side of Eq.(6) equal to 0, and solving the multi-degree homogenerous vibrational equation, the natural circular frequencies ω_r and eigen vectors $\{V_r\}$ may be given. Thus, $\{V_r\}$may be written as follow.

$$\{V\}=\mu_1\{V_1\}q_1(t)+,\cdots,+\mu_r\{V_r\}q_r(t)+,\cdots,+\mu_p\{V_p\}q_p(t) \quad (11)$$

where $q_r(t)$ is the normal coordinate, and the participation factor μ_r is given by following expression.

$$\mu_r=\frac{\{V_r\}^T\{m_0\}}{\{V_r\}^T[M]\{V_r\}} \quad (12)$$

Using the orthogonality of eigen vectors, and introducing the damping factor h_r, the following equation concerning $q_r(t)$ is obtained.

$$\ddot{q}_r(t)+2h_r\omega_r\dot{q}_r(t)+\omega_r^2 q_r(t)=-\ddot{u}_g \quad (13)$$

NUMERICAL EXAMPLE

The analyzed rectangular water tank is assembled by the steel panels in the size of 1m x 1m with bolts and sealing materials. In order to gain the bending rigidity, each panel is pressed with the convex pattern in the depth of 8cm, and welded with the ribs in the height of 8cm along the four edge lines. The test tank in actual size is assembled and installed on the shaking table, shown in Fig. 2 and 3. This shaking table belongs to National Research Center for Disaster Prevention, in Tsukuba Newtown. The water levels employed in this test are 0m, 2m and 3.7m. In each water level, this shaking table is excited by the sinusoidal motions about 50gals with varieties of frequencies, the impulses of step displacements about 1mm and the random motions. The resonance curves neglecting the sloshing are reproduced in Fig. 5. The present analysis method is applied to this test tank. As the weight of panels decreases with height, the unit square weight of walls takes the average value of 45kg/m^2. As the first mode of the empty tank is considered only to exhibit the bending deformation of the pressure wall, the equivalent flexural rigidity is roughly estimated 600t·cm from the natural frequency 9.5Hz. The equivalent shear rigidity k_s of one side wall and the stiffness k_f of the base frame adopt the values $k_s \doteqdot 24$t/cm and k_f=100t/cm that the analytical results would be in good agreements with the test data. The weights are 300kg in the roof and 700kg in the bottom plate.

The decrease of natural frequencies of bulgings with the increase of water levels is shown in Fig. 6. The theoretical resonance curves compared with the test data, on the centroid of the pressure wall, are shown in Figs. 7, 8, 9 and 10. Fig. 11 shows the resonance curve of the hydrodynamic pressures divided by the table accelerations, at the center of the pressure wall. Each resonance curves are in water level 3.7m. Fig. 12 shows the mode shapes along the centroid of the pressure wall. In the theoretical analysis, the dampings of bulgings adopt the values roughly estimated from the test data, and the damping of the first sloshing adopts 0.5%.

Fig. 13 presents the random wave responses of the theoretical and measured relative displacements at the point of height 2m, width 1m of the pressure wall. In this random wave response analysis, the dampings of bulgings are given by the relation $h_r = \alpha_0/(2\omega_r) + \gamma_0\omega_r/2$, the values α_0=3.7 and γ_0=0.001 estimated from the test data are employed. The damping of the first sloshing adopts 0.1%. Considering the influences of the depth of the convex patterns of panels and the thickness of the sealing materials, the length and width of this test tank take 4.18m and 3.016m, respectively.

The present analysis method for rectangular water tanks can be understood to explain the test results in good agreements. Moreover, this method is able to simulate the wall deformations due to the sloshing. This method would be useful in predictions of seismic responses for liquid storage tanks.

REFERENCES

1. Kito, F., "On Vibration of a Rectangular Tank Filled with Water I - VIII", Journal of the Society of Naval Architect of Japan, 1959 - 1964, (in Japanese)

2. Minowa, C., "Dynamic Analysis for Rectangular Water Tanks", Trans. of AIJ, No. 285, Nov., 1979, 23 - 32, (in Japanese)

Fig. 1 Damage of Roof Tank by Off Miyagi Earth.

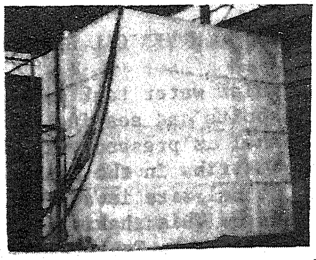


Fig. 2 View of Test tank on Shaking Table

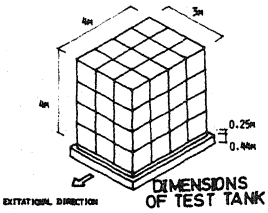
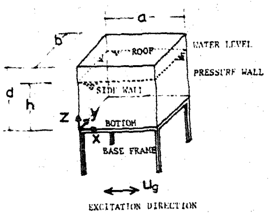


Fig. 3



ANALYTICAL MODEL FOR A RECTANGULAR WATER TANK

Fig. 4

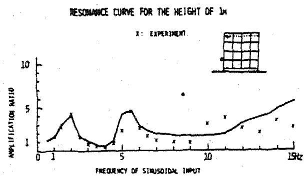


Fig. 7

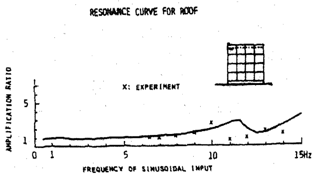


Fig. 10

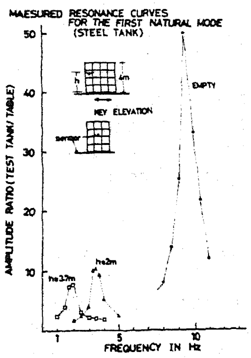


Fig. 5

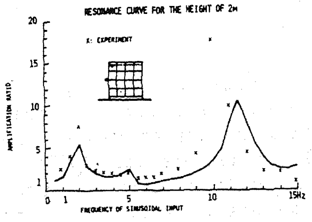


Fig. 8

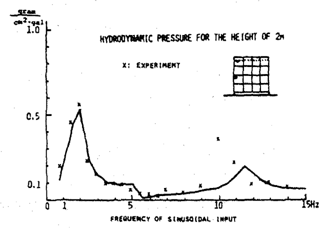


Fig. 11

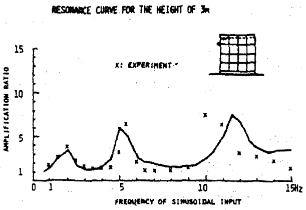


Fig. 9

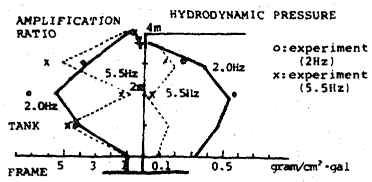


Fig. 12 Vertical Distributions

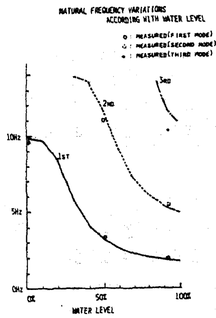


Fig. 6

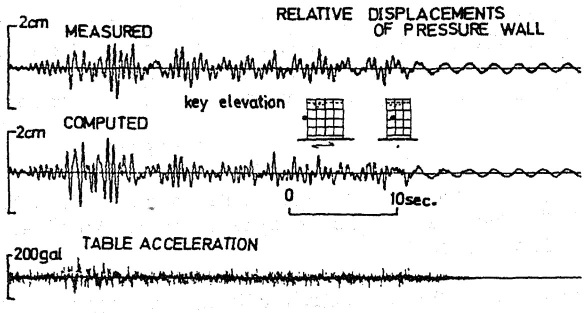


Fig. 13 Random Wave Responses