EARTHQUAKE RESPONSE ANALYSIS

OF TORSIONALLY COUPLED MULTISTOREY BUILDINGS

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SUMMARY

A considerable amount of research on the dynamics of lateral torsionally coupled building response has been carried out in recent years.

The most common idealization assumes the masses lumped at the floor levels.

Aim of this paper is to identify, for an asymmetric building structure, the earthquake horizontal ground motion direction which produces the maximum frame or wall relative displacement.

All the points of the foundation are herein assumed to undergo the same ground acceleration.

However the ground histoty is characterized by a random stationary Gaussian and ergodic process.

Once that direction of the ground motion has been determined the standard thechnique may be adopted.

ANALYSIS OF THE RESPONSE TO GROUND MOTION

Equation of motion in terms of generalized coordinates for a damped, linear, multistorey building having 3n degree-freedom (DOF) (n = number of stories), and subyected to base excitation $\ddot{z}(t)$ have the following matrix form:

Eq.1) $$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = -[M]\{r(\beta)\}\ddot{z}(t).$$

Where $[M]$, $[C]$, $[K]$ are respectively, 3n x 3n generalized mass, damping and stiffness matrices; $\{x(t)\}$ is the generalized relative displacement vector $\{r(\beta)\}$ is a vector whose i-th entry is either a 1 cos β or a 1 sen β or a zero depending on the corresponding DOF being or not coincidental with the direction of the support motion; $\beta$is the direction of $\ddot{z}(t)$, constant respect to time.

Let $[\phi]$ (3n x 3n) the modal matrix O.N. respect to M, i e.:

Eq.2) $$[\phi]^T [M] [\phi] = [I].$$

The usual coordinates transformation gives:

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Eq.3) $$\left[I \right] \{\ddot{y}\} + \left[2\nu\omega \right] \{\dot{y}\} + \left[\omega^2 \right] \{y\} = \{L(\beta)\} \ddot{z}(t).$$

Where $\left[2\nu\omega \right]$ is a matrix obtained by the diagonal elements of the operator $\left[\phi\right]^T \left[C\right] \left[\phi\right]$ and $\{L(\beta)\} = - \left[\phi\right]^T \left[M\right] \{r(\beta)\}$.

The equation of motion for each mode is derived as follows, using the generalized mass and force for the mode, the modal frequency ω_i and a specified value of the modal damping ratio ν_i

Eq.4) $$\ddot{y}_i + 2\,\nu_i \omega_i \dot{y}_i + \omega_i^2 y_i = L_i(\beta)\ddot{z}(t)$$

To calculate the maximum displacement at k-th floor of the s-th frame, let the reference coordinate system (x°, y°) to be as in Fig. 1, the x axis rotated to coincide with the frame horizontal axis

The equation 4) in this reference is rewritten as:

Eq.5) $$\ddot{y}_i^\circ + 2\nu_i \omega_i \dot{y}_i^\circ + \omega_i^2 y_i^\circ = L_i(\beta^\circ)\ddot{z}(t).$$

The Power Spectral Density function (PSD) of $y_i(t)$ is:

Eq.6) $$G^\circ_{yy_i}(\Omega) = \left| H_i(\Omega) \right|^2 \cdot L_i^2(\beta^\circ)\, G_{zz}(\Omega).$$

Where $G_{zz}(\Omega)$ is the PSD of the input, and $H_i(\Omega)$ is the transfer function.

If the input is a random stationary Gaussian and ergodic process, the variance is obtained (weakly damped system):

Eq.7) $$\sigma^{\circ 2}_{yy_i} = \pi\, G_{zz}(\omega_i)/2\,\nu_i \omega_i^3.$$

The mean square response of the system is derived as:

Eq.8) $$\left[\sigma^2_{xx}\right] = \left[\phi^\circ\right] \left[\sigma^{\circ 2}_{yy}\right] \left[\phi^\circ\right]^T.$$

The value of β which maximise the displacement for k-th floor of s-th frame which form an angle zero with x° axis is expressed as:

Eq.10) $$\operatorname{tg} 2\beta^\circ = -2 \frac{\sum_{r=1}^{3n} G_{zz}(\omega_r)\, \xi_r \eta_r \phi^{\circ 2}_{kr}/\omega_r^3 \nu_r}{\sum_{r=1}^{3n} G_{zz}(\omega_r)(\eta_r^2 - \xi_r^2)\phi^\circ_{kr}/\omega_r^3 \nu_r} \qquad k = 1, \ldots, n$$

where

Eq.11) $$\xi_r = \sum_{J=1}^{n} m_j \left[\phi^\circ_{jr} - y^\circ_{GJ}\phi^\circ_{J+2n,r}\right] \qquad \eta_r = \sum_{J=1}^{n} m_j \left[\phi^\circ_{j+n,r} + x^\circ_{GJ}\phi^\circ_{j+2n,r}\right]$$

And x°_{GJ}, y°_{GJ} *are the coordinates of the center of gravity of J-th floor.*

If the r-th mode only is acting, it may be easily shown the β depending on the same r-th mode shape only equation 11) becomes

Eq.12) $$tg\ 2\ \beta = -\ 2\ \xi_r \eta_r / (\eta_r^2 - \xi_r^2)$$

Consider now one storey building, the groun motion in β direction calculated by eq.12) results orthogonal to direction $C_R G$ where C_R is the center of rotation of the rigid-plan deck, and G is the center of gravity.

The geometrical interpretation of this result leads to conclude that in order to maximise the k-th mode it must be input the ground motion in direction β which maximise the moment of inertial forces respect to the center C_R.

NUMERICAL EXAMPLE

Consider a undamped 2-storey building (plane view in Fig.2) table I reports its geometrical and mass properties.

Table I - Properties of sample building

Floor	Mass Kg sec^2/cm	$X_G = Y_G$ cm	$S_X = S_Y$ Kg sec^2	J_o Kg sec^2 cm
2	36	300	10.800	$8,64.10^6$
1	36	300	10.800	$8,64.10^6$

$E = 2.10^5$ Kg/cmq

NUMERICAL RESULTS

The analysis follows the sequence of steps presented in the paper.
In this example a frame 3-4 is selected for study.
The frequencies of the building are:

ω_1 = 8,21 rad/s ω_2 = 10.09 rad/s ω_3 = 17,46
ω_4 = 23,87 rad/s ω_5 = 34,85 rad/s ω_6 = 63,15

The modal matrix

0.174787	-0.093145	-0.112072	0.114671	-0.041673	-0.05264562
0.097605	-0.038897	-0.080302	-0.169945	0.097775	0.11625504
0.015240	0.122817	0.206143	-0.024747	0.050520	0.09202370
0.005544	0.054598	0.087965	-0.017797	-0.117875	-0.21091133
0.000220	-0.000005	-0.000551	0.000222	0.000007	-0.00024885
0.000131	0.000003	-0.000289	-0.000222	0.000003	0.00055955

It is decided in this study to choose a stationary Gaussian process for $\ddot{z}(t)$ with the following spectral properties

Eq.13) $$G_{zz}(\omega) = S_1 \omega_g^4 / \left[(\omega_g^2 - \omega^2) + 4(\rho\omega\omega_g)^2 \right]$$

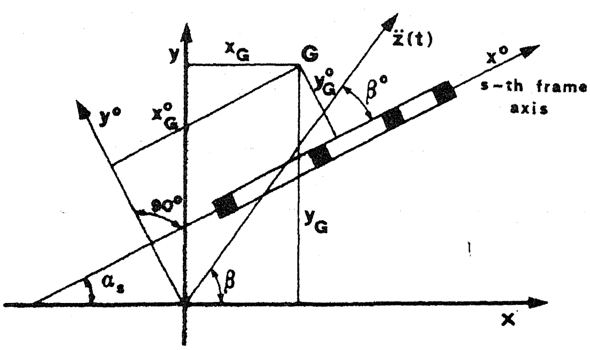
the various constant in this equation have been selected as $\omega_g = 4\pi, \rho=0,5$, $S_1 = 0,02\ \rho\ \pi\omega_g$.

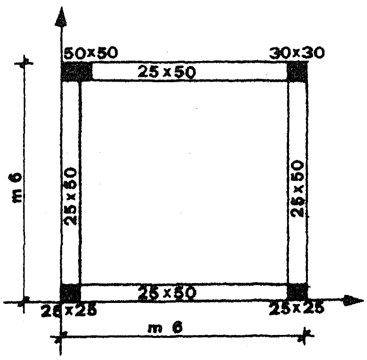
The characteristic angle value, which maximise the relative diplacement for a given frame 3-4 is $\beta=36°$ for both floors.

The relative displacement at each floor for various values of β have been calculated with standard procedure (average response spectrum)and are presented in table II.

Table II - Values of relative displacement for various values of β

Floor	$\beta = 0$ dir. x (cm)	$\beta = 90°$ dir. y (cm)	$\beta = 36°$ max	$\beta = 116°$ min
1	0,66	0,48	0,725	0,286
2	1,11	0,90	1,298	0,599





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