PEAK SEISMIC RESPONSE CHARACTERISTICS OF SECONDARY SYSTEMS

by

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SUMMARY

An analytical formulation is presented for defining the probability distribution functions of the maximum acceleration response of oscillators on ground (representing primary structures) and on floor (representing the supported secondary systems). For a prescribed probability of exceedance, the formulation can be used to obtain the peak factors by which the mean square response should be amplified to obtain the design response. The solution of the equation of motion for an arbitrary ground motion is obtained by the variation of parameter method. This solution is easier to use in further algebraic manipulations which are necessary for the solution of the nonstationary first passage problem. The peak factors are generally different for primary and secondary systems. A scheme for incorporation of these in direct generation of floor response spectra is also presented.

INTRODUCTION

For generation of floor acceleration spectra, direct approaches in which ground response spectra are directly used have been proposed recently (1, 2). In the development of these approaches, it is implicitly assumed that peak factors, the factors by which the root mean square response is multiplied to obtain the design response, are the same for the primary (supporting) and secondary (supported) structures. This peak factor is assumed to be implicitly defined by the ground spectra which represent the maximum (or design) response of a single-degree-of-freedom structure (or oscillator). This assumption tends to overestimate the floor response spectra, as the peak factors for an oscillator supported on a floor is expected to be smaller than that for an oscillator on ground. This is of special significance for a tuned oscillator (an oscillator in resonance with one of the structural frequencies) on a floor. In this paper a methodology is presented to establish the peak factors for oscillators on ground and on floor and their use in direct generation of more realistic floor acceleration spectra is suggested.

RESPONSE OF OSCILLATORS IN CASCADE

A system of two oscillators in cascade has been used earlier (3,4) to study the nonstationary effects of earthquakes on the mean square response and on generation of floor response spectra curves. Herein this system will be used to obtain the peak factors of supported and supporting oscillators.

<u>Supporting Oscillator Response</u>: *The relative displacement response*

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x of an oscillator excited by ground acceleration $\ddot{X}_g(t)$ can be written in the following form

$$x = \{e^{\lambda_1 t} \int_0^t \ddot{X}_g(u) e^{-\lambda_1 u} du - e^{\lambda_2 t} \int_0^t \ddot{X}_g(u) e^{-\lambda_2 u} du\}/(\lambda_1 - \lambda_2) \quad (1)$$

in which $\lambda_1 = -(\beta_s + i\sqrt{1-\beta_s^2})\omega_s$, $\lambda_2 = \lambda_1^*$, β_s = damping coefficient, ω_s = natural frequency and an asterisk * represents a complex conjugate. Using this solution the mean square value of the absolute acceleration response can be obtained in terms of the mean square values of x and $\dot{x}$. The mean square value of, say, x can be written as

$$E[x^2(t)] = \int_{-\infty}^{\infty} \Phi_x(\omega,t)\, d\omega \quad (2)$$

in which $\Phi_x(\omega,t)$ is the evolutionary spectral density function (SDF) of x. Here ground motion is being modeled as $\ddot{X}_g(t) = \ddot{X}_s(t)A(t)$ where $\ddot{X}_s(t)$ is a stationary random process with SDF = $\Phi_g(\omega)$ and A(t) is an intensity modulation function which introduces the nonstationary effect. Such representation of seismic excitation has been used earlier by many researchers. For such an excitation, the evolutionary SDF in Eq. 3 can be written as follows

$$\Phi_x(\omega,t) = \frac{\Phi_g(\omega)}{(\lambda_1-\lambda_2)^2} \int_0^t \int_0^t A(u_1)A(u_2) \ell^{i\omega(u_1-u_2)} \{\exp(2\lambda_1 t - \lambda_1 u_1 - \lambda_1 u_2)$$

$$+ \exp(2\lambda_2 t - \lambda_2 u_1 - \lambda_2 u_2) - \exp(\lambda_1 t + \lambda_2 t - \lambda_1 u - \lambda_2 u_2)$$

$$- \exp(\lambda_1 t + \lambda_2 t - \lambda_1 u_2 - \lambda_2 u_1)\} du_1 du_2 \quad (3)$$

A similar expression is obtained for the SDF of $\dot{x}$ and cross SDF of x and $\dot{x}$. For an arbitrary form of envelope function, it is easier to integrate the expression in Eqs. 2 and 3 than the corresonding expressions written in terms of the impulse or frequency response functions. For this reason, the solution of the equation of motion has been obtained in this form by the variation of parameter method.

To obtain peak factors for a given probability of exceedance, the cumulative probability distribution function (CDF) of the maximum resonse is required. A procedure developed by Vanmarcke (2) which accounts for evolutionary and clumping nature of the narrow band response can be used. The CDF is defined as

$$P(\ddot{X}_m \leq a) = \exp(-\int_0^t \alpha(u,a)du) \quad (4)$$

in which t = earthquake duration and $\alpha(u,a)$ is the barrier crossing rate. The barrier crossing rate is defined in terms of the spectral moments of the evolutionary SDF (2). For a set of oscillator parameters ω_s, β_s, Eq.

4 can be used to establish the CDF numerically. This requires a knowledge of the ground SDF, $\Phi_g(\omega)$. Valuable results can also be obtained by using white-noise representation of the excitation in Eq. 3. See Ref. 3.

Response of Supported Oscillator: The equation of motion defining the acceleration of a supported oscillator can be written as (3):

$$(D^2 + 2\omega_s\beta_s D + \omega_s^2)(D^2 + 2\omega_e\beta_e D + \omega_e^2)\ddot{Y}_e \simeq \omega_e^2\omega_s^2\ddot{X}_g(t) \qquad (5)$$

where $D = d/dt$ and ω_e and β_e are the parameters (frequency and damping) of the supported oscillator. Using variation of parameter method, the acceleration $\ddot{Y}_e$ can be written as

$$\ddot{Y}_e(t) = \sum_{j=1}^{4} C_j \int_0^t S(u)e^{\lambda_j(t-u)} du \qquad (6)$$

in which λ_1 and λ_2 are as defined earlier; $\lambda_3 = -(\beta_e + i\sqrt{1-\beta_e^2})\omega_e$ and $\lambda_4 = \lambda_3^*$; $S(u) = \omega_e^2\omega_s^2\ddot{X}_g(u)$; and C_k's are obtained from the solution of the following simultaneous equations:

$$[V]\{C_j\} = \{d_j\} \qquad (7)$$

in which an element of matrix $V, V(i,j)$, is $= \lambda_j^{i-1}$ and the elements of the right hand side vector are $d_1 = d_2 = d_3 = 0$ and $d_4 = 1$. Using Eq. 6, the evolutionary SDF can be shown to be as:

$$\Phi_y(t,\omega) = \Phi_g(\omega) \sum_j \sum_k C_j C_k \int_0^t \int_0^t A(u_1)A(u_2) \exp\{i\omega(u_1 - u_2) + \lambda_j(t - u_1) + \lambda_k(t - u_2)\}du_1 du_2 \qquad (8)$$

Here again the integrations of this SDF are more easy to obtain than the SDF obtained by the impulse or frequency response function approach. For various parameteric combinations of ω_s, β_s, ω_e and β_e, the CDF for the supported oscillator can be numerically established using Vanmarcke's approach (2). This CDF can then be used to obtain the peak factor, $T = a_p/\sigma$, in which a_p = response corresponding to a probability level p and σ is the root mean square response.

PEAK FACTORS IN FLOOR SPECTRA

For the same probability level, the peak factors for the supporting and supported oscillators are likely to be different. A method to include these different peak factors in direct generation of floor response spectra is described. An equation defining the mean square floor response spectrum value, $\sigma_u^2(\omega_e,\beta_e)$, for an oscillator frequency of ω_e and damping β_e is given in Ref. 1 and can be written in the following form.

$$\sigma_u^2(\omega_e,\beta_e) = \sum_{j=1}^{N} \gamma_j^2\phi_j^2(u)[(A_j + B_j)\sigma_e^2 + (C_j + D_j)\sigma_j^2]$$

+ Double Summation Terms (9)

in which γ_j = jth participation factor of supporting structure, $\phi_j(u)$ = jth relative displacement mode shape of floor u, σ_e^2 = mean square acceleration response of oscillator when excited by ground motion, σ_j^2 = mean square response due to ground motion of an oscillator of frequency equal to the jth structural frequency, and factor A_j, B_j, C_j and D_j are defined in Ref. 1 in terms of frequency ratios, and structural and oscillator damping values. The double summation terms are also defined in Ref. 1.

To obtain the design response, σ_u in Eq. 9 must be multiplied by an appropriate value of the peak factor corresponding to a desired probability level p. However, calculation of such a peak factor for different values of structural and oscillator parameters may be too cumbersome to be adopted in a direct floor spectra generation approach. It is, therefore, proposed to modify each summation term separately to include the peak factor effects. If for a pre-established probability level p, F_e and F_j, respectively, are the peak factors for supporting oscillators with frequencies ω_e and ω_j and T_j is the peak factor of the supported oscillator with parameters ω_e and β_e in cascade with an oscillator with parameters ω_j and β_j, the design floor response spectrum value can be obtained in terms of ground response spectra values as follows:

$$R_u^2(\omega_e,\beta_e) = \Sigma\ \gamma_j^2\psi_j^2(u)\ T_j^2[(A_j + B_j)R^2(\omega_e)/F_e^2 + (C_j + D_j)R^2(\omega_j)/F_j^2] + \text{Double Summation Terms} \qquad (10)$$

in which $R(\omega_e)$ and $R(\omega_j)$ are response spectrum values at frequencies ω_e and ω_j. The methodology for modification of the double summation terms is not clear as yet. However, since these terms are of second order in most cases, their modification may not be necessary.

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