SYNTHESIS OF RESPONSES OF TWO SUBSYSTEMS FOR COUPLED SEISMIC ANALYSIS OF EQUIPMENT MOUNTED ON A STRUCTURE

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SUMMARY

An analytical solution is presented to calculate the coupled response of equipment mounted on a structure which is subjected to earthquake motion. Structure and equipment are taken as two multi-degree of freedom subsystems attached to each other at a point. The response of the coupled system to a seismic input is expressed in terms of the fixed-base modal characteristics of the individual subsystems rather than the modal characteristics of the total system. An estimate of the error introduced by uncoupled analysis of the equipment-structure system is given. In the case of light equipment, estimates obtained for the maximum acceleration response of the equipment are compared to the estimates given in literature.

INTRODUCTION

Functional failure of equipment which is essential to lifeline systems may result in serious consequences after a major earthquake. Hence, there has been a growing interest in the earthquake resistant design of equipment. An estimate of the maximum response of equipment attached to a structure subjected to ground motion is required for its aseismic design.

In general, it is desirable to avoid a coupled time history analysis of the equipment-structure system subject to a variety of specified seismic inputs. This type of analysis may be computationally impracticable due to the large differences between the masses of the equipment and the structure. Other disadvantages of the total dynamic analysis performed by employing conventional computer codes are discussed by Sackman and Kelly [1].

A common approach to the design of equipment is based on the floor response spectrum method [2]. This method neglects the interaction between the structure and the equipment. Sackman and Kelly [1] show that the floor response spectrum method is not valid in the case of tuned equipment-structure systems and propose a design spectrum method.

The authors developed an analytical method in 1973 to investigate the motion of a dynamic system which consists of two coupled subsystems of multi-degree of freedom (equipment and structure or structure and foundation) and is attached to a laterally moving rigid support [3]. In this analysis, quite similar to the analysis given by Sackman and Kelly [1], the dynamic response of subsystems are given in terms of the fixed-base modal characteristics of the two subsystems. The method has been employed previously to investigate the dynamic structure-foundation interaction

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problem [3].

In this paper a more detailed exposition of the method is presented. The method can be employed to obtain the transient response of light equipment for a given earthquake motion as well as to estimate the maximum acceleration response of the equipment when the ground motion is specified by a response spectrum. The estimates for the maximum acceleration of the light equipment obtained from this method are compared to the similar estimates given by Sackman and Kelly [1]. Also, an estimate of the error introduced by uncoupled analysis of equipment-structure systems is given. Some of the parameters appearing in the analysis of the coupled two multi-degree of freedom subsystems are generalizations of the parameters obtained from the analysis of equipment-structure problems modeled as two-degree of freedom systems. The remaining parameters are related to the relative damping characteristics of the equipment and structure, and the location of the attachment point of the equipment on the structure.

THE METHOD OF ANALYSIS

The motion of a dynamic system which is comprised of two subsystems and subjected to a lateral base acceleration will be analyzed in terms of the fixed-base modal properties of the individual subsystems. The subsystem which is attached to the laterally moving base is taken as subsystem 1 (structure). Subsystem 2 (equipment) is attached to subsystem 1 at an interface "s" as shown in Fig. 1. Free body diagrams of the fixed-base subsystems comprising the dynamic system are illustrated in Fig. 2.

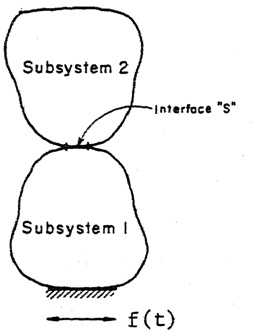

Fig. 1 Dynamic system

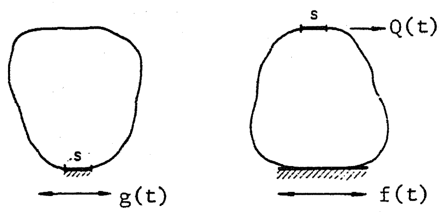

Fig. 2 Free body diagrams of
a) Subsystem 2 b) Subsystem 1

It is assumed that the motion of subsystem 2 is affected mainly by the lateral motion of the interface "s" as the input acceleration is in the lateral direction. Therefore, the moments and the forces other than the lateral force transmitted through interface "s" have a negligible effect on the motion of subsystems 1 and 2. This assumption is appropriate in the analysis of equipment-structure systems, and greatly simplifies the formulation and the interpretation of the results. The designations of the fixed-base modal properties of the two subsystems are given in Table 1. The lateral accelerations of the base and the interface are denoted by $f(t)$ and $g(t)$, respectively. The lateral force exerted on subsystem 1 by subsystem 2 at the interface is denoted by Q(t).

Table 1 SYSTEM CHARACTERISTICS

CHARACTERISTICS	SUBSYSTEM 1	SUBSYSTEM 2
Degrees of Freedom	N	n
Natural Frequencies	$\Omega_1, \Omega_2, \ldots, \Omega_N$	$\omega_1, \omega_2, \ldots, \omega_n$
Modes	$\phi_1, \phi_2, \ldots, \phi_N$	$\psi_1, \psi_2, \ldots, \psi_n$
Generalized Masses	$M_1^*, M_2^*, \ldots, M_N^*$	$m_1^*, m_2^*, \ldots, m_n^*$
Participation Factors	$\Gamma_1, \Gamma_2, \ldots, \Gamma_N$	$\gamma_1, \gamma_2, \ldots, \gamma_n$
Fraction of Critical Damping	$\eta_1, \eta_2, \ldots, \eta_N$	$\beta_1, \beta_2, \ldots, \beta_n$

For brevity in the following analysis, the total acceleration response of a single-degree of freedom system with the natural frequency p, and the fraction of critical damping q, subjected to a base acceleration x(t) will be denoted by I[x(t); p,q]. For small damping, by neglecting terms in the order of q^2, I[x; p,q] can be expressed as

$$I[x(t);\ p,q] = p[\int_0^t x(\tau)\exp[-pq(t-\tau)]\ \sin p(t-\tau)\ d\tau$$

$$+ 2q \int_0^t x(\tau)\exp[-pq(t-\tau)]\ \cos p(t-\tau)\ d\tau]. \qquad (1)$$

Dynamic Analysis of Subsystem 2 (Equipment)

The isolated subsystem 2 is shown in Fig. 2a. The equations of motion of subsystem 2 in normal coordinates $\underset{\sim}{y}$ are

$$\ddot{y}_i + 2\beta_i\omega_i\dot{y}_i + \omega_i^2 y_i = -\gamma' g(t),\quad \dot{y}_i(0) = y_i(0) = 0,\quad i=1,2,\ldots,n \qquad (2)$$

in which

$$\gamma_i' = \underset{\sim}{\psi}_i^T\ [m]\ \underset{\sim}{u}'/m_i^*,\quad u_i' = \begin{cases} 1, & \text{if } i^{th} \text{ degree of freedom is a lateral displacement} \\ 0, & \text{otherwise} \end{cases},\quad i=1,2,\ldots,n. \qquad (3)$$

The dots denote differentiation with respect to time, and other symbols are defined in Table 1. By differentiating twice the solution of Eq. 2, and neglecting terms of the order β_i^2, we obtain

$$\ddot{y}_i = \gamma_i'\ \{I[g(t);\ \omega_i,\beta_i] - g(t)\} \qquad (4)$$

The lateral force Q(t), at the base of subsystem 2 is given by

$$Q(t) = -\sum_{i=1}^{n} u_i' m_i \left[u_i' g(t) + \sum_{j=1}^{n} \psi_{ij} \ddot{y}_j\right] . \tag{5}$$

Here, m_i denotes the discrete mass associated with the i^{th} degree of freedom of subsystem 2. Noting that

$$\sum_{j=1}^{n} \psi_{ij} \gamma_j = 1, \; i=1,2,\ldots,n \quad \text{and} \quad \sum_{i=1}^{n} u_i' m_i \psi_{ij} = m_j^* \gamma_j', \; j=1,2,\ldots,n, \tag{6}$$

in view of Eq. 4, Eq. 5 can be rewritten as

$$Q(t) = -\sum_{j=1}^{n} (\gamma_j')^2 \, m_j^* \, \{I[g(t);\, \omega_j, \beta_j] + \frac{\gamma_j''}{\gamma_j'} g(t)\} \tag{7}$$

in which

$$\gamma_j'' = \gamma_j - \gamma_j', \quad j=1,2,\ldots,n. \tag{8}$$

<u>Dynamic Analysis of Subsystem 1 (Structure)</u>

The isolated subsystem 1 is shown in Fig. 2b. Let the lateral displacement of interface "s" correspond to the s^{th} degree of freedom of subsystem 1. Subsystem 1 is subjected to a lateral base acceleration, f(t), and a force, Q(t), at interface "s". The equations of motion of subsystem 1, in normal coordinates $\underset{\sim}{Y}$ are

$$\ddot{Y}_j + 2\eta_j \Omega_j \dot{Y}_j + \Omega_j^2 Y_j = -\Gamma_j' f(t) + \frac{\phi_{sj}}{M_j^*} Q(t), \; \dot{Y}_j(0) = Y_j(0) = 0, \; j=1,2,\ldots,N \tag{9}$$

in which

$$\Gamma_j' = \underset{\sim}{\phi}_j^T [M] \underset{\sim}{U}' / M_j^*, \quad \underset{\sim}{U}_j' = \begin{cases} 1, & \text{if } j^{th} \text{ degree of freedom is a lateral displacement} \\ 0, & \text{otherwise} \end{cases} \quad , \; j=1,2,\ldots,N \tag{10}$$

and the other symbols are defined in Table 1. In Eq. 9, ϕ_{sj} denotes the relative lateral displacement amplitude corresponding to s^{th} degree of freedom in the j^{th} mode of subsystem 1. By differentiating twice the solution of Eq. 9, we obtain

$$\ddot{Y}_j = \Gamma_j' \, \{I[f(t);\, \Omega_j, \eta_j] - f(t)\} - \frac{\phi_{sj}}{M_j^*} \{I[Q(t), \Omega_j, \eta_j] - Q(t)\} . \tag{11}$$

By definition, the total lateral acceleration corresponding to the s^{th} degree of freedom is g(t), and therefore

$$g(t) = \sum_{j=1}^{N} \phi_{sj} \ddot{Y}_j + f(t) \tag{12}$$

By substituting $\ddot{Y}_j$ from Eq. 11 into Eq. 12, we obtain

$$g(t) = \sum_{j=1}^{N} \Gamma_j' \phi_{sj} \{I[f;\Omega_j,\eta_j] + \frac{\Gamma_j''}{\Gamma_j'} f\} - \sum_{j=1}^{N} \frac{\phi_{sj}^2}{M_j^*} \{I[Q;\Omega_j,\eta_j] - Q\} \tag{13}$$

in which

$$\Gamma_j'' = \Gamma_j - \Gamma_j', \quad j=1,2,\ldots,N. \tag{14}$$

Synthesis of Dynamic Analysis of Two Subsystems

The analysis of two subsystems developed previously will be combined to express the lateral acceleration of the interface, $g(t)$, in terms of the dynamic properties of the fixed-base subsystems. Substituting the right-hand side of Eq. 5 for $Q(t)$ into Eq. 13, we obtain

$$g(t) = \sum_{j=1}^{N} \Gamma_j' \phi_{sj} \{I[f;\Omega_j,\eta_j] + \frac{\Gamma_j''}{\Gamma_j'} f(t)\} + \sum_{j=1}^{N} \sum_{i=1}^{n} (\gamma_i')^2 \phi_{sj}^2 \frac{m_i^*}{M_j^*} \{F[g;\omega_i,\beta_i;\Omega_j,\eta_j] - I[g;\omega_i,\beta_i] + \frac{\gamma_i''}{\gamma_i'} \{I[g;\Omega_j,\eta_j] - g\}\}. \tag{15}$$

in which

$$F[g;\omega,\beta;\Omega,\eta] = I[I[g(t),\omega,\beta];\Omega,\eta] \tag{16}$$

Here, $F[g;\omega,\beta;\Omega,\eta]$ denotes the total acceleration response of a single degree of freedom system with the natural frequency Ω and the fraction of critical damping η, subjected to a base acceleration $I[g;\omega,\beta]$ which is the acceleration response of a single degree of freedom system with the natural frequency ω and the fraction of critical damping β subjected to a base acceleration $g(t)$.

EFFECT OF COUPLING IN EQUIPMENT-STRUCTURE SYSTEMS

In many equipment-structure problems, it is sufficient to consider only the first N' modes of the structure and the first n' modes of the equipment in the analysis. In many practical problems $\Gamma'/\Gamma' << 1$ and $\gamma''/\gamma' << 1$ for these modes. Hence, in this case, Eq. 15 reduces to

$$g(t) = g_u(t) + E(g) \tag{17}$$

in which

$$g_u(t) = \sum_{j=1}^{N'} \Gamma'_j \phi_{sj} \, I[f;\Omega_j,\eta_j] \tag{18}$$

$$E(g) = \sum_{j=1}^{N'} \sum_{k=1}^{n'} (\gamma_k')^2 \phi_{sj}^2 \frac{m_k^*}{M_j^*} \{F[g;\omega_k,\beta_k;\Omega_j,\eta_j] - I[g;\omega_k,\beta_k]\} \tag{19}$$

The function $g_u(t)$ can easily be identified as the uncoupled lateral acceleration of the interface. The function $E(g)$ represents the effect of the coupling between the equipment and the structure. The terms of the sum expressing $E(g)$ (Eq. 19) are the contributions of the mode pairs of the individual subsystems to the coupling. It is clear that the coefficients

$$(\gamma_k')^2\phi_{sj}^2 \; m_k^*/M_j^*, \quad k=1,2,\ldots,n', \quad j=1,2,\ldots,N'$$

in this sum are important parameters determining the degree of coupling between the two subsystems. By comparison to two-degree of freedom models used for the equipment-structure problems, the ratio $m_k^* M_j^*$ can be identified as the generalized mass ratio associated with the mode pair j,k. The parameters Γ_j' and γ_k' express the effect of the direction of the input motion. The effect of the attachment point "s" is reflected by the parameter ϕ_{sj}.

A first approximation to the solution of Eq. 15 can be expressed as

$$g(t) \cong g_u(t) + E(g_u) \tag{20}$$

If an uncoupled analysis of the equipment-structure problem is performed, an error estimate in the lateral acceleration a_i corresponding to the i^{th} degree of freedom due to uncoupling is given by

$$\text{Error in } a_i = \sum_{j=1}^{n'} \psi_{ij}\gamma_j' \;\; I[E(g_u);\omega_j,\beta_j] \tag{21}$$

The integral equation (Eq. 15) in unknown g(t) can be solved step by step in time performing iterations at each time step if an exact coupled transient response analysis is required.

UNCOUPLED ANALYSIS FOR LIGHT EQUIPMENT

In many practical problems, the generalized mass ratio m_k^*/M_j^* associated with all the mode pairs is small and, therefore, an uncoupled analysis is justified. By taking $g(t) = g_u(t)$, the lateral acceleration of the equipment a_i corresponding to the i^{th} degree of freedom becomes

$$a_i(t) = \sum_{j=1}^{N'} \sum_{k=1}^{n'} \gamma_k'\Gamma_j'\psi_{ik}\phi_{sj} \;\; F[f;\Omega_j,\eta_j;\omega_k,\beta_k] \tag{22}$$

The definition of the function F is given by Eq. 16. Employing Laplace transform, the function $F[g;\Omega,\eta;\omega,\beta]$ can be expressed as follows:

i) If the conditions $\Omega=\omega$ and $\beta=\eta$ are not satisfied simultaneously, we have

$$F[f;\Omega,\eta;\omega,\beta] = aU[f;\Omega,\eta] + bU[f;\omega,\beta] + cJ[f;\Omega,\eta] + dJ[f;\omega,\beta] \tag{23}$$

in which

$$U[f;p,q] = p\int_0^t f(\tau)\; \exp[-pq(t-\tau)]\; \sin p(t-\tau)\; d\tau \tag{24}$$

$$J[f;p,q] = 2pq\int_0^t f(\tau)\; \exp[-pq(t-\tau)]\; \cos p(t-\tau)\; d\tau \tag{25}$$

and the coefficients a,b,c,d can be expressed in terms of the frequency ratio and the critical dampings as

$$a = \{(1+2\beta\lambda)(\lambda^2+\varepsilon^2+1) - 2\varepsilon(\varepsilon-2\eta\lambda) + 4\beta\eta\varepsilon(\lambda^2+\varepsilon^2-1)\}/\Delta,$$
$$b = \varepsilon\{(\varepsilon-2\eta\lambda)(\lambda^2+\varepsilon^2+1) - 2\varepsilon(1+2\beta\lambda) + 4\beta\eta(\lambda^2-\varepsilon^2+1)\}/\Delta, \quad (26)$$
$$c = \{(2\beta\eta\lambda-\beta\varepsilon+\eta)(\lambda^2+\varepsilon^2+1) + \varepsilon\lambda\}/(\eta\Delta), \quad d = -\varepsilon\eta c/\beta$$

where

$$\Delta = [\lambda^2+(1+\varepsilon)^2][\lambda^2+(1-\varepsilon)^2], \quad \varepsilon = \Omega/\omega, \quad \lambda = \beta-\eta\varepsilon. \quad (27)$$

ii) If the conditions $\Omega=\omega$ and $\beta=\eta$ are satisfied simultaneously, by dropping terms of order β^2 and β, we obtain

$$F[f;\omega,\beta;\omega,\beta] = \frac{1}{2} U[f;\omega,\beta] - \frac{1}{2} V[f;\omega,\beta] \quad (28)$$

in which

$$V[f;\omega,\beta] = \omega \int_0^t f(\tau) \exp[-\omega\beta(t-\tau)]\, \omega(t-\tau) \cos\omega(t-\tau)\, d\tau. \quad (29)$$

In order to evaluate the maximum acceleration response of the equipment employing Eq. 22, it is required to evaluate the contribution of each mode pair to the maximum response. For this purpose, it is assumed that the fraction of critical dampings of each mode pair are small and of the same order. We consider contributions into the maximum acceleration of the equipment from highly untuned, slightly tuned and tuned mode pairs separately.

i) Highly untuned mode pairs

In this case, $|1-\varepsilon| >> \beta$, and dropping terms of second order in damping in Eq. 26, we obtain

$$a = 1/(1-\varepsilon^2), \quad b = \varepsilon^2/(\varepsilon^2-1), \quad c = (1-\beta\varepsilon^3/\eta)/(1-\varepsilon^2)^2, \quad d = (\varepsilon^3-\eta/\beta)/(1-\varepsilon^2)^2 \quad (30)$$

By substituting these coefficients into Eq. 23, maximum of the function F can be expressed as

$$\max F[f;\Omega,\eta;\omega,\beta] = \max\{\frac{1}{1-\varepsilon^2} U(f;\Omega,\eta) + \frac{1}{1-(1/\varepsilon)^2} U(f;\omega,\beta)\} \quad (31)$$

Denoting the acceleration response spectrum for the ground motion f(t) by S_A, we have

$$\max U(f,\Omega,\eta) = S_A(\Omega,\eta), \quad \max U(f;\omega,\beta) = S_A(\omega,\beta) \quad (32)$$

Hence, if all the mode pairs are highly untuned in view of Eq. 22, the maximum acceleration response of the equipment can be obtained by root mean square superposition of the contributions from mode pairs as

$$\max a_i = \{\sum_{j=1}^{N'} \sum_{k=1}^{n'} (G_{kj}^{is})^2 [\frac{S_A^2(\Omega_j,\eta_j)}{[1-(\Omega_j/\omega_k)^2]^2} + \frac{S_A^2(\omega_k,\beta_k)}{[1-(\omega_k/\Omega_j)^2]^2}]\}^{\frac{1}{2}} \quad (33)$$

in which

$$G_{kj}^{is} = \gamma_k' \Gamma_j' \psi_{ik} \phi_{sj} . \tag{34}$$

In the case of single-degree of freedom equipment, Eq. 33 reduces to the maximum acceleration estimate given by Sackman and Kelly [1].

ii) Slightly tuned mode pairs

Defining

$$\delta = 1-\varepsilon = \frac{\omega-\Omega}{\omega} , \tag{35}$$

and assuming that $|\delta|$ is of the order of β or smaller, approximate expressions for the coefficients a,b,c,d given by Eq. 26 are obtained. Substituting these into Eq. 23, and neglecting higher order terms in β,η and δ, we obtain

$$F[f;\Omega,\eta;\omega,\beta] = [\frac{1}{2} + \frac{2(\beta^2-\eta^2)}{\delta^2+\lambda^2}] \; U[f;\; \frac{\Omega+\omega}{2},\; \frac{\eta+\beta}{2}] - \frac{1}{2}\frac{\delta^2}{\delta^2+\lambda^2} \; V[f;\; \frac{\Omega+\omega}{2},\; \frac{\eta+\beta}{2}] + \frac{1}{2}\frac{\delta(\eta-\beta)}{\delta^2+\lambda^2} \; w[f;\; \frac{\Omega+\omega}{2},\; \frac{\eta+\beta}{2}] \tag{36}$$

in which U and V are defined by Eq. 24 and Eq. 29 respectively, and

$$W[f;p,q] = p \int_0^t f(\tau) \; \exp[-pq(t-\tau)]p(t-\tau) \; \sin p(t-\tau) \; d\tau . \tag{37}$$

iii) Tuned mode pairs

In this case, $\delta=0$ and $\beta=\eta$ simultaneously. The function F is given by Eq. 28. It should be noted that

$$V[f;p,q] = \lim_{\alpha\to 0} \{\frac{p}{\alpha} \int_0^t f(\tau) \; \exp[-pq(t-\tau)]\sin\alpha p(t-\tau) \; \cos p(t-\tau) \; d\tau\} \tag{38}$$

and W can be expressed as a limit of a similar integral. Hence, the second term in Eq. 28 and second and third terms in Eq. 36 can be identified as representing a beat motion [1] for tuned and slightly detuned mode pairs.

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