GENERALIZED MUTO METHOD
FOR STRUCTURES COMPOSED OF NON-ORTHOGONAL FRAMES

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SUMMARY

A simplified method is developed for the lateral load analysis of multistorey buildings composed of non-orthogonal frames with columns whose principal axes are in arbitrary directions. The method is applied independently to each storey by constructing and solving a set of linear equations with three unknowns. Principal displacement directions and principal lateral stiffnesses of columns are obtained by taking both column and non-orthogonal floor beam characteristics into account. The assumptions made by Muto are appropriately interpreted and adopted in the derivation of the lateral stiffness formulae for topmost, intermediate and lowermost storey columns. Thus the conventional Muto method is generalized for the analysis of non-orthogonal space structures. The results of the error analysis carried on several numerical examples are presented.

1- INTRODUCTION

Lateral load analysis for multistorey buildings is one of the subjects on which a large number of studies have been devoted, due to both the importance of the subject and various complex special cases met in practical applications. A number of simplified methods have been developed by various authors [1] , [2] , [3] , which reduce this highly statically indeterminate problem to a statically determinate one.

At present, simplified methods are used for both preliminary and final stages of analysis in the design of multistorey structures. They are also used for checking the order of the solutions obtained by exact methods when ill-conditioning is expected.

In this paper, a simplified method is developed for the analysis of multistorey structures composed of non-orthogonal frames. As in most of other simplified methods, it is assumed that the behaviour of a particular storey is independent of the behaviour of other storeys. The assumptions of Muto [1] are suitably interpreted and adopted in calculating the lateral stiffnesses of columns. Thus the Muto method for lateral load analysis of plane frames is generalized and extended for the analysis of non-orthogonal space structures.

It must be mentioned that, since direct substructuring is not possible for non-orthogonal structures, the exact solution i.e. computer analysis may only be carried out for relatively high costs. The method presented herein, on the other hand, offers an approximate but highly economic solution to this problem.

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2- ASSUMPTIONS

The following assumptions are made in the analysis :

1- The material is linear elastic.
2- Floors are infinitely rigid in their planes.
3- Axial deformations of columns are neglected.
4- Relative displacements of the columns of a particular storey are independent of shear forces and stiffnesses of other storeys.

It is well known that the last assumption, which is correct in the case of infinitely rigid beams, is common to all the simplified methods [1] ,[2]. In these methods the lateral stiffness D of a column is defined as

$$D = \frac{T}{\delta} \tag{1}$$

where T and δ denote the shear force and relative displacement of the column respectively, (Fig.1a).

In all the simplified methods, after distributing the storey shears among the storey columns in proportion to their lateral stiffnesses, column end moments are obtained by

$$\left.\begin{aligned} M_o &= (1-y)Th \\ M_u &= yTh \end{aligned}\right\} \tag{2}$$

as shown in Fig. 1b . Here h is the storey height and y is a dimensionless parameter defining the point of contraflexure. Lateral load analysis of plane frames by simplified methods is completed by distributing the sum of the column end moments at the joints to obtain the end moments of beams. In general, the simplified methods differ from each other in the assumptions and definitions associated with the D and y values.

In this paper the assumptions of Muto [1] are adopted and interpreted as follows.

A multistorey frame under lateral loading, shown in Fig. 2a, may be idealized as being composed of a number of single-storey frames as shown in Fig. 2b. The columns of the idealized frames are the same as those of the original frame. The beam stiffnesses for intermediate and lowermost storeys, however, are taken as shown in Fig. 2b and it is assumed that all the beams undergo antisymmetrical deformations. It may readily be shown that, under these assumptions Muto's formulae for column lateral stiffnesses are obtained.

3- STOREY EQUILIBRIUM EQUATIONS

The typical floor plan of a multistorey building whose columns and beams are in arbitrary directions is shown in Fig. 3a. The relative rigid body motion of the floor with respect to the floor below, may be expressed by the relative displacement components

$$[\delta] = \begin{bmatrix} \delta_x \\ \delta_y \\ \delta_\theta \end{bmatrix} \tag{3}$$

of an arbitrarily chosen point 0, (Fig. 3b). Since it is assumed that the relative displacements of a particular storey are independent of other storeys, storey equilibrium equations are also independent of other storeys i.e. the analysis of each storey may be carried out by means of the three relative displacement components of that storey.

There exist two principal displacement directions for each column at a storey as will be shown presently. One of these directions is indicated by i in Fig. 3a. The angle between direction i and the coordinate axis x is designated by α_i which is assumed to be positive in the counterclockwise direction. The distance of point 0 to the directional vector i is shown by r_i which is assumed to be positive when the directional vector tends to turn counterclockwise about point 0. Column shear force T_i in the direction of axis i for any column may be obtained by simple superposition as

$$T_i = D_i(\delta_x \cos\alpha_i + \delta_y \sin\alpha_i + \delta_\theta r_i) \qquad (4)$$

in terms of relative displacements. Here D_i represents the lateral stiffness of the column in direction i.

The storey equilibrium equations may be written as

$$\begin{bmatrix} \Sigma D_i\cos^2\alpha_i & \Sigma D_i\sin\alpha_i\cos\alpha_i & \Sigma D_i r_i\cos\alpha_i \\ & \Sigma D_i\sin^2\alpha_i & \Sigma D_i r_i \sin\alpha_i \\ (\text{Sym.}) & & \Sigma D_i r_i^2 \end{bmatrix} \begin{bmatrix} \delta_x \\ \delta_y \\ \delta_\theta \end{bmatrix} - \begin{bmatrix} T_x \\ T_y \\ T_m \end{bmatrix} = 0 \qquad (5)$$

where the summations are to be performed on storey columns taking each of the principal directions seperately. The constants of Eqs. 5 are the contributions of external loads; T_x and T_y denote the storey shears in x and y directions respectively and T_m is the sum of the external moments about point 0, of the loads above the storey under consideration.

Once Eqs. 5 are solved for relative displacements $[\delta]$, column shear force T_i in the direction of axis i for any column may be obtained by substituting them into Eq. 4 which will be applied twice, i.e. once for each principal displacement direction. Lateral load analysis of the structure is completed by computing end moments of columns and beams which is carried out in a very similar manner to the conventional Muto method, [4].

4- PRINCIPAL AXES AND PRINCIPAL STIFFNESSES

It is seen from Eqs. 4 and 5 that in order to construct the equilibrium and superposition equations, principal displacement directions and lateral stiffnesses in these directions must be determined for each column. In the following, first the principal axes for beam groups will be considered and later principal displacement directions and principal stiffnesses of columns will be determined.

4.1- Principal axes and principal stiffnesses of beam groups

A beam group consisting of beams joining at either top or bottom end of a column and the end moments of an arbitrary beam ij which is inclined by an angle β with respect to axis ξ are shown in Fig. 4a and 4b respectively. The sign convention for beam end moments are chosen in accordance with the sign convention of plane frame analysis by slope deflection equations. When joint i is under the effect of external bending moments M_ξ and M_η as shown in Fig. 4a, the moment equilibrium equations may be written as

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \theta_\xi \\ \theta_\eta \end{bmatrix} = \begin{bmatrix} M_\xi \\ M_\eta \end{bmatrix} \qquad (6)$$

where θ_ξ and θ_η are the rotations in the directions of M_ξ and M_η respectively and the matrix of coefficients is

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} \sum_j k_b\cos^2\beta + k_t\sin^2\beta & \sum_j (k_b-k_t)\sin\beta\cos\beta \\ \sum_j (k_b-k_t)\sin\beta\cos\beta & \sum_j k_b\sin^2\beta + k_t\cos^2\beta \end{bmatrix} \qquad (6')$$

Here, k_b and k_t denote the bending and torsional stiffnesses of a beam respectively. Assuming that the beams undergo antisymmetrical bending deformations, the stiffnesses are given by

$$k_b = \frac{6EI}{L} \qquad k_t = \frac{GI_t}{L} \qquad (7)$$

where EI and GI_t are the bending and torsional rigidities and L is the span of the beam.

The principal stiffnesses k are defined by

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \theta_\xi \\ \theta_\eta \end{bmatrix} = k \begin{bmatrix} \theta_\xi \\ \theta_\eta \end{bmatrix} \qquad (8)$$

which yields

$$\det \begin{bmatrix} a_{11}-k & a_{12} \\ a_{21} & a_{22}-k \end{bmatrix} = 0 \qquad (9)$$

or

$$k_{1,2} = \frac{1}{2}\cdot(a_{11}+a_{22}) \pm \sqrt{\left[\frac{a_{11}-a_{22}}{2}\right]^2 + a_{12}^2} \qquad (10)$$

The angle ϕ_o between the axis ξ and principal axis 1 is given by

$$\tan 2\phi_o = \frac{2a_{12}}{a_{11}-a_{22}} \qquad (11)$$

In certain special cases the principal axes of beam groups coincide with the principal axes of column cross sections [4]. In these cases the principal displacement directions of columns also coincide with these axes.

hence principal lateral stiffnesses may be computed by means of classical Muto formulae. The analysis may then be carried out by means of Eqs. 4 and 5.

4.2. Principal displacement directions and principal lateral stiffnesses of columns

In the general case i.e. when the principal axes of beam groups do not coincide with the principal axes of column cross sections, the principal displacement directions and principal lateral stiffnesses of columns need be determined. If the structure is assumed to be idealized as explained in Section 2, it may be shown that the column deformations are antisymmetrical. The equilibrium equations at one end of a column may be written as

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} & 0 \\ A_{21} & A_{22} & 0 & A_{24} \\ A_{31} & 0 & A_{33} & 0 \\ 0 & A_{42} & 0 & A_{44} \end{bmatrix} \begin{bmatrix} \theta_\xi \\ \theta_\eta \\ \delta_\xi \\ \delta_\eta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ T_\xi \\ T_\eta \end{bmatrix} \qquad (12)$$

$$[A]\ [d] = [F]$$

Here, axes ξ and η are taken as the principal axes of column cross section and θ_ξ , θ_η and δ_ξ , δ_η denote rotations and relative displacements respectively in the vertical planes passing through these axes. T_ξ and T_η are the column shear forces acting in these planes respectively. The coefficients of Eq. 12 may be expressed as

$$\begin{aligned} A_{11} &= a_{11} + k_{11} , & A_{12} &= a_{12} & A_{13} &= k_{13} \\ A_{21} &= a_{21} & A_{22} &= a_{22} + k_{22} , & A_{24} &= k_{24} \\ A_{31} &= k_{31} & A_{33} &= k_{33} & A_{42} &= k_{42} , \ A_{44} = k_{44} \end{aligned} \qquad (12')$$

where stiffness coefficients a_{ij} related to the beam groups are given by (6'). As explained in Section 2, in determining these coefficients, beam stiffnesses k_b and k_t should be computed as follows :

- At uppermost and intermediate storeys, one fourth of the sum of beam rigidities at top and bottom,
- At lowermost storey with fixed support, one third of the beam rigidities of first storey,
- At lowermost storey with hinged support, the whole of the beam rigidities of first storey,

will be taken into account.

The coefficients denoted by k_{ij} in Eqs. 12' represent the stiffness coefficients of the column and are given by the following formulae :

At uppermost and intermediate storeys

$$k_{11}=\frac{6EI_\xi}{h}\ ,\quad k_{13}=\frac{6EI_\xi}{h^2}\ ,\quad k_{31}=\frac{12EI_\xi}{h^2}\ ,\quad k_{33}=\frac{12EI_\xi}{h^3}$$
$$k_{22}=\frac{6EI_\eta}{h}\ ,\quad k_{24}=\frac{6EI_\eta}{h^2}\ ,\quad k_{42}=\frac{12EI_\eta}{h^2}\ ,\quad k_{44}=\frac{12EI_\eta}{h^3} \qquad (13)$$

At lowermost storey with fixed support

$$k_{11}=\frac{4EI_\xi}{h}\ ,\quad k_{13}=\frac{6EI_\xi}{h^2}\ ,\quad k_{31}=\frac{6EI_\xi}{h^2}\ ,\quad k_{33}=\frac{12EI_\xi}{h^3}$$
$$k_{22}=\frac{4EI_\eta}{h}\ ,\quad k_{24}=\frac{6EI_\eta}{h^2}\ ,\quad k_{42}=\frac{6EI_\eta}{h^2}\ ,\quad k_{44}=\frac{12EI_\eta}{h^3} \qquad (13')$$

At lowermost storey with hinged support

$$k_{11}=\frac{3EI_\xi}{h}\ ,\quad k_{13}=\frac{3EI_\xi}{h^2}\ ,\quad k_{31}=\frac{3EI_\xi}{h^2}\ ,\quad k_{33}=\frac{3EI_\xi}{h^3}$$
$$k_{22}=\frac{3EI_\eta}{h}\ ,\quad k_{24}=\frac{3EI_\eta}{h^2}\ ,\quad k_{42}=\frac{3EI_\eta}{h^2}\ ,\quad k_{44}=\frac{3EI_\eta}{h^3} \qquad (13'')$$

In Eqs. 13, 13' , 13" , EI_ξ and EI_η are the bending rigidities of the column in the directions ξ and η respectively.

The lateral stiffness matrix $[D]$ of the column is obtained by eliminating the first two rows and columns of matrix $[A]$. Thus

$$\begin{bmatrix} D_{\xi\xi} & D_{\xi\eta} \\ D_{\eta\xi} & D_{\eta\eta} \end{bmatrix} \begin{bmatrix} \delta_\xi \\ \delta_\eta \end{bmatrix} = \begin{bmatrix} T_\xi \\ T_\eta \end{bmatrix} \qquad (14)$$
$$[D]\,[\delta] = [T]$$

is obtained. Due to the special form of matrix $[A]$, the coefficients of matrix $[D]$ are readily obtained as

$$D_{\xi\xi} = A_{33} - \frac{A_{22}A_{13}A_{31}}{p}\ ,\quad D_{\xi\eta} = D_{\eta\xi} = \frac{A_{12}A_{24}A_{31}}{p}$$
$$D_{\eta\eta} = A_{44} - \frac{A_{11}A_{24}A_{42}}{p}\ ,\quad (p = A_{11}A_{22} - A_{12}^2) \qquad (15)$$

Principal lateral stiffnesses D_1 and D_2 of the column may be determined using the definition of principal axes for matrix $[D]$. Thus

$$D_{1,2} = \frac{1}{2} \cdot (D_{\xi\xi} + D_{\eta\eta}) \pm \sqrt{\left[\frac{D_{\xi\xi} - D_{\eta\eta}}{2}\right]^2 + D_{\xi\eta}^2} \qquad (16)$$

is obtained. The angle between axis ξ and the principal direction 1 is given by

$$\tan 2\phi_o = \frac{2D_{\xi\eta}}{D_{\xi\xi} - D_{\eta\eta}} \qquad (17)$$

The principal displacement directions and principal stiffnesses D_1 and D_2 thus obtained will be used to construct Eqs. 4 and 5. D_1 and D_2 are the generalized form of the D values given by Muto for lateral load analysis of plane frames, [1].

5- ERROR ANALYSIS AND CONCLUSIONS

Several numerical examples have been analysed by the simplified method presented above and the results have been compared with the exact solution, [4]. The error analysis have revealed the following results :

(i) The order of the errors are the same as those obtained by the conventional Muto method for plane frames.

(ii) Generally, the errors at 1st storey are smaller than the errors at upper storeys.

(iii) The errors become smaller when the beams are more rigid when compared with the columns.

(iv) In cases when the beam rigidities are of the same order of the column rigidities, overall average of the errors in column shears is less than 8%. This value is about 3% at 1st storey.

It must be mentioned that, the errors of the presented method are solely due to the assumptions introduced in computing the D values. The exact solution, on the other hand, is obtained by taking a large number of unknowns, namely, two rotations at each joint and three displacements components at each storey, into account simultaneously. Hence, the cost of the analysis is increased manyfold.

A research programme utilizing some of the concepts developed herein, is being carried out with the purpose of developing a simplified method which is more accurate and at the same time economic.

6- REFERENCES

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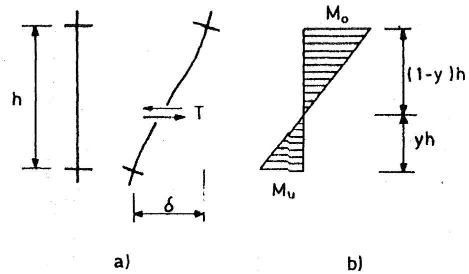

Fig.1_ Simplified lateral load analysis

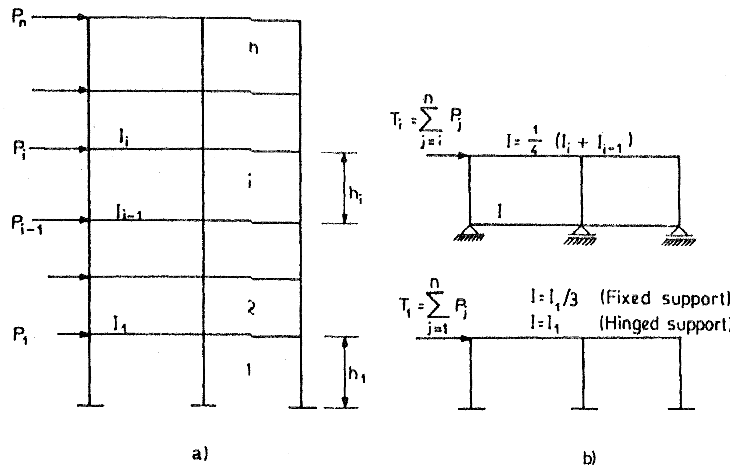

Fig.2_Idealization of plane frames

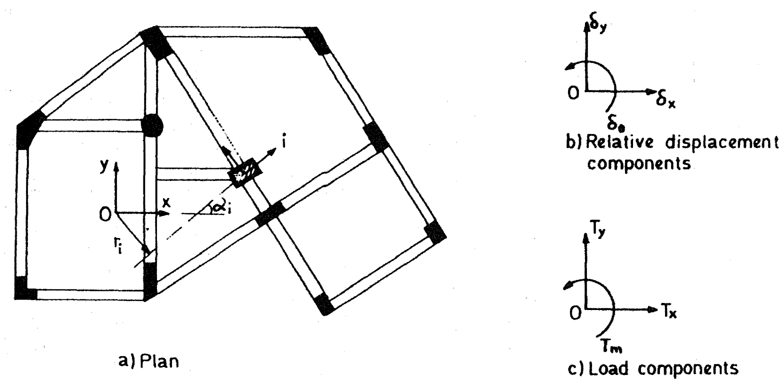

Fig. 3_Structure composed of non-orthogonal frames

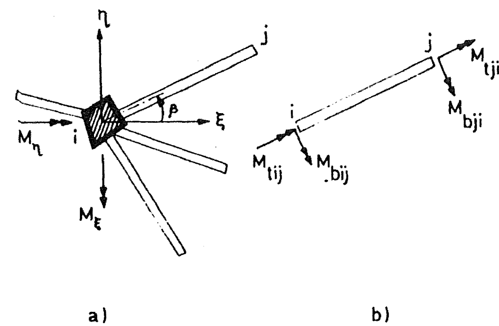

Fig.4_ Beam group and end moments