VIBRATIONAL CHARACTERISTICS AND EARTHQUAKE RESPONSES OF THE ASYMMETRIC MULTISTORY BUILDINGS

By

Joji Sakurai*

SUMMARY

Serial lumped mass systems modeling asymmetric multistory buildings of low and middle rise with eccentricity have been studied for comparison with symmetric buildings. In the analysis the Laplace transformed transfer matrices are used for calculation of frequency responses and mode superposition method for earthquake responses. differences of response characteristics of frequency responses have been shown of several models. Also models with eccentricity in various conditions have been analyzed of earthquake responses, of which comparative investigation has been made on base shear coefficient, base torsional moment and stress of frames.

PREFACE

Since most buildings are not built in symmetry, their centers of gravity and rigidity of each story in the structure rarely gather to one point, hence occurs eccentricity. Under earthquakes they transform not only in horizontal direction in parallel but twist with respect to vertical axis, where eccentricity is assumed one of the important factors that would make buildings suffer serious damages. We have had many papers on this problem, but still having left some ambiguous points to be studied. This paper proposes a calculation method as to frequency responses and shows several examples of asymmetrical buildings, and in the analysis of vibrational characteristics and earthquake responses of model buildings with eccentricity of various conditions some aseismic measures have been sought for planning buildings with eccentricity.

ANALITICAL METHOD

A typical model building for analysis is shown in Fig. 1. This model has ordinal beams and columns, together with aseismic elements whose arrangement is not always symmetric in the plane view, where the centers of gravity and of rigidity on a certain story on the building have been assumed not to agree to one point. Also vibration models of Fig. 2 have been set in order to analyze frequency responses of the building, in which vertical members are resistant elements to the horizontal displacement and to the rotation about vertical axis. The horizontal members are rigid, representing rigid floor. Symbols used in the Figs. represent the followings, suffix of which means 'in the th story'.

M_i : mass
K_i : shear stiffness
C_i : constant of viscous damping to shear deformation
${}_TK_i$: torsional stiffness
X_i : relative displacement between the bottom of structure and the bottom of the i+1th vertical member
${}_TC_i$: constant of viscous damping to torsional deformation

*Professor, College of Technology, Waseda University

θ_i : rotation angle of horizontal member

E_i : horizontal distance between vertical member in the i +1th story mass in the i th story

$\bar{E}_i$: horizontal distance of vertical members between in the i +1th story and in the i th story

Fig. 3 expresses the condition of displacement of the models in Fig. 2 projected into the plane.

Referring to Figs. 2 and 3, equations of motion in the i th story are given by Eq. (1) and Eq. (2).

$$M_i(\ddot{x}_i + e_i\ddot{\theta}_i) + \left(C_i\frac{d}{dt} + K_i\right)(x_i - \bar{e}_i\theta_i - x_{i-1}) - \frac{1}{K_{i+1}}\left(C_{i+1}\frac{d}{dt} + K_{i+1}\right)Q_{i+1} = 0 \tag{1}$$

$$I_i\ddot{\theta}_i + M_i(\ddot{x}_i + e_i\ddot{\theta}_i)(e_i + \bar{e}_i) + \left({}_TC_i\frac{d}{dt} + {}_TK_i\right)(\theta_i - \theta_{i-1}) - \frac{1}{K_{i+1}}\left({}_TC_{i+1}\frac{d}{dt} + {}_TK_{i+1}\right)T_{i+1} - \bar{e}_i Q_{i+1} = 0 \tag{2}$$

in which symbols are Q_i : shear force in the i th story, T_i : torsional moment in the i th story and T : time. Eq. (1) shows equilibrium of force in the horizontal direction and Eq. (2) equilibrium of moment. The Laplace transformed Eqs. (1) and (2), being rearranged in the expression of transfer matrix, introduces Eq. (3).

$$\begin{Bmatrix} x_{i-1}(s) \\ \theta_{i-1}(s) \\ Q_i(s) \\ T_i(s) \end{Bmatrix} = \begin{bmatrix} \frac{M_i s^2 + C_i s + K_i}{C_i s + K_i} & \frac{M_i e_i s^2 - C_i\bar{e}_i s - K_i\bar{e}_i}{C_i s + K_i} & -\frac{C_{i+1}s + K_{i+1}}{C_i s + K_i} & 0 \\ \frac{M_i(e_i+\bar{e}_i)s^2}{{}_TC_i s + {}_TK_i} & \frac{\{I_i + M_i(e_i+\bar{e}_i)\}s^2 + {}_TC_i s + {}_TK_i}{{}_TC_i s + {}_TK_i} & -\frac{\bar{e}_i}{{}_TC_i s + {}_TK_i} & -\frac{{}_TC_{i+1}s + {}_TK_{i+1}}{{}_TK_{i+1}({}_TC_i s + {}_TK_i)} \\ -\frac{M_i K_i s^2}{C_i s + K_i} & -\frac{M_i e_i K_i s^2}{C_i s + K_i} & \frac{K_i(C_{i+1}s + K_{i+1})}{K_{i+1}(C_i s + K_i)} & 0 \\ -\frac{M_i(e_i+\bar{e}_i){}_TK_i s^2}{{}_TC_i s + {}_TK_i} & -\frac{\{I_i + M_i(e_i+\bar{e}_i)e_i\}{}_TK_i s^2}{{}_TC_i s + {}_TK_i} & \frac{\bar{e}_i\,{}_TK_i}{{}_TC_i s + {}_TK_i} & \frac{{}_TK_i({}_TC_{i+1}s + {}_TK_{i+1})}{{}_TK_{i+1}({}_TC_i s + {}_TK_i)} \end{bmatrix} \begin{Bmatrix} x_i(s) \\ \theta_i(s) \\ Q_{i+1}(s) \\ T_{i+1}(s) \end{Bmatrix} \tag{3}$$

Similarly it is expressed by Eq. (4) at the top story of n-storied model.

$$\begin{Bmatrix} x_{n-1}(s) \\ \theta_{n-1}(s) \\ Q_n(s) \\ T_n(s) \end{Bmatrix} = \begin{bmatrix} \frac{M_n s^2 + C_n s + K_n}{C_n s + K_n} & \frac{M_n e_n s^2}{C_n s + K_n} & -\frac{1}{C_n s + K_n} & 0 \\ \frac{M_n e_n s^2}{{}_TC_n s + {}_TK_n} & \frac{\{I_n + M_n e_n^2\}s^2 + {}_TC_n s + {}_TK_n}{{}_TC_n s + {}_TK_n} & 0 & -\frac{1}{{}_TC_n s + {}_TK_n} \\ -\frac{M_n K_n s^2}{C_n s + K_n} & -\frac{M_n K_n e_n s^2}{C_n s + K_n} & \frac{K_n}{C_n s + K_n} & 0 \\ -\frac{M_n {}_TK_n e_n s^2}{{}_TC_n s + {}_TK_n} & -\frac{(I_n + M_n {}_TK_n e_n^2)s^2}{{}_TC_n s + {}_TK_n} & 0 & \frac{{}_TK_n}{{}_TC_n s + {}_TK_n} \end{bmatrix} \begin{Bmatrix} x_n(s) \\ \theta_n(s) \\ Q_{n+1}(s) \\ T_{n+1}(s) \end{Bmatrix} \tag{4}$$

Here in Eqs. (3) and (4), S is Laplace transformation operator and (S) is Laplace transformed amount. Expressing Eqs. (3) and (4) in the form of Eqs. (5) and (6) respectively,

$$\{U_{i-1}\} = [A_i]\{U_i\} \quad (5) \qquad \{U_{n-1}\} = [A_n]\{U_n\} \quad (6)$$

When substitution is made in order,

$$\{U_{i-1}\} = [A_i][A_{i+1}]\cdots\cdots[A_{n-1}][A_n]\{U_n\} \quad (7)$$

Further Eq. (7) is expressed as

$$\{U_{i-1}\} = [\bar{A}_i]\{U_n\} \qquad [\bar{A}_i] = \prod_{k=i}^{n}[A_k] \quad (8)$$

By use of Eq. (8) the relation from the top story to the bottom story can be obtained as Eq. (9).

$$\{U_0\} = |\bar{A}_1|\{U_n\} \quad (9)$$

The left side of Eq. (9) is

$$\{U_0\} = \{X_0(S),\ \theta_0(S)\ Q_1(S),\ T_1(S)\}^T \quad (10)$$

and $\{U_n\}$ in the right side is

$$\{U_n\} = \{X_n(S),\ \theta_n(S),\ Q_{n+1}(S),\ T_{n+1}(S)\}^T \quad (11)$$

Generally as it can be considered that $\theta_0(S)=0,\ Q_{n+1}(S)=0,\ T_{n+1}=0$ if $X_0(S)$ is already known, $X_n(S)$ and $\theta_n(S)$ are obtained by Eq. (9). Now the element of $[\bar{A}_1]$, being expressed by ${}_1\bar{a}_{jK}$

$$X_n(S) = {}_1\bar{a}_{22}\,X_0(S)/\Delta \qquad \theta_n(S) = -{}_1\bar{a}_{21}\,X_0(S)/\Delta$$
$$\Delta = {}_1\bar{a}_{11}\,{}_1\bar{a}_{22} - {}_1\bar{a}_{21}\,{}_1\bar{a}_{12} \quad (12)$$

Since the value of $X_n(S)$, $\theta_n(S)$ is obtained from Eq. (12), the value of becomes Eq. (13) by use of Eq. (8).

$$\begin{Bmatrix} X_{i-1}(S) \\ \theta_{i-1}(S) \\ Q_i(S) \\ T_i(S) \end{Bmatrix} = \begin{bmatrix} {}_i\bar{a}_{11} & {}_i\bar{a}_{12} \\ {}_i\bar{a}_{21} & {}_i\bar{a}_{22} \\ {}_i\bar{a}_{31} & {}_i\bar{a}_{32} \\ {}_i\bar{a}_{41} & {}_i\bar{a}_{42} \end{bmatrix} \begin{Bmatrix} X_n(S) \\ \theta_n(S) \end{Bmatrix} \quad (13)$$

Therefore, the maximum value of steady state response in the case to which ground motion $X_0 = \bar{X}_0 \sin \omega t$ is added can be obtained by Eq. (14).

$$
\begin{aligned}
X_{i-1}(j\omega) &= \left| ({}_i\bar{a}_{11}\,{}_i\bar{a}_{22} - {}_i\bar{a}_{12}\,{}_i\bar{a}_{21}) \,/\, \Delta \right| \bar{X}_0 \\
\theta_{i-1}(j\omega) &= \left| ({}_i\bar{a}_{21}\,{}_i\bar{a}_{22} - {}_i\bar{a}_{22}\,{}_i\bar{a}_{21}) \,/\, \Delta \right| \bar{X}_0 \\
Q_i\ (j\omega) &= \left| ({}_i\bar{a}_{31}\,{}_i\bar{a}_{22} - {}_i\bar{a}_{32}\,{}_i\bar{a}_{21}) \,/\, \Delta \right| \bar{X}_0 \\
T_i\ (j\omega) &= \left| ({}_i\bar{a}_{41}\,{}_i\bar{a}_{22} - {}_i\bar{a}_{42}\,{}_i\bar{a}_{21}) \,/\, \Delta \right| \bar{X}_0 \\
\Delta &= {}_i\bar{a}_{11}\,{}_i\bar{a}_{22} - {}_i\bar{a}_{21}\,{}_i\bar{a}_{12}
\end{aligned}
\qquad (14)
$$

As for calculation of Vibrational characteristics and earthquake responses in the following paragraph the method so far mentioned has not been applied but stiffness method has been used instead.

MODEL

The model buildings used in the following analysis are 2, 4, 6 and 8 storied as shown in Fig. 4, in which the plane is 10m x 10m and the weight of one story is 100 ton. Of this building, when no eccentricity exists, the shear rigidity has been decided as the fundamental period of the first normal mode T becomes T = 0.08N (N : number of story), that increases linearly toward upper stories. Also the rotary mass moment of inertia I of each story has been obtained assuming that weight distribution is almost uniform on the floor. Torsional stiffness $_TK$ has been obtained of the frames set at every five meters. Damping coefficient hasbeen given where n = 5% to first vibration mode. Eccentricity is expressed by the ratio of the distance between the centers of gravity and rigidity and the half width of the building.

FREQUENCY RESPONSES

In order to see differences of response characteristics of symmetrical and asymmetrical buildings, a six mass model has been taken up from many analyzed examples, and its displacement responses at the top story have been shown in Figs. 5 and 6 as some examples. Circular frequency in those Figs. is the value normalized by the first natural circular frequency of symmetrical building. Both Figs. are of eccentricity 50%, direction of which is same in each floor in Fig. 5, but reverse in each floor in Fig. 6. Characteristics have appeared through those calculations of frequency responses of vibrational systems.

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Fig. 7 relates condition of eccentricity with numbers of model types, taking example by four mass models. In calculation of vibrational characteristics Givens-Householder Method is applied, neglecting damping. Fig. 8 shows the relation between first natural period of the model and eccentricity. The value of natural period does not increase under eccentricity ratio 10% but rapidly increases more than 10%. The natural period of Types 6, 7 and 8 in contrast appears rather similar to Types 1, 2 and 3. Of earthquake responses, mode superposition method is used for analysis considering damping in proportion to natural circular frequency. In relation to eccentricity ratio, four Figs. 9 12 are displayed for response results of the cases having EL CENTRO 1940 NS components input at the maximum acceleration 0.33G at the lower end of the model. Fig. 9 shows maximum displacement at the top story of the model. The relation to base shear coefficient C_B is shown in Fig. 10, where along with increase of eccentricity ratio the C_B value of two-storied models (Types in 20s) shows tendency of increase, but of taller models than four-storied it does not increase or rather shows tendency of decrease instead. Fig. 11 is the relation to torsional moment of the first story of each model. As for torsional moment, being different from base shear coefficient, its value increases as eccentricity does; but increasing rate decreases of taller cases. Concerning asymmetrical and symmetrical models, Fig. 12 is the ratio of maximum stress of frame in the first story. As eccentricity increases, stress of frame always increases, and even as much as 10%, the ratio becomes about 1.1 1.2. Responses of Types 6, 7 and 8 with complicated eccentricity are rather similar to those of Type 3.

CONCLUSION

The investigation of frequency responses, vibrational characteristics and earthquake response analysis of model buildings with eccentricity has resulted to bring forth the following conclusion.

1) As much as eccentricity ratio 10% (5% if width of the building based), responces little differ from those of no eccentricity.
2) Of those buildings whose first natural period is shorter than predominant period of input waves, base shear coefficient and base torsional moment rapidly increase along with increase of eccentricity ratio.
3) When first natural period is longer than predominant period, base shear coefficient do not constantly increases or rather decreases in some cases, but torsional moment always increases.
4) In order not to have base shear coefficient and torsional moment increase, it is better to lessen eccentricity in the lower part of buildings if of similar scale.

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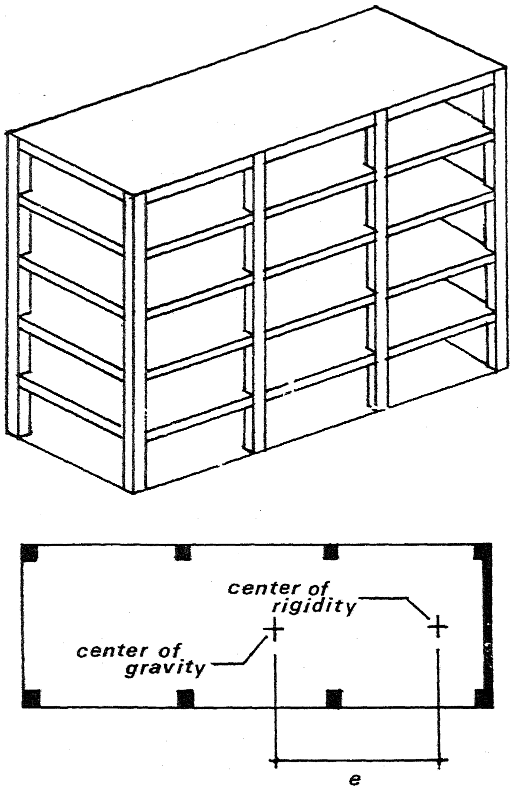

Fig. 1

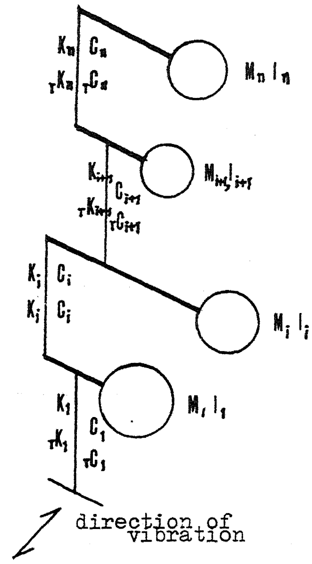

Fig. 2

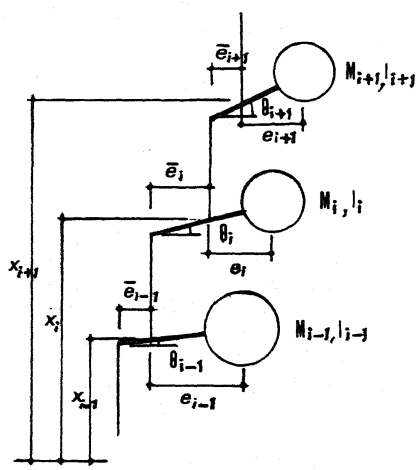

Fig. 3

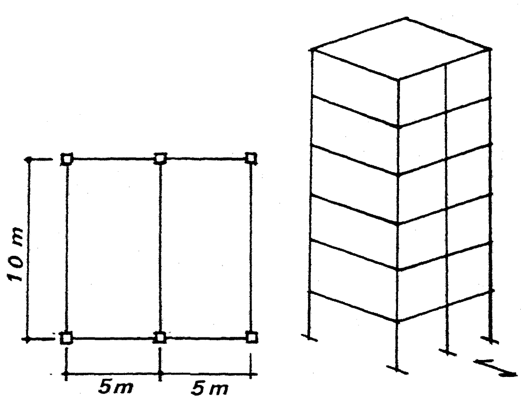

Fig. 4

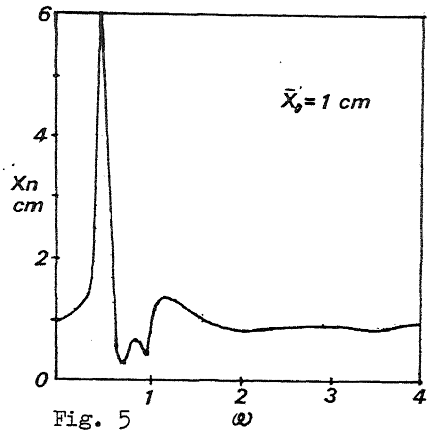

Fig. 5

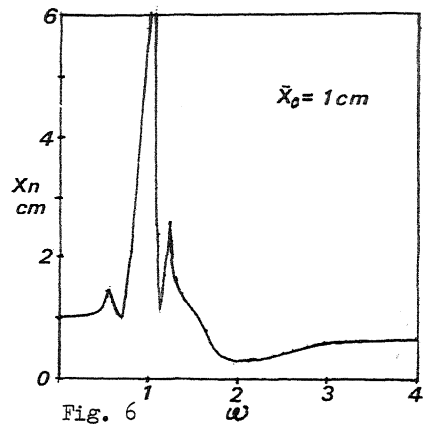

Fig. 6

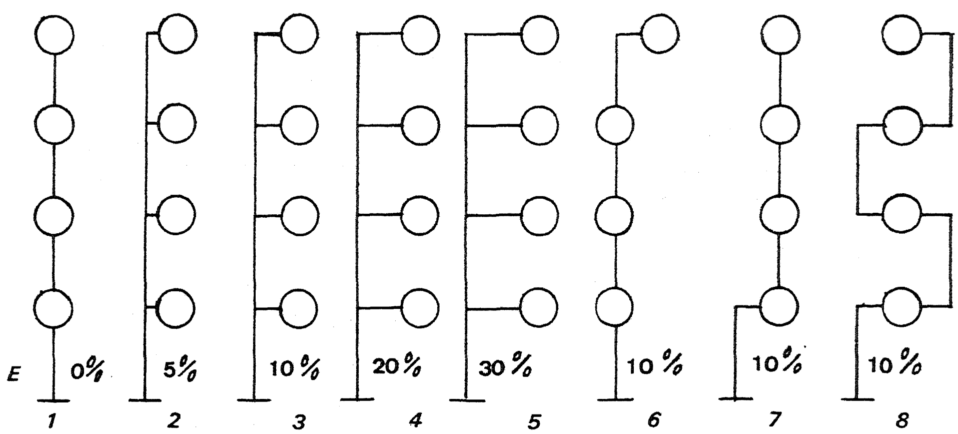

Fig. 7

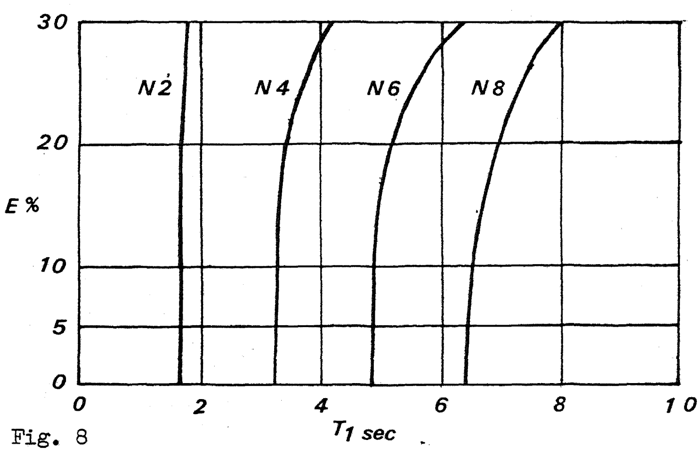

Fig. 8

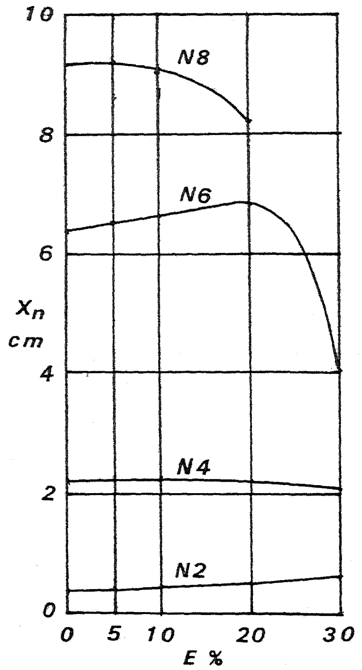

Fig. 9

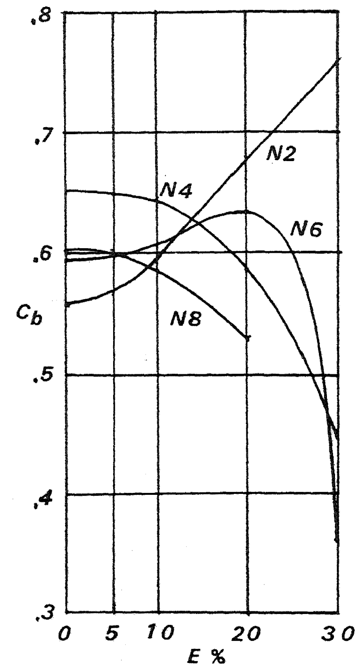

Fig. 10

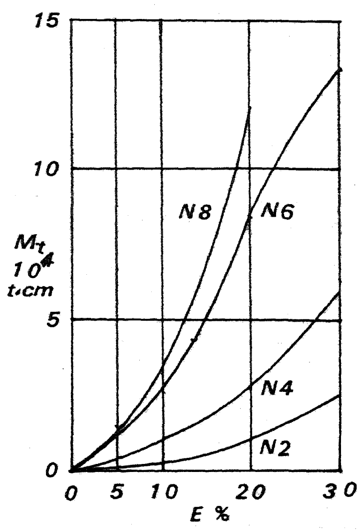

Fig. 11

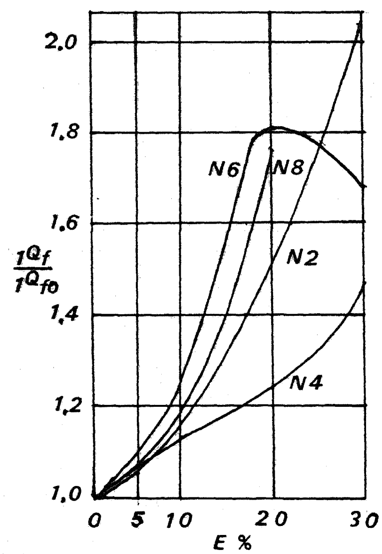

Fig. 12