DYNAMIC CHARACTERISTICS OF NONSYMMETRIC BUILDING STRUCTURES

Riko Rosman[I]

SUMMARY

Simple formulae and a coefficient table are given for the fundamental dynamic characteristics of four types of contemporary building structures the stiffness and mass axes of which do not coincide. Applying the formulae straightforward investigations of the buildings' response to earthquakes can be made in engineering offices.

INTRODUCTION

Four types of contemporary buildings are dealt with:

1. buildings the vertical structure of which consists of columns,
2. buildings the vertical structure of which consists of shear walls and/or shear-wall assemblies with or without additional columns (Fig. 1),
3. buildings the vertical structure of which consists of trusses and/or truss assemblies with or without additional columns (Fig. 2) and
4. buildings the structure of which is a rigid frame (Fig. 3) consisting of plane component frames.

The floor slabs are assumed to be rigid in their plane. In the vertical direction the structure is supposed regular: all the story heights are equal and the cross-sectional properties and the material moduli are constant. At their bottom ends all the vertical elements are fixed into a rigid foundation, a grid of basement walls or the like. With the building types 1, 2 and 3 the floor slabs are supposed pin-connected to the vertical elements, so that no frame action develops. With the building type 2 the height of the shear walls should be at least twice their width and with the building type 3 the number of the trusses' panels along the height should be at least five, so that the bending action of the structure dominates over its shear action. Pin columns with the building types 1, 2 and 3 do not contribute to their lateral stiffness and are, hence, ignored in dynamic analyses. With the building type 4 all the component frames are assumed proportioned in the sense defined by Fritz and Csonka (4). Moreover, the lintels'-flexibility coefficients of all component frames are equal.

The vertical structures of all four building types can be thought of as a cantilever fixed at its bottom and working in bending and torsion. The cantilever modeling the building type

---

I
Professor of Structural Engineering, Faculty of Architecture, Zagreb, Yugoslavia

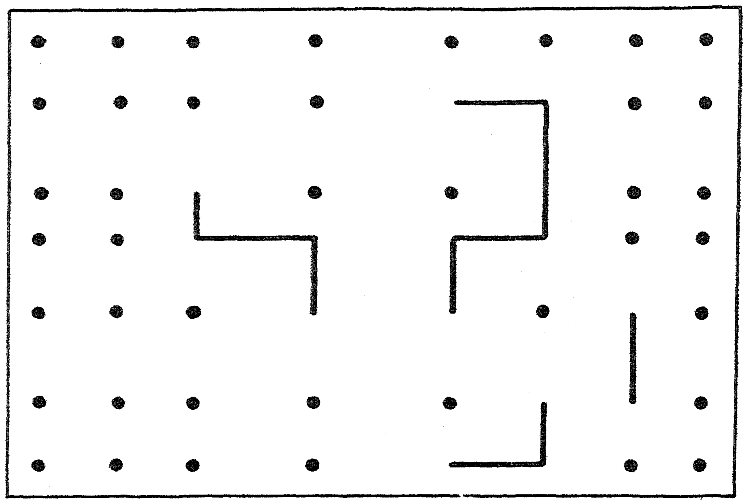

Fig. 1.
Shear-wall
structure

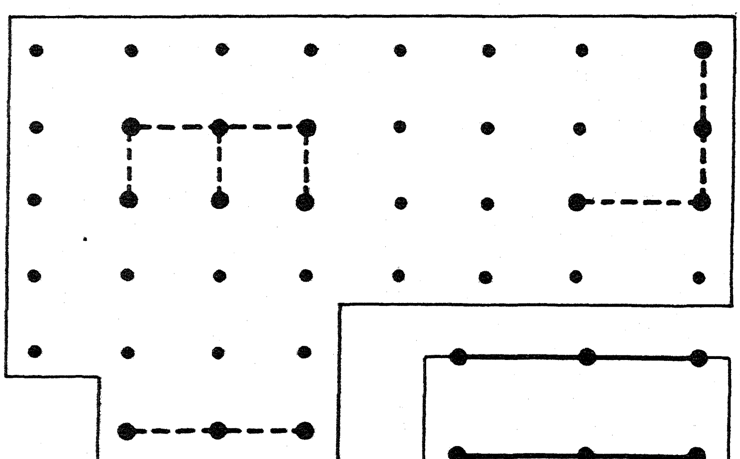

Fig. 2.
Truss
structure

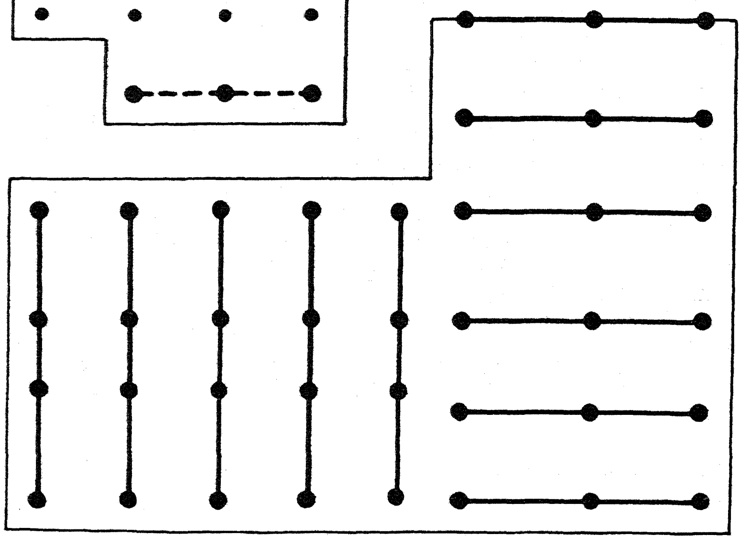

Fig. 3.
Rigid-frame
structure

4 is, at the floor levels, partly restrained against angular deflections due to the effect of the lintels. Because the cross-sectional properties of the structure are constant along the height, the cantilever has a vertical stiffness axis.

On all floors the mass distribution is assumed to be equal, so that the building has a vertical mass axis. Moreover, all the floor masses let be equal. In general, the mass axis does not coincide with the stiffness axis.

A hand method is described, which enables a simple, rapid determination of the mode shape and the period of the fundamental vibrations of the considered building types. It is a generalization of previous author's investigations (1, 3, 4, 5).

LATERAL STIFFNESS CHARACTERISTICS

The location of the shear center S of the structures' cross section and herewith of the buildings' stiffness axis has been determined by methods of the theory of structures (2).

The results for shear-wall structures consisting of mutually orthogonal single shear walls, for example, are as follows.

The principal axes x and y through S of the substitute cantilever's cross section are parallel to the walls. The z axis coincides with the stiffness axis and is oriented from the structure's bottom end upwards, so that x,y,z is a right-handed system.

Let denote

x', y' arbitrary auxiliary coordinate axes parallel to the walls

I'_{xi}, I'_{yi} moments of inertia of the i-th wall parallel to the x direction and of the j-th wall parallel to the y direction

x'_j, y'_i coordinates of the center of the j-th wall in the y direction and of the i-th wall in the x direction

The sectorial products of inertia of the walls amount to

$$I'_{xzi} = - y'_i \, I'_{xi} \; , \quad I'_{yzj} = x'_j \, I'_{yj} \; . \tag{1}$$

Superposing the contributions of all the walls one gets the sectorial products of inertia of the cantilever,

$$I'_{xz} = \sum_i I'_{xzi} \; , \quad I'_{yz} = \sum_j I'_{yzj} \; . \tag{2}$$

The coordinates of the building's stiffness axis follow to be

$$x'_S = I'_{yz}/I_y \ , \quad y'_S = - I'_{xz}/I_x \ ; \tag{3}$$

herein

$$I_x = \sum_i I'_{xi} \ , \quad I_y = \sum_j I'_{yj} \tag{4}$$

are the moments of inertia of the cantilever for the x and y directions, respectively.

The sectorial moment of inertia of the cantilever about its stiffness axis amounts to

$$I_z = \sum_i (y'_i - y'_S)^2 \, I'_{xi} + \sum_j (x'_j - x'_S)^2 \, I'_{yj} \ . \tag{5}$$

The results for shear wall structures listed above are valid also for <u>rigid-frame structures</u>, provided I'_{xi} and I'_{yi} denote the sum of the moments of inertia of all the columns of the i-th component frame in the x direction and of the j-th component frame in the y direction, respectively.

The lateral flexibility of a rigid-frame structure is governed not only by the moments of inertia I_x, I_y and I_z of its substitute cantilever but, further, by its dimensionless lintels'-flexibility coefficient C. In the frequent case when the bay widths of any component frame are equal and the moments of inertia of the interior columns are twice the moments of inertia of the exterior columns, there is

$$C = lJ/(hJ') \ , \tag{6}$$

where l is the bay width, J the moment of inertia of an exterior column and J' the moment of inertia of the lintels (4).

UNCOUPLED VIBRATIONS

Let denote

n number of stories
h, H story height and height of the structure
E modulus of elasticity
q weight of the building, a reasonable part of the occupancy load included, per unit height
i mass radius of gyration of the building about its stiffness axis

Dimensionless building parameters are defined according to

$$d_y = I_x/I_y \ , \quad d_z = i^2 \, I_x/I_z \ . \tag{7}$$

In the case the mass axis coincides with the stiffness axis, the building's vibration modes are uncoupled. There exist n plane vibration modes in the x direction, n plane vibration modes in the y direction and n rotational vibration modes about the z axis. The largest of the n periods corresponding to the flexural vibrations in the x direction, the largest of the n periods corresponding to flexural vibrations in the y direction and the largest of the n periods corresponding to the torsional vibrations about the z axis were found (3, 4, 5) to be

$$T_x = d\, H^2 \sqrt{q/(EI_x)} \;, \quad T_y = \sqrt{d_y}\, T_x \;, \quad T_z = \sqrt{d_z}\, T_x \;. \qquad (8)$$

The uncoupled period coefficient d depends on the number n of stories and the lintels'-flexibility coefficient C. With the building types 1, 2 and 3 there is $C = \infty$. Numerical values of d are listed in Table 1.

The structure's largest, i.e. fundamental vibration period is equal to the largest of the three uncoupled vibration periods T_x, T_y and T_z.

COUPLED VIBRATIONS

If the mass axis G of the building does not coincide with its stiffness axis z, the flexural and torsional vibrations are coupled. The building vibrates in flexural-torsional modes.

The period of the building's fundamental vibrations is conveniently related to its uncoupled period T_x according to

$$T = \sqrt{t}\, T_x \;, \qquad (9)$$

t being the - dimensionless - period coefficient.

When the building vibrates, the floor slabs undergo plane motions. These may be regarded as a rotation about a zero-displacement point. The author has proved (5) that, with the fundamental mode, the zero-displacement points of all the floor slabs lie on a vertical line, the building's rotation axis. Hence, the flexural-torsional vibration can be considered as a torsional vibration about a - hitherto unknown - rotation axis R. The rotation axis is that axis with respect to which the torsional vibration has the largest period.

Applying the mathematical extremum requirement, the coordinates of the rotation axis are expressed through the coordinates x_G, y_G of the mass axis and the period coefficient t,

$$x_R = \frac{x_G}{1 - t/d_y} \;, \quad y_R = \frac{y_G}{1 - t} \;. \qquad (10)$$

Table 1. Period coefficients d ($m^{-1/2}sec$)

n	C = 0	0.125	0.25	0.50	1	2	4
1	0.579	0.597	0.613	0.643	0.692	0.766	0.859
2	0.234	0.249	0.263	0.287	0.326	0.384	0.461
3	0.145	0.156	0.167	0.185	0.215	0.260	0.321
4	0.104	0.114	0.122	0.137	0.161	0.198	0.248
5	0.0814	0.0891	0.0961	0.108	0.129	0.160	0.203
6	0.0667	0.0733	0.0793	0.0898	0.107	0.134	0.171
7	0.0565	0.0623	0.0675	0.0766	0.0918	0.115	0.148
8	0.0490	0.0541	0.0587	0.0668	0.0803	0.101	0.131
9	0.0433	0.0478	0.0520	0.0592	0.0713	0.0901	0.117
10	0.0388	0.0429	0.0466	0.0532	0.0642	0.0812	0.106
11	0.0351	0.0388	0.0423	0.0483	0.0583	0.0740	0.0968
12	0.0320	0.0355	0.0386	0.0442	0.0535	0.0679	0.0891
13	0.0295	0.0327	0.0356	0.0407	0.0493	0.0627	0.0824
14	0.0273	0.0303	0.0330	0.0378	0.0458	0.0583	0.0767
15	0.0254	0.0282	0.0308	0.0352	0.0427	0.0545	0.0718

n	C = 8	16	∞
1	0.954	1.03	1.16
2	0.552	0.645	0.859
3	0.397	0.483	0.762
4	0.312	0.389	0.714
5	0.258	0.327	0.685
6	0.221	0.283	0.666
7	0.193	0.249	0.652
8	0.171	0.223	0.642
9	0.154	0.202	0.634
10	0.140	0.185	0.628
11	0.128	0.170	0.623
12	0.119	0.157	0.618
13	0.110	0.147	0.615
14	0.103	0.137	0.612
15	0.0962	0.129	0.609

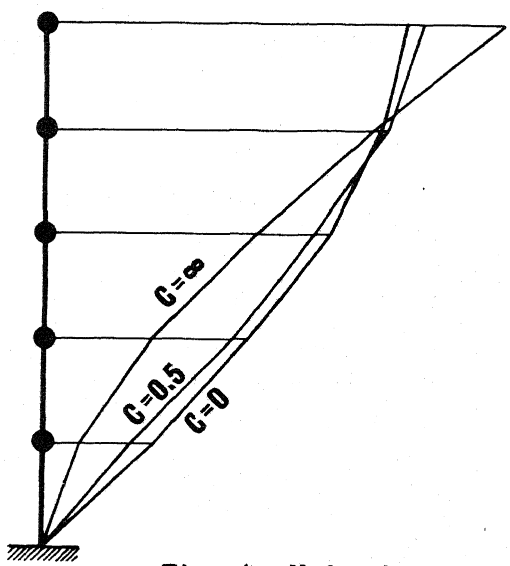

Fig. 4. Mode shapes

The governing period coefficient is the largest of the three roots of the cubic period-coefficient equation

$$t^3 - B_2 t^2 + B_1 t - B_0 = 0 \; . \tag{11}$$

The equation coefficients amount to

$$B_2 = 1 + d_y + d_z \; , \quad B_0 = k \, d_y \, d_z \; ,$$
$$B_1 = d_y + (1 - y_G^2/i^2) \, d_z + (1 - x_G^2/i^2) \, d_y \, d_z \; , \tag{12}$$

where

$$k = 1 - (x_G^2 + y_G^2)/i^2 \; . \tag{13}$$

The period coefficient t is larger than the largest of the three building parameters 1, d_y and d_z. It can easily be determined solving Eq. (11) by iteration, using a pocket calculator, proceeding from the largest of the three building parameters to larger values.

After t has been determined, the building's fundamental period is obtained by Eq. (9) and the rotation axis by Eqs. (10). The mode shape is described by the vibration amplitudes (3, 4, 5) of the uncoupled torsional vibrations measured from the rotation axis. Fig. 4 shows the mode shapes for three values of the lintels'-flexibility coefficient C.

In the special case when the building's lateral stiffness is equal for all directions ($I_x = I_y$), one of the roots of Eq. (11) becomes 1. The corresponding vibration is a plane vibration in the SG direction, i.e. in the plane through the stiffness and mass axes. The larger of the two other roots is

$$t = (1 + d_z)/2 + \sqrt{(1 + d_z)^2/4 - k \, d_z} \; . \tag{14}$$

The corresponding rotation axis lies in the SG plane.

In the special case when the structure and the mass have a common symmetry plane, say xz, one of the roots of Eq. (11) again becomes 1. The corresponding vibration is the plane vibration in the symmetry plane. The larger of the other two roots is

$$t = (d_y + d_z)/2 + \sqrt{(d_y + d_z)^2/4 - k \, d_y \, d_z} \; . \tag{15}$$

The corresponding rotation axis lies in the symmetry plane.

In the special case when the stiffness and mass axes coincide ($x_G = y_G = 0$), the period-coefficient equation yields the three roots 1, d_y, and d_z. The fundamental vibration is that uncoupled vibration which corresponds to the largest of the three uncoupled periods T_x, T_y and T_z (Eq. 8).

NUMERICAL EXAMPLE

Determine the fundamental vibration period and the corresponding mode shape of the type 3 building shown in Fig. 5. Assume that the floor masses are uniformly distributed over the plan area. Let b denote the bay width in both directions and A the cross-sectional area of a column.

The coordinates of the stiffness axis S with respect to the left bottom corner of the plan are found to be -0.4 b and 1.667 b. Moments of inertia of the substitute cantilever: $I_x = 3\ Ab^2$, $I_y = 2.5\ Ab^2$, $I_z = 19.27\ Ab^4$. Mass axis: $x_G = -0.6$ b, $y_G = 1.333$ b. Square of mass radius of gyration: $i^2 = 5.47\ b^2$. Auxiliary value: k = 0.609. Period coefficient: t = 1.549. Vibration period: $T = 1.245\ T_x$, where T_x has to be determined by Eq. 8. Rotation axis: $x_R = 2.062$ b, $y_R = -2.428$ b.

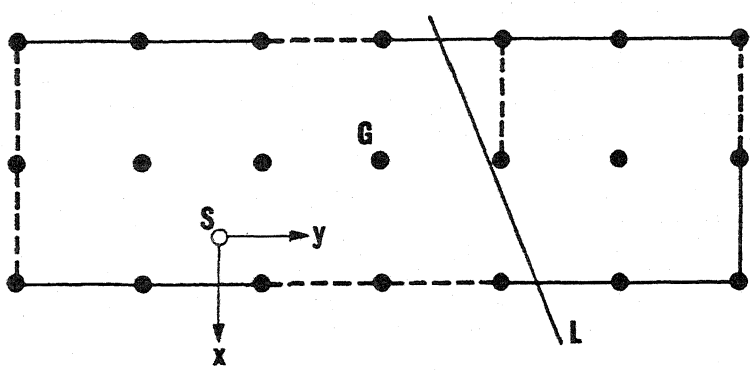

Fig. 5. Example structure

REFERENCES

1. Rosman, R., 1968, Statik und Dynamik der Scheibensysteme des Hochbaues, Springer-Verlag Berlin Heidelberg New York.
2. Rosman, R., 1971, Statik unsymmetrischer Stabsysteme, Bautechnik V. 48, pp. 94-98.
3. Rosman, R., 1975, Berechnung gekoppelter Stützensysteme des Hochbaues, 2nd edition, Ernst & Sohn Berlin München.
4. Rosman, R., 1977, Aproximate Analysis of Regular Spatial Frameworks, Building and Environment Vol. 12, pp. 97-110.
5. Rosman, R., 1979, Schwingungen im Grundriß unsymmetrischer Stützen- und Wandscheibensysteme, Beton- und Stahlbetonbau V. 74, pp. 271-277, 307-309.