EFFECTS OF INITIAL OUT-OF-ROUNDNESS ON SEISMIC RESPONSE OF CYLINDRICAL LIQUID STORAGE TANKS

by

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SUMMARY

Factors that must be considered in the analysis of the seismic response of initially out-of-round cylindrical liquid storage tanks are discussed, and an attempt is made to explain some unexpected results observed in a recent experimental program on model tanks.

INTRODUCTION

The behavior during earthquakes of large-diameter cylindrical liquid storage tanks has been a topic of considerable interest in recent years, and several analytical studies have been reported concerning the effects of tank flexibility on the magnitude and distribution of the hydrodynamic forces induced and the response of the tanks (e.g. Refs. 1,2).

The results of such analyses reveal that, for tanks of truly circular cross section, the circumferential distributions of both the hydrodynamic pressures and the radial displacements of the tank wall are proportional to $\cos\theta$, where θ is the circumferential angle measured from the vertical plane of excitation. However, the results of a comprehensive experimental program conducted recently under Professor R. W. Clough's direction at the University of California at Berkeley (Refs. 3,4) have revealed response mechanisms that cannot be fully explained by the analyses referred to above. More specifically, the circumferential distributions of the radial displacements measured in this test program were not proportional to $\cos\theta$, but instead were dominated by functions proportional to $\cos 3\theta$ and $\cos 4\theta$. Obviously, these results raise serious questions about the reliability of the theoretical predictions.

While several factors may be responsible for the difference between the observed and predicted response, one of the more important contributing factors appears to be the initial out-of-roundness of the test tanks.

The objective of this paper is to assess the possible effects of such an out-of-roundness on the seismic response of cylindrical tank-fluid systems. Undertaken with a view toward understanding and interpreting the unexpected results of the Berkeley tests, this study is of an exploratory nature and the analyses employed are approximate.

Following the approach normally used in studies of tank-fluid systems (Refs. 1,2,5), the hydrodynamic effects will be expressed as the sum of two parts: an impulsive part, which represents the effects of the portion

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of the liquid that moves as a rigid body with the tank; and a convective part, which represents the effects of the sloshing action of the liquid. Because they are associated with oscillations of much longer periods than those corresponding to the impulsive effects, the convective effects cannot be affected materially by the flexibility of the tank or its initial out-of-roundness, and can be determined with reasonable accuracy by the procedure applicable to rigid tanks. The following discussion will, therefore, be limited to consideration of the impulsive effects only.

SYSTEM CONSIDERED

A slightly out-of-round cylindrical tank of uniform thickness filled with liquid to a height H is considered. The liquid is presumed to be incompressible and inviscid, and its upper surface is considered to be free. The tank is presumed to be fixed at the base and excited by a horizontal component of ground shaking. The radial surface of the unloaded tank (i.e. of the tank at rest without the fluid) is taken as

$$R(\theta) = a[1 + \varepsilon \cos n\theta], \quad (1)$$

where a is the nominal tank radius, ε is a small number much less than unity, and n is an integer that defines the order of the out-of-roundness. Note that the irregularity is considered to be independent of the vertical coordinate.

The Berkeley test structure considered is described in Ref. 3. It is a 12-ft. diameter by 6-ft. high aluminum model filled with water to a height of 5 ft. The average wall thickness is 0.065 in., leading to a thickness-to-radius ratio of approximately 0.001. The tank was tested for two different conditions of support, of which only the one associated with a clamped condition is considered. The base excitation was the NS component of the 1940 El Centro earthquake record speeded up by a factor of 1.73 and amplified to produce a peak acceleration of 0.5g, where g is the acceleration of gravity.

Radial displacements were measured at midheight and near the top using eight sensors at each level. The sensors were uniformly spaced around the circumference with one located at $\theta = 0$. The instantaneous values of the displacements measured at each level were then expressed in the form of a truncated Fourier series as follows:

$$w(\theta) = A_o + A_1 \cos\theta + \ldots + A_4 \cos 4\theta + B_1 \sin\theta + \ldots + B_3 \sin 3\theta, \quad (2)$$

where A_o through B_3 are time dependent coefficients the values of which at any time were determined from the set of measured displacements by solving a system of eight simultaneous algebraic equations. With the experimental data interpreted in this matter, the absolute maximum values of A_3 and A_4 were found to be approximately 3.5 times as great as the corresponding value of A_1.

FACTORS AFFECTING RESPONSE

Consider that the initially out-of-round empty tank is being filled with liquid. The hydrostatic pressures induced in this process develop circumferential tensile forces in the tank which tend to reduce but not completely eliminate the initial irregularity, with the result that the tank continues to be out-of-round when full.

Two distinct factors must be considered in the analysis of the dynamic response of such a tank-fluid system. The first concerns the effects of the out-of-roundness that remains in the filled tank. Such out-of-roundness can be shown to produce small hydrodynamic pressure components which are of higher order circumferential distribution than $\cos\theta$, and these pressure components, in turn, induce radial displacement components of the same circumferential distribution.

The second factor concerns the effects that the hydrodynamic pressure has on the portion of the initial out-of-roundness that is eliminated during the process of filling the tank with liquid. If it is assumed, as it is reasonable to do, that the liquid pressure induced by the ground shaking is dominated by a component proportional to $\cos\theta$, it may be concluded that the total fluid pressure (sum of hydrostatic and hydrodynamic) increases during shaking on one side of the tank and decreases on the opposite side. On the side where the total pressure is relieved, there is a partial recovery of the part of the initial irregularity that was removed by the original filling of the tank. This localized partial recovery of the originally removed irregularity manifests itself as a response of some high-order circumferential distribution.

The reduction in the initial irregularity due to the filling process and its subsequent partial recovery depends on the properties of the tank, the geometry of the initial irregularity, and, of course, on the magnitudes of the hydrostatic and hydrodynamic pressures. The evaluation of these factors and their relative importance are considered in the following sections.

EFFECTS OF REMAINING IRREGULARITY

These effects were evaluated on the assumption that the lateral surface of the filled tank is of the form

$$R(\theta) = a[1 + \tilde{\varepsilon}\cos n\theta], \qquad (3)$$

where $\tilde{\varepsilon}$ is a dimensionless factor smaller than the factor ε in Eq. 1.

The hydrodynamic pressure induced by the ground shaking was computed on the assumption that the tank is rigid. The analysis involved the solution of Laplace's equation in polar coordinates using a perturbation technique to satisfy the conditions along the irregular boundary. It was found that a $\cos n\theta$ irregularity produces an impulsive pressure component that is proportional to $\cos(n-1)\theta$ and one of smaller magnitude that is proportional to $\cos(n+1)\theta$. These pressure components are, of course, to be added to the pressure for the truly circular tank which is proportional to $\cos\theta$.

The amplitudes of these high-order pressure components are generally of the order of the initial irregularity, and hence quite small compared to that of the primary component associated with a $\cos\theta$ distribution. However, since the tank is considerably more flexible for a high order $\cos n\theta$ pressure distribution than a $\cos\theta$ distribution of the same amplitude, it is conceivable that these small amplitude, high order pressure components may produce responses of the order of those measured in the test program.

This possibility was explored by first evaluating the static radial displacements of the tank corresponding to the above-designated pressures. These displacements were computed by application of the Rayleigh-Ritz procedure using strain energy expressions compatible with Flügge's shell theory for circular cylindrical shells. Approximations to the dynamic displacements were then obtained by multiplying the static displacements by an appropriate amplification factor, which is a function of the fundamental natural frequency of the tank-fluid system, the associated damping factor, and the characteristics of the ground motion. For the El Centro earthquake ground motion used in the Berkeley test program, the absolute maximum amplification factor is 4 for a system with 2 percent of critical damping.

From the results of such analyses, it has been concluded that the irregularity remaining in the filled tank cannot, by itself, be responsible for the behavior observed in the Berkeley test program. For realistic distributions and amplitudes of out-of-roundness, the higher order displacement components computed in this manner were generally a small fraction of those actually measured in the test program.

EFFECT OF PARTIAL RECOVERY OF IRREGULARITY REMOVED

This effect was examined in two steps: first, the part of the initial out-of-roundness removed by filling the tank with liquid was computed, and then, the effect that the hydrodynamic pressure has on the part of initial out-of-roundness removed by the filling process was evaluated.

Loss of Irregularity Due to Filling

For an initial irregularity, w_i, of the form

$$w_i = \varepsilon a \cos n\theta, \tag{4}$$

the part, w_ℓ, which is lost by filling the tank can be expressed in the form

$$w_\ell = (LF)_n w_i = (LF)_n \varepsilon a \cos n\theta, \tag{5}$$

where the loss factor, $(LF)_n$, depends on the magnitude of the liquid pressure, the number of circumferential waves, and the characteristics of the tank.

An approximation for $(LF)_n$ was obtained from consideration of the behavior of an initially out-of-round simply supported cylindrical shell subjected to a uniform internal pressure, p. If the lengthwise variation of the initial irregularity is considered to be a half-sine wave and its circumferential variation is considered to be proportional to $\cos n\theta$, it can be shown that the loss factor is given by the equation

$$(LF)_n = \frac{p/(p_{cr})_n}{1 + p/(p_{cr})_n}, \qquad (6)$$

in which $(p_{cr})_n$ is the buckling pressure of the shell corresponding to a mode associated with a single half-sine wave in the longitudinal direction and n full waves in the circumferential direction.

Equation 6 was considered to be applicable to a hydrostatically loaded cantilever tank subject to the following interpretations for the quantities p and $(p_{cr})_n$. The pressure p was taken as the average hydrostatic pressure exerted on the tank, and $(p_{cr})_n$ was taken as 60% of the buckling pressure for a simply supported tank of the same dimensions. The latter buckling pressure was determined from a solution given in Ref. 6.

The interpretation given to the quantity $(p_{cr})_n$ in Eq. 6 is justified by the results of the study reported in Ref. 7, in which comparisons were made of the lowest buckling pressures for circular cylindrical tanks clamped at one end and free at the other, and tanks of the same total length simply supported at both ends. The 60% value was found to be a good approximation over a wide range of H/a ratios and thickness-to-radius ratios of the order of 0.001, which is the value applicable to the test tank under study.

The values of $(LF)_n$ computed in this manner for the test tank are summarized below for several values of n:

Value of n	4	6	8	12	16
Value of $(LF)_n$ . . .	0.01	0.07	0.25	0.71	0.83

It can be seen that the higher order irregularities, i.e. those associated with small wave lengths, are affected significantly more than the lower order irregularities. Note, in particular, that 1% of a $\cos 4\theta$ out-of-roundness and 71% of a $\cos 12\theta$ out-of-roundness are removed by filling the test tank with water.

Recovery Due to Hydrodynamic Pressure

For the purpose of assessing the effect that the hydrodynamic pressure has on the out-of-roundness removed by the filling of the tank, it is adequate to consider only the contribution of the primary pressure component which is proportional to $\cos\theta$. As in the previous section, we shall deal with the average values of the pressure exerted over the tank height and the average values of the resulting displacements.

The impulsive component of the hydrodynamic pressure was determined from the rigid tank solution, and the absolute maximum value of the average pressure over the tank height was found to be 0.62 psi; this compares with an average hydrostatic pressure of p_o = 1.08 psi. The total average pressure on the tank wall may then be expressed as

$$p(\theta) = p_o(1 + 0.57 \cos\theta). \qquad (7)$$

Because of the importance of the circumferential hoop forces in the tank wall, the relationship between pressure and deflections is nonlinear. The radial displacements w produced by the hydrodynamic pressure were evaluated from the expression

$$w = \varepsilon a\,[(DLF)_{n,\theta} - (LF)_n] \cos n\theta, \tag{8}$$

in which the first term represents the contribution of the total average pressure defined by Eq. 7, and the second term represents the contribution of the average hydrostatic pressure. The component displacements are measured from the empty configuration of the tank, and, as already noted, represent average values over the tank height. It is important to note that since the factor $(DLF)_{n,\theta}$ is a function of θ, the circumferential distribution of w is generally not proportional to $\cos n\theta$.

The right term within the brackets in Eq. 8 is defined by Eq. 6, with p interpreted as p_o, and the left term is approximated by an equation similar to Eq. 6 as follows:

$$(DLF)_{n,\theta} = \frac{p(\theta)/(p_{cr})_n}{1 + p(\theta)/(p_{cr})_n} \tag{9}$$

Implicit in the use of this equation is the assumption that the displacements of the tank along a vertical line are proportional to the average pressure on that line. This is admittedly an approximation, but it is considered to be adequate for the purpose of this study, particularly since the circumferential variation of the total pressure is gradual and not very great.

Note should also be made of the fact that the values of $(p_{cr})_n$ in Eq. 9 were taken equal to those used in conjunction with Eq. 6. Since the circumferential distribution of the pressure is not uniform in this case, this step is not strictly valid. However, it is justified by the results of the study reported in Ref. 8, in which the lowest critical pressure for simply supported shells was evaluated for a pressure of the form $p_o(1 + \alpha \cos\theta)$. For shell dimensions corresponding to those of the test tank considered herein and for values of $\alpha \leq 0.7$, the critical condition was found to occur when the value of the peak pressure, $(1 + \alpha)p_o$, attained the buckling pressure for a uniformly compressed shell. It may be recalled that the absolute maximum value of α was 0.57 for the test tank.

The values of w for a unit value of εa computed in this manner are tabulated below for several values of n and the values of θ where the experimental data were measured:

Value θ	n = 4	n = 6	n = 9	n = 10	n = 11	n = 12
0°	0.007	0.036	0.111	0.111	0.099	0.084
45°	-0.005	0.000	0.059	0.000	-0.054	-0.065
90°	0	0	0	0	0	0
135°	0.005	0.000	0.080	0.000	-0.091	0.118
180°	-0.007	-0.040	0.173	-0.204	0.210	-0.199

As would be expected, the larger values occur at $\theta = 180^o$, where the hydrostatic pressure is relieved the greatest amount.

For a direct comparison of these results with the experimental data as reported in Ref. 3, it is necessary to compute the Fourier coefficients corresponding to the computed displacements. Since the calculated response is symmetric about $\theta = 0$, the coefficients B_1 through B_3 in Eq. 2 vanish. The remaining coefficients for a unit value of εa are as follows:

Value A_i	n = 4	n = 6	n = 9	n = 10	n = 11	n = 12
A_0	0.000	0.001	-0.070	0.012	-0.002	0.001
A_1	0.000	-0.019	0.023	-0.079	0.015	-0.006
A_2	0.000	0.001	-0.071	0.023	-0.077	0.029
A_3	-0.007	-0.019	0.008	-0.079	0.041	-0.135
A_4	0.000	0.001	-0.001	0.012	-0.075	0.028

It is important to note that a large value of A_i does not necessarily imply a large displacement component of $\cos i\theta$ distribution, but may be due to a component of higher order circumferential distribution.

Analysis of Results

There are no measurements available of the initial irregularities present in the test tank considered in this study. However, from measurements of the radial displacements produced in another tank by filling it with water, the peak amplitude of the initial irregularity was estimated to be approximately 1/4 inch. This corresponds to about 3.5 wall thicknesses or to 1/300 of the tank radius. Incidentally, the latter values are larger than those normally expected in full scale structures.

For an initial out-of-roundness with an amplitude $\varepsilon a = 0.25$ inches and a circumferential distribution in the range between $\cos 9\theta$ and $\cos 12\theta$, the radial displacements calculated for $\theta = 180^o$ range from $0.173(0.25) = 0.043$ inches to $0.210(0.25) = 0.052$ inches, and the maximum values of A_3 and A_4 are 0.034 inches and 0.019 inches, respectively. These values are of the same order of magnitude as those determined experimentally for a section near the top of the tank, particularly if provision is made for the fact that the calculated values represent average displacements over the tank height.

The results of these analyses further reveal that the circumferential distributions of the responses reported for the test tanks in the Berkeley program could not have been accurately defined by the number of sensors used. With the eight radial displacement sensors per elevation employed, it is possible to define correctly only displacement components with circumferential distributions up to $\cos 4\theta$. The higher order components, which based on the results of this study, are believed to have been major contributors to the response, must, therefore, have been interpreted incorrectly as being due to lower order components.

CONCLUSION

The analyses presented herein suggest that the responses of high-order circumferential distribution determined in the Berkeley test program must have been due to an initial out-of-roundness with wave lengths of 1/9 the circumference or less. More specifically, the measured displacements must have been caused by the reduction of the initial out-of-roundness due to filling the tank with water and its subsequent partial recovery during the ground shaking.

This study was limited to circumferential irregularities of the form defined by Eq. 1. However, similar results would also be expected for irregularities directed along the tank height.

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