COMPUTATION OF SEISMIC RESPONSE OF LIGHT ATTACHMENTS TO BUILDINGS

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SUMMARY

A simple approximate procedure is introduced for computing the maximum response of light secondary systems attached to buildings subjected to earthquakes. A numerical example is provided to describe its use, and the results of a comparative study are presented to illustrate its accuracy.

INTRODUCTION

Ordinarily, there are a variety of attachments to the floors and walls of large and complex buildings which, because of their different characteristics and functions, may not be considered as part of the structures that support them, but because of their low mass, stiffness, and damping values may be particularly vulnerable to the effects of earthquakes. Examples of these attachments are the piping system, electrical or mechanical equipment, and parapets that are usually present in multi-story buildings, industrial plants, or nuclear power facilities.

In principle, the analysis of such building attachments -- also called secondary systems -- may be carried out in conjunction with the analysis of the primary structures to which they are connected. However, the computational difficulties, the excessive number of degrees of freedom involved, and the problem of schedule and efficiency introduced by having to analyze together two systems that are customarily analyzed separately, make this procedure costly, cumbersome and impractical.

Upon recognition of the necessity of a simplified method to facilitate their analysis, several authors have suggested in the past few years several approximate methods. But although some of these methods have been proved to give reasonable results for some particular cases, such as for secondary systems with very small masses and natural frequencies far off the natural frequencies of their supporting systems, it is believed that, in general, all of them are either inaccurate or impractical, and that there is still a need for a simple and reliable procedure for the seismic analysis of secondary systems.

In this paper, then, an alternative approximate method is proposed to estimate, accurately and in a simple manner, the maximum response of secondary systems attached to buildings subjected to earthquakes. The method, derived from the analysis by a variation of the response spectrum method [1] of the composite system formed by a primary and a secondary system, and from the development of simplified analytical expressions for each of the steps that are needed to carry out such an analysis, is described in detail in Ref. 2 and presented here in a summarized form for its direct application in the solution of practical problems.

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SCOPE AND LIMITATIONS

As presented, the method may be applied for the analysis of a multi-degree-of-freedom secondary system connected to arbitrary points of a multi-degree-of-freedom primary structure. It exhibits, in addition, the following characteristics: (1) It is simple enough to carry out the necessary computations by hand. (2) It fully takes into account the interaction between a secondary system and its supporting structure, including the damping effect that each system exerts upon each other. (3) It is formulated in terms of the natural frequencies, mode shapes, and damping ratios of independent primary and secondary systems. (4) It uses the response spectrum of the ground motion prescribed for the analysis of its primary system to define the earthquake input to a secondary system. (5) It may be employed to analyze secondary systems that are near or in resonance with their supporting systems.

The method, however, is limited to those cases in which the separate primary and secondary systems are linear elastic systems with classical modes of vibration. In addition, it is restricted to the analysis of secondary systems that are connected to a primary system at no more than two points, and have small masses in comparison with the masses of their supporting structures.

RECOMMENDED APPROXIMATE PROCEDURE

Consider a secondary system attached to one or two arbitrary points of a supporting primary system. Let the independent primary system be described by its matrix of unit-participation-factor mode shapes $[\Phi]$, its natural frequencies ω_{p_i}, $i = 1, 2, \ldots, N_p$, its generalized masses† M_i^*, $i = 1, 2, \ldots, N_p$, and its modal damping ratios ξ_{p_i}, $i = 1, 2, \ldots, N_p$, where N_p represents the number of degrees of freedom of the system. Similarly, let the independent secondary system be fixed at its point of attachment with the primary system, and let it be characterized by its modal matrix $[\phi]$ (mode shapes also with unit participation factors), its natural frequencies ω_{s_j}, $j = 1, 2, \ldots, N_s$, its generalized masses m_j^*, $j = 1, 2, \ldots, N_s$, and its modal damping ratios ξ_{s_j}, $j = 1, 2, \ldots, N_s$ in which N_s denotes the number of degrees of freedom of such an independent secondary system.

Let, then, the following variables be defined as follows:

$SD(\omega, \xi)$ = response spectrum displacement ordinate corresponding to a natural frequency ω and damping ratio ξ.

γ_{ij} = m_j^*/M_j^* = generalized mass ratio in jth secondary and ith primary modes.

†The ith generalized mass of a system with N degrees of freedom is defined as

$$M_i^* = \sum_{n=1}^{N} M_n \Phi^2{}_n (i)$$

where Φ_n (i) is the amplitude of the ith mode shape of the system at the level of its nth mass, and M_n represents the value of such a nth mass.

$\phi_n(j)$ = amplitude of the jth mode shape of the independent secondary system at the level of its nth mass,

$\{d\phi\}^{(j)} = \{\phi_1(j) \quad \phi_2(j)-\phi_1(j) \quad \ldots \quad -\phi_{N_s+1}(j)\}^T$ = vector of element distortions in the jth mode of the independent secondary system,

$\{\frac{df}{f_{cc}}\} = \frac{1}{1/k_1+1/k_2+\ldots 1/k_{N_s+1}} \{1/k_1 \quad 1/k_2 \quad \ldots \quad 1/k_{N_s+1}\}^T$ = vector of normalized differential flexibilities

$\beta_j = k_{N_s+1} \phi_{N_s}(j)/(\omega_{s_j}^2 m_j^*)$

$\Phi_o(i,j) = \Phi_k(i) + \beta_j[\Phi_\ell(i) - \Phi_k(i)]$ = central value of the amplitudes of the points of attachment in the ith primary and jth secondary modes.

In the above expressions, k_j, j=1,2, ..., N_s+1, represents the jth stiffness constant of the secondary system, and $\Phi_k(i)$ and $\Phi_\ell(i)$ are the amplitudes of the kth and ℓth primary masses - the masses of the primary system to which the secondary system is attached - in the ith mode of the independent primary system. For secondary systems with a single point of attachment, $\{df/f_{cc}\} = \{0\}$, $\beta_j = 0$, and $\Phi_o(i,j) = \Phi_k(i)$.

Assume now that the assembled system (primary and secondary systems together) is an N_p+N_s degree-of-freedom system whose natural frequencies are the frequencies of its independent primary and secondary components. Classify a mode of this assembled system as a resonant mode if its natural frequency is a frequency common to both independent components, and as a nonresonant mode if its frequency is any other. Let R denote the number of these resonant modes, and for each mode of the assembled system identify with subscripts I and J the parameters of the separate primary and secondary systems, respectively, whose frequencies are the closest to or coincide with the frequency of such a mode.

Thus, if the base of the primary system is excited by a ground motion, and if this ground motion is specified by its response spectra, the vector of maximum distortions* of the secondary system may be calculated by

$$\{X_s\}_{max} = \sqrt{\sum_{s=1}^{R/2} X_s^{(s)^2} \sum_{r=1}^{N_p+N_s-R} \{X_s\}^{(r)^2}} \tag{1}$$

where $\{X_s\}^{(s)}$, which represents the combined maximum response of the secondary system in two resonant modes with equal frequency, and $\{X_s\}^{(r)}$, which denotes the maximum secondary response in the rth nonresonant mode, may be determined as follows:

*As presented in this paper, the method is formulated to obtain specifically maximum distortions. It is important to note, however, that if few adjustments are made, it may be used as well to estimate other responses (Ref. 2).

Resonant modes

$$\{X_s\}^{(s)} = \psi_R^{(s)} \{d\phi\}^{(J)} SD(\omega_o, \xi_o) \quad (2)$$

where

$$\omega_o = \omega_{p_I} = \omega_{s_J} \quad (3)$$

$$\xi_o = \frac{1}{2} (\xi_{p_I} + \xi_{s_J}) \quad (4)$$

and

$$\psi_R^{(s)} = \sqrt{\frac{\frac{1}{2}(1-\alpha_{mn})\Phi_o^2(I,J)}{|\Phi_o^2(I,J)\gamma_{IJ}-(\xi_{p_I} - \xi_{s_J})^2|}} \quad (5)$$

in which, depending on the relation between the damping and mass ratios of the separate primary and secondary systems, α_{mn} is given by one of the following formulas:

CASE I: $\left|\Phi_o(I,J) \sqrt{\gamma_{IJ}}\right| \leq \left|\xi_{p_I}-\xi_{s_J}\right|$

$$\alpha_{mn} = 2\sqrt{\xi'_m\xi'_n} \,/\, (\xi'_m + \xi'_n) \quad (6)$$

where ξ'_m and ξ'_n are of the form

$$\xi'_r = \xi_r + 2/[\omega_r S(\xi_r)], \quad r = m, n \quad (7)$$

and ξ_m and ξ_n are given by

$$\xi_{\substack{m\\n}} = \xi_o \mp \sqrt{(\xi_{p_I}-\xi_{s_J})^2 - \Phi_o^2(I,J)\gamma_{IJ}} \quad (8)$$

CASE II: $|\Phi_o(I,J) \sqrt{\gamma_{IJ}}| \geq |\xi_{p_I}-\xi_{s_J}|$

$$\alpha_{mn} = 1/\{1 + [\Phi_o^2(I,J)\gamma_{IJ}-(\xi_{p_I}-\xi_{s_J})^2]/(4\xi_o'^2)\} \quad (9)$$

where

$$\xi'_o = \xi_o + 2/[\omega_o S(\xi_o)] \quad (10)$$

In Eqs. 7 and 10, $S(\xi_r)$, a function of ξ_r, $r = 0, m, n$, stands for the duration of the equivalent white noise that best represents the ground motion under consideration. It may be calculated as suggested in Ref. 2, or it may be assumed on the basis of the characteristics of the expected average earthquakes and site conditions in the area of interest. Thus, for example, it is reported in Ref. 3 that for a group of earthquakes recorded in relatively firm ground along the west coast of the United States such an equivalent earthquake duration may be taken as 12.5 sec.

Notice that when $|\Phi_o(I,J) \sqrt{\gamma_{IJ}}| = |\xi_{p_I} - \xi_{s_J}|$, the amplification factor given by Eq. 5 becomes indeterminate. However, it is shown in Ref. 2 that if some of the second-order terms that were neglected in the derivation of that amplification factor were included in this equation, its denominator would be different from zero. Consider, thus, that

$$\psi_R^{(s)} = 0 \text{ when } |\Phi_o(I,J)\sqrt{\gamma_{IJ}}| = |\xi_{p_I} - \xi_{s_J}|.$$

Nonresonant modes with a frequency of the primary system

$$\{X_s\}^{(r)} = \psi_p^{(r)}[r_c\{\frac{df}{f_{cc}}\} + \sum_{j=1}^{N_s} r_j\{d\phi\}^{(j)}]\ SD(\omega_{p_I}, \xi_{p_I}) \tag{11}$$

where

$$r_c = \{[\Phi_\ell(I) - \Phi_k(I)]/A_o(J)\}\sqrt{1+\delta_J^2} \tag{12}$$

$$r_j = \mathrm{sgn}(1-\delta_j)[A_o(j)/A_o(J)][(1+\delta_J^2)/(1+\delta_j^2)] \tag{13}$$

$$\psi_p^{(r)} = A_o(j)/\sqrt{1+\delta_J^2} \tag{14}$$

in which

$$A_o(j) = \Phi_o(I,j)\ \omega_{p_I}^2/(\omega_{s_j}^2 - \omega_{p_I}^2) \tag{15}$$

$$\delta_j = (\xi_{p_I}\omega_{p_I} - \xi_{s_j}\omega_{s_j})/(\omega_{p_I} - \omega_{s_j}) \tag{16}$$

and "sgn" is a function that reads as "the sign of."

Nonresonant Modes with a frequency of the secondary system

$$\{X_s\}^{(r)} = \psi_s^{(r)}\{d\phi\}^{(J)}\ SD\ (\omega_{s_J}, \xi_{s_J}) \tag{17}$$

where

$$\psi_s^{(r)} = \{[1 + \sum_{i=1}^{N_p} B_o'(i)]^2 + [\sum_{i=1}^{N_p} B_o'(i)\ \delta_i]^2\}^{1/2} \tag{18}$$

in which

$$B_o'(i) = [\Phi_o(i,J)\omega_{s_J}^2/(\omega_{p_i}^2 - \omega_{s_J}^2)]\ /\ (1 + \delta_i^2) \tag{19}$$

$$\delta_i = (\xi_{s_J}\ \omega_{s_J} - \xi_{p_i}\ \omega_{p_i})\ /\ (\omega_{s_J} - \omega_{p_i})\ . \tag{20}$$

It is important to note that the above expressions have been presented in its most general form and that, consequently, their application does not necessarily require the use of such a general form. Rather, without overlooking that this is an approximate method, one should interpret these expressions and use a simplified version of them. Thus, for example, although Eq. 1 indicates the use of all the assembled system modes, and Eqs. 18 and 11 consider respectively all the modes of the primary and secondary systems, in similarity with a conventional modal analysis one should take into account only those of such modes which significantly affect the response of the secondary system under consideration

ILLUSTRATIVE EXAMPLE

To clarify the use of the procedure established above, the maximum distortions of the secondary system shown in Fig. 1 are here calculated for the case in which its primary supporting structure is subjected to a portion of El Centro (May 18, 1940) earthquake whose response spectrum is shown in Fig. 2. (The units of the values indicated in Fig. 1 are T-sec^2/m for the masses and T/m for the stiffnesses). The damping matrices of the independent primary and secondary systems are assumed proportional to their respective stiffness matrices, and the damping ratios in their fundamental modes are considered to be 2 and 0.1%, respectively. Their modal matrices, natural frequencies, modal damping ratios, and generalized masses are as:

<u>Primary System</u>

$$[\Phi] = \begin{bmatrix} 0.5 & 0.4 & 0.1 \\ 1.0 & 0.2 & -0.2 \\ 1.5 & -0.6 & 0.1 \end{bmatrix} \quad \begin{matrix} f_{p_1} = 1.0 \text{ c.p.s} & \xi_{p_1} = 0.02 & M_1^* = 4.5 \\ f_{p_2} = 2.0 \text{ c.p.s} & \xi_{p_2} = 0.04 & M_2^* = 0.9 \\ f_{p_3} = 3.0 \text{ c.p.s} & \xi_{p_3} = 0.06 & M_3^* = 0.1 \end{matrix}$$

<u>Secondary System</u>

$$[\phi] = \begin{bmatrix} 0.5 & 0.5 \\ 1.5 & -0.5 \end{bmatrix} \quad \begin{matrix} f_{s_1} = 1.0 \text{ c.p.s} & \xi_{s_1} = 0.001 & m_1^* = 0.0045 \\ f_{s_2} = \sqrt{2} \text{ c.p.s} & \xi_{s_2} = 0.00141 & m_2^* = 0.0015 \end{matrix}$$

Thus, in accordance with the procedure introduced above, in this example the secondary system and its supporting structure give rise to a five-degree-of-freedom assembled system whose natural frequencies in cycles per second are:

f_1	f_2	f_3	f_4	f_5
1.0	1.0	$\sqrt{2}$	2.0	3.0

Then, this assembled system has two resonant and three nonresonant modes. The first two are the resonant ones; the third is a nonresonant mode with a frequency of the secondary system whereas the last two are nonresonant modes with frequencies of the primary system. By inspection, however, of the above frequency values, it may be established beforehand that the resonant modes dominate the response of the secondary system herein being considered. Hence, its maximum response will be approximated by the maximum response in such resonant modes as follows:

Observe that the frequency and damping ratio of the resonant modes are (see Eqs. 3 and 4)

$$\omega_o = 2\pi \text{ rad/sec} ; \quad \xi_o = \frac{1}{2}(0.02+0.001) = 0.0105$$

Observe also that

$$\beta_1 = 0.00075(1.5)/[1.0(0.0045)] = 0.25$$

and that the central value of the modal amplitudes of the points of attachment is

$$\Phi_o(1,1) = 0.5 + 0.25(1.5-0.5) = 0.75.$$

Consequently the values of $|\xi_{p_I} - \xi_{s_J}|$ and $|\Phi_o(I,J)\sqrt{\gamma_{IJ}}|$ are

$$\xi_{p_1} - \xi_{s_1} = 0.02 - 0.001 = 0.019$$

$$\Phi_o(1,1)\sqrt{\gamma_{11}} = 0.75\sqrt{0.0045/4.5} = 0.02372.$$

Evidently, in this case $|\Phi_o(I,J)\ \sqrt{\gamma_{IJ}}\ |$ is greater than $|\xi_{p_I} - \xi_{s_J}|$, and hence α_{mm} in Eq. 5 should be calculated by means of Eq. 9. For this purpose, then, assume in accordance with the values calculated in Ref. 2 that the equivalent earthquake duration for $\xi_o = 0.0105$ is S(0.0105) = 17.2 sec. Thus, Eq. 10 yields

$$\xi_o' = 0.0105 + 2/2\pi(17.2) = 0.02901$$

and consequently from Eqs. 9 and 5 one obtains

$$\alpha_{11} = 1/\{1 + [0.02372)^2 - (0.019)^2]/[4(0.02901)^2]\} = 0.94349$$

$$\Psi_R^{(1)} = \sqrt{\frac{\frac{1}{2}(1.0-0.94349)(0.75)^2}{(0.02372)^2 - (0.019)^2}} = 8.87815$$

With this value of $\Psi_R^{(1)}$ and since from Fig. 2 one has that SD(2 ,0.0105)= 0.201 m,
Eq. 2 leads therefore to

$$\{X_s\}^{(1)} = 8.87815\begin{Bmatrix} 0.5 \\ 1.5 - 0.5 \\ -1.5 \end{Bmatrix}0.201 = \begin{Bmatrix} 0.892 \\ 1.784 \\ -2.677 \end{Bmatrix}\text{m}$$

from which one may conclude that if the response in all the other modes is neglected, the maximum distortions of the secondary system result approxmately as

$$\{X_s\}_{max} = \begin{Bmatrix} 0.892 \\ 1.784 \\ 2.677 \end{Bmatrix}\text{m}.$$

COMPARATIVE STUDY

The accuracy of the procedure presented above is evaluated in Ref. 2 by means of a comparative study with "exact" time-history solutions. In this comparative study, a number of secondary systems with different mass ratios, frequency distributions, damping characteristics, location of the points of attachment, and number of these points of attachment, are analyzed for three different earthquakes, and the averages of their approximate and exact maximum distortion responses to these three earthquakes are compared. The results of the study are statistically summarized in Table 1, where some statistics of the obtained approximate to exact maximum distortion ratios are shown. For each of the categories and damping characteristics considered, the statistics furnished in this table indicate the mean *value, coefficient of variation (c.o.v.), and the maximum and minimum* values of the sample formed by all the approximate to exact maximum distortion ratios of all the analyzed systems with one of such damping characteristics and within one of such categories.

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REFERENCES

1. Villaverde, R., 1980, "Earthquake Response of Systems with Nonproportional Damping by the Conventional Response Spectrum Method," 7th World Conf. on Earthq. Engrg., Istanbul, Turkey.
2. Villaverde, R., 1980, "Seismic Response of Light Attachments to Buildings," Ph.D. Thesis, University of Illinois, Urbana, Illinois.
3. Newmark, N.M. and Rosenblueth, E., 1971, Fundamentals of Earthquake Engineering, Prentice Hall.

TABLE 1 GROUP AVERAGE STATISTICS OF APPROXIMATE TO EXACT MAXIMUM DISTORTION RATIOS OF SECONDARY SYSTEMS

CATEGORY	DAMPING	MEAN	C.O.V.	MAX	MIN
Assembled Systems with Resonant Modes	$\zeta_{p_1} \geq 4\%$, $\zeta_{s_1} = 0\%$	0.973	0.103	1.117	0.784
	$\zeta_{p_1} = 0\%$, $\zeta_{s_1} \geq 4\%$	0.945	0.130	1.123	0.646
	$\zeta_{p_1} = 2\%$, $\zeta_{s_1} = 0.1\%$	0.989	0.104	1.155	0.811
Assembled Systems with No Resonant Modes	$\zeta_{p_1} = 2\%$, $\zeta_{s_1} = 0\%$	1.018	0.062	1.129	0.902

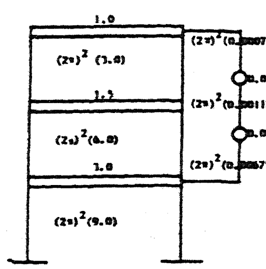

FIG. 1 SECONDARY AND PRIMARY SYSTEMS IN ILLUSTRATIVE EXAMPLE

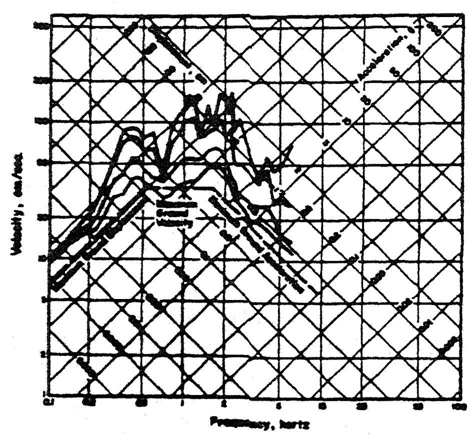

FIG. 2. RESPONSE SPECTRA, EL CENTRO, MAY 18, 1940, COMP S00E, DURATION = 10 SEC—0, 2, 10 AND 20 PERCENT DAMPING