# IMPULSIVE PRESSURE ACTING ON THE TANK ROOFS CAUSED BY SLOSHING LIQUID

by

Nobuyuki Kobayashi*

## SUMMARY

The behavior of fluid forces acting on tank roofs caused by sloshing liquid during an earthquake is investigated with experiments in which two kinds of model tanks with various shapes of roofs are used: a cylindrical tank (970 mm in diameter), and a rectangular tank (4,800 mm in length). The following results are obtained: (1) the pressure caused on the roofs is separated into inpulsive pressure and bourrage hydrodynamic pressure. (2) approximate formulae to estimate impulsive pressure and bourrage hydrodynamic pressure caused by sloshing liquid are proposed.

## 1. INTRODUCTION

One of the urgent problems concerning earthquakes is to clarify the safety of storage tanks. It has been revealed recently that strong earthquakes have rather long period waves (5 to 10 seconds). If these waves coinside with the primary natural period of the liquid in a large tank, the liquid sways to a considerable degree, and then causes what is known as "sloshing". The design criteria of the sloshing are proposed in Japan that earthquake wave is sinusoidal wave, of which period coinsides with the primary natural period of the storage liquid, which continues three times in waves (hereinafter referred to as 3-waves resonance), and of which displacement amplitude is 60 cm.(1)

When sloshing happens in a storage tank with a floating roof or without a roof, the liquid may overflow from the top of the side wall. Moreover, if the roof is fixed to the side wall of the tank, the liquid may hit the roof, restraining the movement of the liquid's surface. In the latter case, the secondary pressure (hereinafter referred to as impulsive pressure) is caused, and then the roof or the fixed parts of the roof may be damaged. Judging from the records of damage during the Niigata earthquake (June 1964) and others, impulsive pressure may be caused by earthquakes. Therefore, the behavior of impulsive pressure should be clarified.

However, only few researches have been made on this problem. The author conducted experiments and tried to clarify the behavior of fluid pressure acting on tank roofs caused by sloshing during an earthquake.

## 2. LIQUID FORCE ACTING ON THE ROOF

Two kinds of pressures are caused by sloshing on the fixed roof: impulsive pressure and bourage hydrodynamic pressure (hereinafter referred to

---

*Research member, Research Institute, Ishikawajima-Harima Heavy Industries Co., Ltd.

as hydrodynamic pressure) (Fig. 4).

For the gradient roof, impulsive pressure ($P_i$) may be obtained as in the following,(2) (3) approximately,

$$P_i = \rho \frac{\pi}{2} \cot \theta \, (\dot{\zeta}_r)^2 , \tag{1}$$

where, $\theta$ : the angle of the roof to the horizontal line,
$\rho$ : the density of the contained liquid in the tank,
$\dot{\zeta}_r$: the elevating velocity of the liquid bumping against the roof of the tank.

In equation (1), when $\theta$ becomes zero, impulsive pressure $P_i$ at the roof increases infinitely. The impulsive pressure will have finite value, actually, because there is no time interval for the air to escape around the roof and the air will be rolled into the liquid. No examination had been conducted about this problem in Reference (2). In section 3, an experimental formula will be established about this problem referring to a study on the impulse of a plate falling on the surface of water.

Hydrodynamic pressure ($P_h$) may be obtained as in the following, because hydrodynamic pressure is nearly equal to the static pressure from the maximum liquid level to point A in Fig. 1.

$$P_h = \rho g \, (\zeta_R - h) , \tag{2}$$

where, $\zeta_R$: the maximum height of liquid elevation,
$h$ : the distance between the static liquid level and the point A at the roof.

The liquid elevation ($\zeta_r$) and the liquid elevation velocity ($\dot{\zeta}r$) can be obtained as in the following. When there is no constraint by the roof, ($\zeta_r$) can be inferred from the velocity potential theory, and especially, ($\zeta_{r(n)}$) is given by equation (3) for the tank exerted by the sinusoidal waves which continue $n$ cycles.

$$\zeta_{r(n)} = \frac{\alpha}{g} RA_{(n)} \beta_P \frac{J_1(1.841\frac{\gamma}{R})}{J_1(1.841)} \cos \theta , \tag{3}$$

where, $\alpha$ : exciting acceleration,
R : the radius of the tank,
($\gamma, \theta, z$) : cylindrical coordinate,
$A_{(n)}$: magnification factor for $n$ wave, $n\pi$ for resonance condition,
$J_1$: regular Bessel function of first kind and of first order, (Fig. 12),
$\beta_P$: participation factor, $\beta_P = 0.8371$.

($\zeta_R$) is given by putting r=R, $\theta=0°$ in equation (3).

The responding wave height is in sinusoidal vibration with the same frequency to that of the exciting force of sinusoidal waves. Hence, it is possible to approximate as in the following, when its circular frequency is taken to be $\omega_f$,

$$h = \zeta_r \sin \omega_f t . \tag{4}$$

The velocity of the liquid surface ($\dot{\zeta}_r$) immediately before the liquid bumps against the roof of the tank is given as following.

$$\dot{\zeta}_r = \zeta_r \omega_f \cos\left(\sin^{-1}\frac{h}{\zeta_r}\right) . \qquad (5)$$

The distribution of impulsive pressure is obtained by calculating the liquid elevation velocity with varying ($\gamma$) in equation (5).

The natural circular frequency $\omega_1$ is expressed in equation (6), so, in the case of resonance ($\omega_f$) equal to ($\omega_1$).

$$\omega_1 = \sqrt{1.841 \frac{g}{R} \tanh\left(1.841 \frac{H}{R}\right)} , \qquad (6)$$

where, H : liquid height.

## 3. EXPERIMENTS

3.1 *Dome Roof and Cone Roofs* The experimental model shown in Fig. 2 is a cylindrical tank mainly made of transparent acrylic plate with a thickness of 15 mm, being 970 mm in diameter (1,000 mm in external diameter), 1,000 mm in height, and 15 mm in thickness. To investigate the effect of the shape of roofs on the impulsive pressure, five types of roofs were prepared; three types of cone roofs (roof angle of 7, 15, or 25 degrees), a plane roof, and an 800R dome roof.

Fig. 2 also shows the position where the pressure gauges and the accelerometers were attached. Experiments were conducted with seven kinds of the liquid levels (70, 80, 85, 90, 95 and 100 cm), three kinds of oscillating amplitudes (5, 10 and 15 mm) and roof angles as a parameter, exciting the natural frequency of the contained water. Fig. 3 shows an example of the pressure shapes recorded by the transducers for the tank with the 800R dome roof. The idealized pressure shape exerted on the roofs is shown in Fig. 4, and ($P_i$) is called impulsive pressure; ($P_h$) is called hydrodynamic pressure. In these pressures, ($P_i$) is great in its value and the working time is showt (shorter than one-tenth second); On the contrary, ($P_h$) is not greater than ($P_i$) and the working time is long (about a few seconds). Fig. 5 shows the relationship between the liquid elevation velocity bumping at the roof for the first time and the impulsive pressure ($P_i$) obtained by the experiments. The lines in the figure are the values calculated by equation (1).

Impulsive pressure ($P_i$) calculated by equation (1) and impulsive pressure ($P_{i\ell}$) obtained by experiments are compared; ($P_{i\ell}$) is the maximum value during excitement within the 3-waves resonance. Fig. 6 shows the results of various shapes of roofs. The ordinate stands for the ratio of impulsive pressure, while the abscissa stands for the liquid level. Fig. 5 shows the impulsive pressure ($P_{i\ell}$) follows equation (1), on the whole, except the plane roof. And Fig. 6 shows large scattering in impulsive pressure at full tank, but small scattering for roofs having larger angles than 15°. It is considered that the degree of the restriction by the roofs is small in the case of 3-waves resonance.

From these results, impulsive pressure caused during 3-waves resonance can be calculated by using equation (1) for roofs which have angles larger than 7°. Impulsive pressure for roofs whose angle is about 7° may be calculated by equation (1), but it will be overestimated for the liquid elevation velocity at 3-waves resonance.

3.2 Plane Roofs and Approximate Plane Roof In order to estimate impulsive pressure ($P_i$) acting on plane roofs, the experiments are conducted with a rectangular model tank, being 4,800 mm in length, 1,000 mm in width and 1,700 mm in height, mainly made of acrylic plates and steel plates (Fig. 7). The experimental method is similar to that of the cylindrical tank; exciting 3 or 6 cycles at the natural period of the contained liquid. The experiments are conducted with three kinds of displacement amplitudes: 22.5, 45, 67.5 mm, and six kinds of liquid levels: 144.7, 142, 138, 130, 123, 116 cm.

Fig. 8 shows the sloshing wave bumping on the plane roof, while Fig. 9 shows the example of the wave forms on the plane roof; impulsive pressure doesn't increase in value, even though the tank holds on being excited. Hence, impulsive pressure is examined, paying attention to the first significant wave bumping on the roof (excepting the wave grazing the roof).

Fig. 10 shows the relationship between impulsive pressure ($P_i$) and the calculated liquid elevation velocity at the time of bumping the roof. Impulsive pressure is obtained by the experimental results of the cylindrical model, the results of the rectangular model and the experimental results of the falling tests of the flat plate on the water surface.(4),(5) From this figure, the experimental formula (7) is obtained for first bumping on the plan roof.

$$P_i = 225\rho\ (\dot{\zeta})^{1.6} . \tag{7}$$

Fig. 11 shows the relationship between the inpulsive pressure obtained in each experiment and the calculated impulsive pressure. Where, the former is the maximum value during 2.5-waves or 3-waves resonance, and the latter is obtained from formula (7) given on the assumption that the sloshing waves grow without restriction from a roof. Impulsive pressure doesn't increase even though the calculated elevation velocity increases. Impulsive pressure can be calculated by formula (7) using the elevated velocity of the first wave bumping at the roof, since the maximum impulsive pressure occurs at the first wave bumping at the roof.

For roofs having angles of about 7°, impulsive pressure may be obtained from equation (1) using the elevation velocity bumping for the roof at first time, because the growth of the liquid elevation is restricted by the roof as in the plane roof.

No consideration will be given to hydrodynamic pressure, because there is no hydrodynamic pressure in the case of the plane roof and the approximate plane roof shown as Fig. 9.

## 4. RESPONSE CALCULATION

There is a difference in characteristics between impulsive pressure and hydrodynamic pressure, and then attention must be paid in calculating the response of the tank with the experimental result as mentioned before.

Hydrodynamic pressure, which is often smaller than impulsive pressure, may be treated as static liquid pressure, because the duration time is long. Impulsive pressure must be treated as dynamic response because the duration time is short. Though impulsive pressure is great in value, the impule of it is small.

From the characteristics of the working time of the pressure, hydrodynamic pressure should be used for the whole stress analysis tank, while impulsive pressure should be used for the local stress analysis of the tank roof.

## 5. CONCLUSION

Experiments were conducted on the problem of fluid forces on fixed roofs caused by sloshing, which is one of the hydrodynamic forces exerting the cylindrical liquid storage tank. The following conclusion is obtained from the examination of the results of experiments.

1) There are two categories of pressure: impulsive pressure and hydrodynamic pressure.
2) In impulsive pressure, the pressure value is often very great and the duration time is very short (less than one-tenth second).
3) The hydrodynamic pressure is caused in the case of roofs which have some roof angle, but no hydrodynamic pressure is caused in the case of the plane roofs and approximate plane roofs.
4) The hydrodynamic pressure works a few seconds and is smaller than impulsive pressure.
5) The simplified formulae are proposed for impulsive pressure and hydrodynamic pressure.

## REFERENCES

(1) H. Shibata, et al.: Design Criteria for Oil and Liqufied Gas Storages, Proc. of 6th European Conference on Earthquake Engineering (DUBROVNIK), Vol. 2, Sep. 1978, p509-516.
(2) Y. Ochi, N. Kobayashi: Sloshing Experiment of a Cylindrical Storage Tank, IHI Eng. Review, Vol. 11, No. 2, (1978), p36-46.
(3) Y. Yamamoto: Sloshing of the Liquid Surface and Impulsive Pressure in Petroleum Tanks in case of Earthquarkes, J. JHPI (Japan), Vol. 3, No. 1, Jan. 1965, p370-376.
(4) Y. Fujita, et al.: On the Impulsive Force of Flat Plate Falling upon the Water Surface, J. of Zosen Kiokai (Japan) No. 90, June, 1956, p69-75.
(5) S.L. Chuang: Investigation of Impact of Rigid and Elastic Bodies with Water, Navel Ship Research and Development Center AD702-727 (1970).

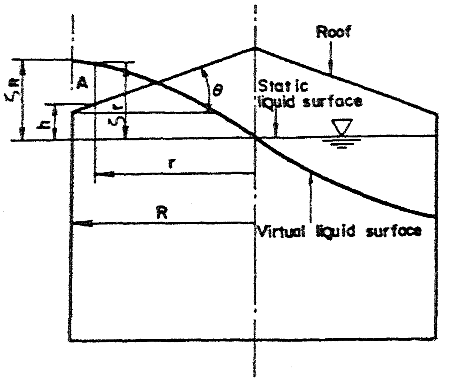


Fig.1 Sloshing model

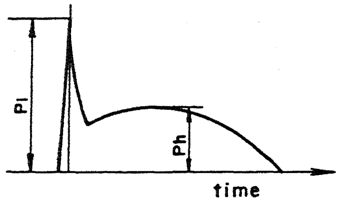
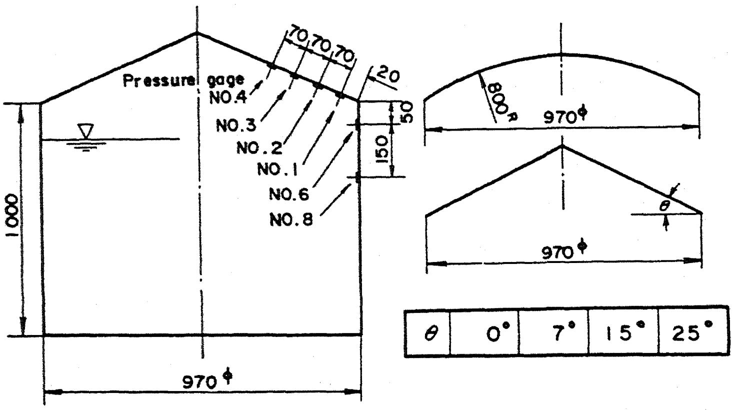


Fig.4 Idealized pressure shape



| θ | 0° | 7° | 15° | 25° |
|---|---|---|---|---|

Fig.2 Cylindrical model tank

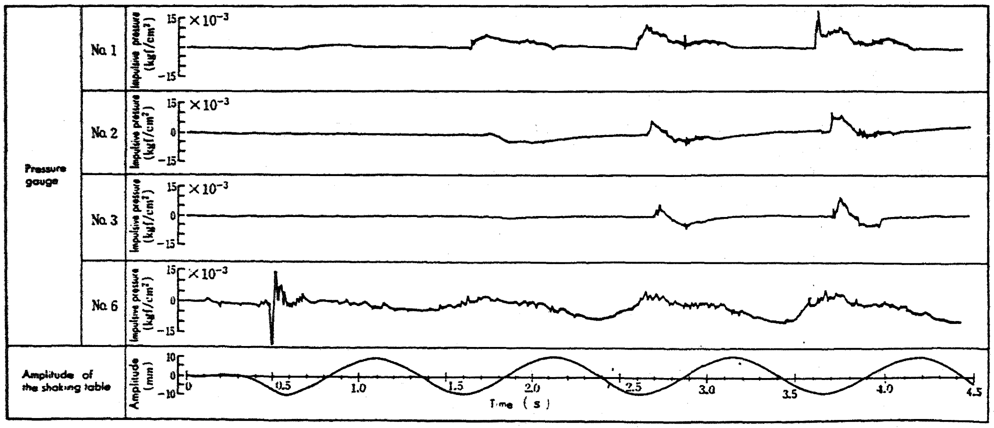


Fig.3 Oscillogram of impulsive press. on dome roof
( H = 100 cm, Disp. = 10 mm )

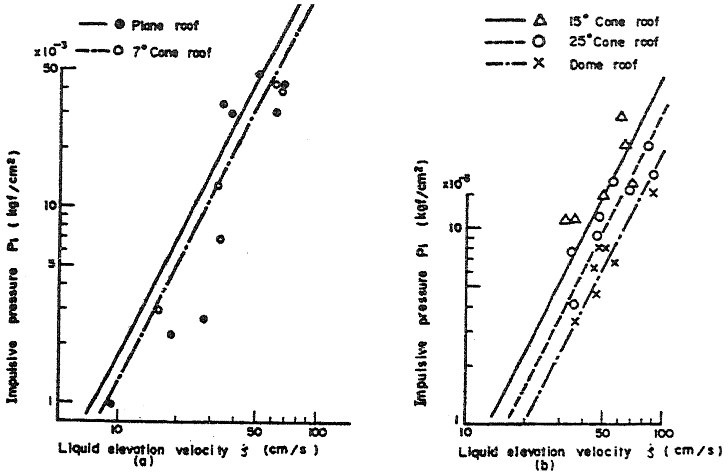


Fig.5 Impulsive pressure

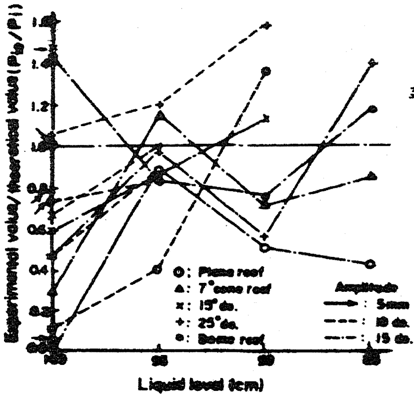


Fig.6 Impulsive pressure of 3waves(from 1st to 3rd)

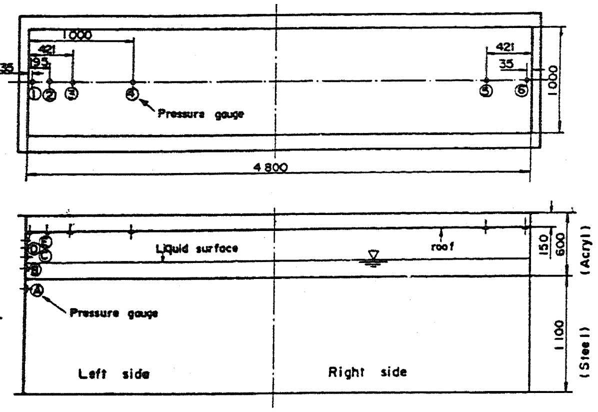


Fig.7 Rectangular model tank

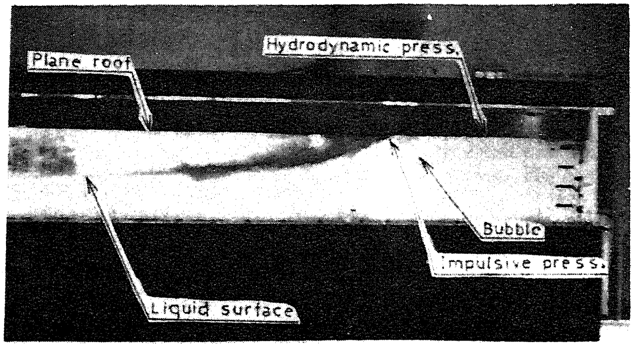


Fig.8 Sloshing wave bumping on the roof

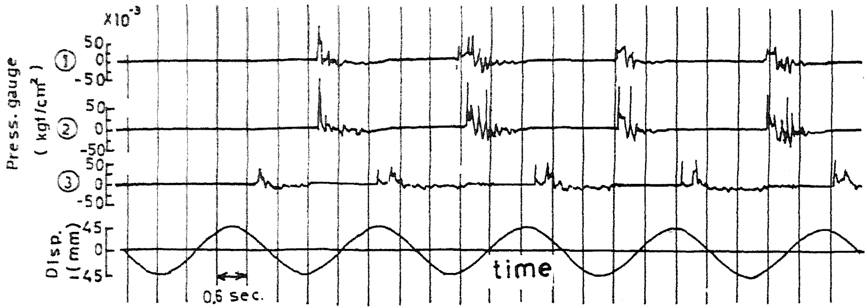


Fig.9 Oscillogram of impulsive press. on plane roof
( H = 144.7 cm, Disp. = 45 mm )

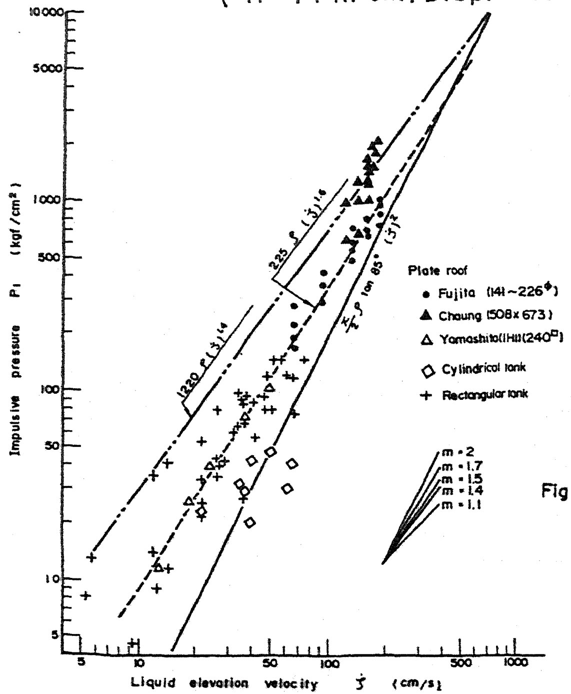


Fig.10 Impulsive prssure VS bumping velocity

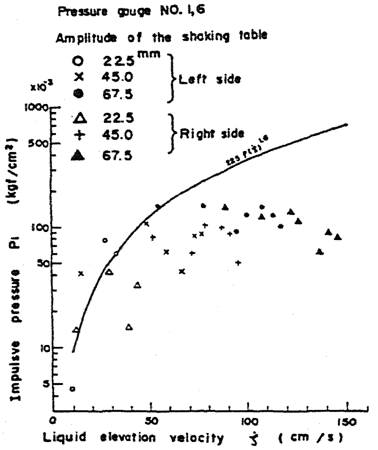


Fig.11 Impulsive pressure(The maximum pressure occured between 1st and 3rd wave )

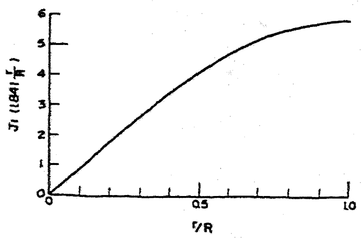


Fig.12 Value of J₁