TWO-DIMENSIONAL INELASTIC SEISMIC RESPONSE OF STEEL FRAMES INCLUDING AXIAL DEFORMATION OF COLUMNS

by

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SYNOPSIS

The restoring force characteristics of an inelastic steel columns, which is fixed at one end and subjected to horizontal load, vertical load and bending moment at the other end, are formulated by basing on the bi-linear hysteretic stress-strain relation. Using this stiffness matrix, a method of two-dimensional inelastic seismic response analysis is presented. Example studies are made to clarify the usefullness of this analysis and, from these results, it is emphasized that the plastic axial deformation of some columns are accumulated in one direction, which causes the increase of the horizontal displacement and P-Δ effect of gravity.

INTRODUCTION

P-Δ effect of gravity on the inelastic response of steel frames under strong ground motions is not negligible in some cases, as studied by P.C.Jennings et al.[1], S.C.Goel,[2] C.K.Sun et al.[4] and S.Tani et al.[5] by using single-degree-of-freedom models or multi-story frames with elastic columns.

In steel frames, when subjected to strong ground motion at the base, some columns deform plastically. Especailly in slender frames, the plastic axial deformation of these columns is accumulated and results to increase the horizontal displacement and P-Δ effect of gravity. From this reason, to examine the P-Δ effect of gravity in the seismic response of steel frames, it is required to study the restoring force characteristics of columns including the plastic axial deformation, which are simultaneously subjected to horizontal force, bending moment and axial force at the ends.

The object of this paper is to formulate the restoring force characteristics of inelastic steel columns and to present a method of two-dimensional inelastic seismic response analysis of steel frames in vertical plane to study the plastic axial deformation of columns and P-Δ effect of gravity.

STIFFNESS MATRIX OF A COLUMN

Assumptions The behavior of a steel member is formulated under the following assumptions:

a) The member is fixed at one end and subjected to horizontal load H, vertical load P and bending moment M_c at the other end and deforms only in this plane, as shown in Fig.1.
b) The section of this member replaced with two-flange-section at a distance of $2i_Y$ (i_Y: radius of gyration of the member). The sectional area of each flange is A/2. (A : sectional area of the member

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c) Shear strains are neglected and bi-linear hysteretic stress-strain relation is used, as shown in Fig.2.

Incremental Deformation and Incremental Strain Two kinds of incremental curvatures and incremental average strains, as shown in Fig.3, are used. The incremental curvature and the incremental average strain between O and B are denoted by $\Delta u_p''+\Delta u_{1e}''$ and $\Delta\varepsilon_{op}+\Delta\varepsilon_{oe}$ respectively and those between B and C are denoted by $\Delta u_{2e}''$ and $\Delta\varepsilon_{oe}$. (' : differentiation with respect to Z, Δ : increment) The following conditions are given to them.

a) With respect to Z, $\Delta u_p''$, $\Delta u_{1e}''$ ($0\leq Z\leq Z_B$) are constant and $\Delta u_{2e}''$ ($Z_B\leq Z\leq L$) is linear. With respect to Z, $\Delta\varepsilon_{op}$($0\leq Z\leq Z_B$), $\Delta\varepsilon_{oe}$ ($0\leq Z\leq L$) are constant.

b) They satisfy the following boundary conditions and conditions of continuity.

$$\Delta u_p=0,\quad \Delta u_p'=0,\quad \Delta u_{1e}=0,\quad \Delta u_{1e}'=0 \qquad \text{at } Z=0$$
$$\Delta u_{1e}=\Delta u_{2e},\quad \Delta u_{1e}'=\Delta u_{2e}',\quad \Delta u_{1e}''=\Delta u_{2e}'' \qquad \text{at } Z=Z_B \tag{1}$$

The incremental displacements and incremental rotations at the end C, defined in Fig.4, are represented in the following form

$$\Delta\delta_{He}=\Delta u_{2e\,(Z=L)},\qquad \Delta\delta_{HP}=\Delta u_{P\,(Z=Z_B)}+(L-Z_B)\,\Delta u_{P\,(Z=Z_B)}',$$
$$\Delta\delta_{Ve}=-\int_0^L\Delta\varepsilon_{oe}\,dZ+\int_0^{Z_B}U'\,\Delta u_{1e}'\,dZ+\int_{Z_B}^L U'\,\Delta u_{2e}'\,dZ,$$
$$\Delta\delta_{VP}=-\int_0^{Z_B}\Delta\varepsilon_{op}\,dZ+\int_0^{Z_B}U'\,\Delta u_P'\,dZ+\int_{Z_B}^L U'\,\Delta u_{P\,(Z=Z_B)}'\,dZ, \tag{2}$$
$$\Delta\phi_e=\Delta u_{2e\,(Z=L)}',\qquad \Delta\phi_P=\Delta u_{P\,(Z=Z_B)}'$$

where $\Delta\delta_{He},\Delta\delta_{Ve},\Delta\phi_e$ are the incremental displacements and rotations caused by $\Delta u_{1e},\Delta u_{2e},\Delta\varepsilon_{oe}$ and $\Delta\delta_{HP},\Delta\delta_{VP},\Delta\phi_P$ are those caused by $\Delta u_P,\Delta\varepsilon_{op}$. U denotes the total deflection in the X direction. The incremental strains $\Delta\varepsilon_{1e},\Delta\varepsilon_{2e},\Delta\varepsilon_P$ are expressed in the following form

$$\Delta\varepsilon_{1e}=\Delta\varepsilon_{oe}-\Delta u_{1e}''X \qquad (0\leq Z\leq Z_B)$$
$$\Delta\varepsilon_{2e}=\Delta\varepsilon_{oe}-\Delta u_{2e}''X \qquad (Z_B\leq Z\leq L) \tag{3}$$
$$\Delta\varepsilon_P=\Delta\varepsilon_{op}-\Delta u_P''X \qquad (0\leq Z\leq L)$$

where $\Delta\varepsilon_{1e},\Delta\varepsilon_{2e}$ are the incremental strains caused by $\Delta u_{1e},\Delta u_{2e}$ and $\Delta\varepsilon_{oe}$, and $\Delta\varepsilon_P$ is the incremental strain caused by Δu_P and $\Delta\varepsilon_{op}$. From Eq.(2) and Eq.(3), we obtain

$$\Delta\varepsilon_{1e}=\mathbb{C}_1\,\Delta\mathrm{d}_e \qquad (0\leq Z\leq Z_B)$$
$$\Delta\varepsilon_{2e}=\mathbb{C}_2\,\Delta\mathrm{d}_e \qquad (Z_B\leq Z\leq L) \tag{4}$$
$$\Delta\varepsilon_P=\mathbb{C}_P\,\Delta\mathrm{d}_P \qquad (0\leq Z\leq Z_B)$$

where

$$\Delta\mathrm{d}_e^T=\{\Delta\delta_{He}/L\ \ \Delta\delta_{Ve}/L\ \ \Delta\phi_e\},\quad \Delta\mathrm{d}_P^T=\{\Delta\delta_{HP}/L\ \ \Delta\delta_{VP}/L\ \ \Delta\phi_P\}$$

$\mathbb{C}_1$, $\mathbb{C}_2$, $\mathbb{C}_P$ are listed in Table 1.

Relation between $\Delta\mathrm{d}_e$ and $\Delta\mathrm{d}_P$ The relation between $\Delta\varepsilon_1$ and $\Delta\varepsilon_2$, defined in Fig.5, is expressed in the following form

$$e\,\Delta\varepsilon_2=m_3\,(1-e)\{\Delta\varepsilon_1+m_1\,(m_2\sigma_0+\sigma)/E\} \tag{5}$$

The quantity m_k (k = 1,2,3) in Eq.(5) are defined in the following manner:

$m_1 = 1$ where $\Delta\sigma$-$\Delta\varepsilon$ relation is bi-linear.
$m_1 = 0$ where $\Delta\sigma$-$\Delta\varepsilon$ relation is linear.
$m_2 = -1$ where $\Delta\sigma>0$, $m_2 = 1$ where $\Delta\sigma<0$,
$m_3 = 0$ where $\Delta\varepsilon_2=0$, $m_3 = 1$ where $\Delta\varepsilon_2\neq0$.

By expressing $\Delta\varepsilon_1$ by $\Delta\varepsilon_{1e}$ and $\Delta\varepsilon_2$ by $\Delta\varepsilon_p$, we obtain

$$\Delta \mathrm{dl}_P = \mathbb{t}_2 \Delta \mathrm{dl}_e + \mathbb{t}_3 \tag{6}$$

where

$$\mathbb{t}_2 = \begin{bmatrix} \xi_1 & \xi_2 & \xi_3 \\ \eta_1 & \eta_2 & \eta_3 \\ \frac{2}{2-\beta}\xi_1 & \frac{2}{2-\beta}\xi_2 & \frac{2}{2-\beta}\xi_3 \end{bmatrix} \qquad \mathbb{t}_3 = \begin{Bmatrix} \xi_4 \\ \eta_4 \\ \frac{2}{2-\beta}\xi_4 \end{Bmatrix}$$

and ξ_k, η_k (k = 1,2,3,4) are listed in Table 2.

Equilibrium Equation   Using loads H,P,M_c shown in Fig.1 and the incremental deformation shown in Fig.3 and Fig.4, the virtual work equation may be written

$$\mathbb{P}^T \Delta \mathrm{dl} = \int_{V_1} \sigma (\mathbb{e}_P \Delta \mathrm{dl}_P + \mathbb{e}_1 \Delta \mathrm{dl}_e) dV + \int_{V_2} \sigma \mathbb{e}_2 \Delta \mathrm{dl}_e \, dV \tag{7}$$

where $\mathbb{P}^T = \{H \;\; P \;\; M_c\}$, $\Delta \mathrm{dl} = \Delta \mathrm{dl}_P + \Delta \mathrm{dl}_e$, $\int_{V_1} dV = \int_0^{z_B}\int_A dA\,dz$, $\int_{V_2} dV = \int_{z_B}^{L}\int_A dA\,dz$

Substituting Eq.(6) in Eq.(7), we obtain

$$\begin{aligned} \Big\{ \mathbb{P}^T - \int_{V_1} \sigma (\mathbb{e}_P \mathbb{t}_0^{-1} \mathbb{t}_2 + \mathbb{e}_1 \mathbb{t}_0^{-1}) dV - \int_{V_2} \sigma \mathbb{e}_2 \mathbb{t}_0^{-1} dV \Big\} \Delta \mathrm{dl} \\ - \Big\{ \int_{V_1} \sigma (\mathbb{e}_P - \mathbb{e}_1) dV - \int_{V_2} \sigma \mathbb{e}_2 \, dV \Big\} \mathbb{t}_0^{-1} \mathbb{t}_3 = 0 \end{aligned} \tag{8}$$

where $\mathbb{t}_0 = \mathbb{t}_2 + \mathbb{1}$. Since Eq.(8) must hold for any $\Delta \mathrm{dl}$

$$\mathbb{P}^T \mathbb{t}_0 = \int_{V_1} \sigma (\mathbb{e}_P \mathbb{t}_2 + \mathbb{e}_1) dV + \int_{V_2} \sigma \mathbb{e}_2 \, dV \tag{9}$$

From the rate equation of Eq.(9), we finally obtain

$$\Delta \mathbb{p} = \mathbb{K} \Delta \mathrm{dl} + \mathbb{K}_0 \tag{10}$$

in which

$$\mathbb{K} = (\mathbb{t}_0^T)^{-1} (\mathbb{k}_1 + \mathbb{k}_2 + \mathbb{k}_3) \mathbb{t}_0^{-1}$$

$$\mathbb{K}_0 = -\mathbb{K} \mathbb{t}_3 + (\mathbb{t}_0^T)^{-1} \mathbb{k}_{30}$$

$$\Delta \mathbb{p}^T = \{ \Delta H/P_y \;\; \Delta P/P_y \;\; \Delta M_c/(P_y L) \}$$

P_y : yield axial load and $\mathbb{k}_1$, $\mathbb{k}_2$, $\mathbb{k}_3$, $\mathbb{k}_{30}$ are listed in Table 3.

Plastic Zone   By expressing the length by Z_B ($=\beta L$) where the maximum stress of the section are greater than σ_0, defined in Fig.2, we obtain

$$\beta = \pm \Big\{ -\frac{\sigma_0}{\sigma_y} + \frac{P}{P_y} \pm \Big(\frac{M_c}{P_y L} + \frac{H}{P_y} + \frac{P}{P_y}\phi_P \Big) \lambda \Big\} \Big/ \Big\{ \Big(\frac{H}{P_y} + \frac{P}{P_y}\phi_P \Big) \lambda \Big\} \tag{11}$$

where λ: slenderness ratio, σ_y: yield stress. In Eq.(11) δ_{He} and ϕ_e are neglected as a small value comparing with δ_{HP} and ϕ_P respectively.

EQUATIONS OF MOTION

The equations of motion of a steel frame is obtained by basing on the following assumptions:

a) The steel frame is a single-bay, multi-story and rigid-beam frame.
b) The mass of the structure is concentrated at the floor levels, and uniformly distributed along the beams.
c) Damping resistance is not considered.

The equations of motion of the i-th floor with the lower half-columns of (i+1)-th story and the upper half-columns of i-th story are expressed, using the notations defined in Fig.6, in the following form

$$\begin{aligned} & -m_i(\ddot{u}_i + \ddot{u}_g) + {}_1H_{i+1} + {}_2H_{i+1} - {}_1\overline{H}_i - {}_2\overline{H}_i + (-{}_1P_{i+1} - {}_2P_{i+1} + {}_1\overline{P}_i + {}_2\overline{P}_i)\,\theta_i = 0 \\ & -m_i(\ddot{w}_i + \ddot{w}_g + g) - {}_1P_{i+1} - {}_2P_{i+1} + {}_1\overline{P}_i + {}_2\overline{P}_i + (-{}_1H_{i+1} - {}_2H_{i+1} + {}_1\overline{H}_i + {}_2\overline{H}_i)\,\theta_i = 0 \\ & -I_i\ddot{\theta}_i + {}_1H_{i+1}\{L_{i+1} + \tfrac{1}{2}d_i - {}_1(\delta_V)_{i+1}\} + {}_2H_{i+1}\{L_{i+1} + \tfrac{1}{2}d_i - {}_2(\delta_V)_{i+1}\} \\ & \quad + {}_1\overline{H}_i\{L_i + \tfrac{1}{2}d_i - {}_1(\overline{\delta}_V)_i\} + {}_2\overline{H}_i\{L_i + \tfrac{1}{2}d_i - {}_2(\overline{\delta}_V)_i\} \\ & \quad + {}_1P_{i+1}\{-\tfrac{1}{2}B + {}_1(\delta_H)_{i+1}\} + {}_2P_{i+1}\{\tfrac{1}{2}B + {}_2(\delta_H)_{i+1}\} \\ & \quad + {}_1\overline{P}_i\{\tfrac{1}{2}B + {}_1(\overline{\delta}_H)_i\} + {}_2\overline{P}_i\{-\tfrac{1}{2}B + {}_2(\overline{\delta}_H)_i\} \\ & \quad + {}_1M_{i+1} + {}_2M_{i+1} + {}_1\overline{M}_i + {}_2\overline{M}_i = 0 \end{aligned} \tag{12}$$

where $\Delta\ddot{u}_g$, $\Delta\ddot{w}_g$: the horizontal and vertical incremental ground acceleration, respectively, m_i : mass of i-th floor, I_i : rotatory inertia of i-th floor, g : acceleration of gravity. By using the conditions of continuity of the deformations at the center of columns, shown in Fig.7, and the conditions of equilibrium of the forces at the center of columns, shown in Fig.8, also by applying Eq.(10) to each half-column of the frame, the motion equations of the frame are now represented in the following form

$$\mathbb{M}_m \Delta\ddot{\mathbb{D}} + \mathbb{K}_t \Delta\mathbb{D} = -\mathbb{M}_m \Delta\mathbb{A}_g - \mathbb{K}_{ot} \tag{13}$$

where $\Delta\mathbb{D}^T = \{ \Delta u_1\ \Delta u_2 \cdots \Delta u_i \cdots \Delta u_n\ \Delta w_1\ \Delta w_2 \cdots \Delta w_i \cdots \Delta w_n\ \Delta\theta_1\ \Delta\theta_2 \cdots \Delta\theta_i \cdots \Delta\theta_n \}$

$\mathbb{M}_m$: mass matrix

$\mathbb{K}_t$: tangent stiffness matrix of the frame

$\mathbb{K}_{ot}$: complemental force in case of bi-linear $\Delta\sigma$-$\Delta\varepsilon$ relation

$\Delta\mathbb{A}_g$: incremental acceleration of the base

NUMERICAL EXAMPLES

The response of a 2-story steel frame and a 5-story steel frame subjected to strong ground shakings are calculated using Wilson θ Method ($\theta = 2.0$). The properties of the frames are given in Table 4. The N-S component of El Centro,40 earthquake is chosen as a accelerogram, and the acceleration ordinates of the El Centro record are multiplied by a factor of α (α=1.0,2.0). In these calculation to decrease the computation time, Eq.(14) is used.

$$\Delta\phi = \frac{2}{2-\beta}\frac{1}{L}\Delta\delta_H \tag{14}$$

Eq.(14) is obtained by neglecting $\Delta\delta_{HE}$ and $\Delta\phi_e$, as a small value comparing with $\Delta\delta_{HP}$ and $\Delta\phi_P$, respectively.

The numerical results are shown in Fig.(9) and Fig.(10). Fig.(9) shows horizontal displacement response of the top floor and Fig.(10) shows the axial deformations of the two half-columns of the same story. From these results it is noticed that the axial deformation of columns, which is mostly due to the plastic strain, increases when the columns deform plastically. In the 5-story frame , the difference of the axial deformation between the two half-columns of the same story also increases when the columns deform plastically, and the horizontal displacement of the frame gradually increases in one direction and tends not to return to the zero displacement position. This accumulation of the horizontal displacement in one direction is corresponding to the difference of the axial deformation between the two columns of the same story.

No general conclusion should be drawn from these limited numerical results, nevertheless it is indicated that the plastic axial deformation of columns effects considerably to the horizontal displacement response and P-Δ effect of gravity even in the response of the 5-story steel frame under strong ground motion, and also it is clarified that the analysis presented in this paper is usefull to calculate the two-dimensional inelastic seismic response of steel frames under strong ground motion.

REFERENCES

1) P.C.Jennings and R.Husid, "Collapse of Yielding Structures during Earthquakes", J. Eng. Mech. Div., ASCE, Vol.94, No.EM5, Oct.,1968, pp.1045-1065

2) S.C.Goel, "P-Δ and Axial Column Deformation in Aseismic Frames", J. Struct. Div., ASCE, Vol.95, No.ST8, Aug.,1969, pp.1693-1711

3) C.K.Sun, G.V.Berg and R.D.Hanson, "Gravity effect on Single-Degree Inelastic System", J. Eng. Mech. Div., ASCE, Vol.99, No.EM1, Feb.,1973, pp.183-200

4) B.Kato, H.Akiyama and K.Inoue, "Post-Buckling Behavior of Short Steel Columns", Trans. of AIJ, No.229, March,1975, pp.67-76 (in Japanese)

5) S.Tani and S.Soda, "Vertical Load Effect on Structural Dynamics", Proc. 6-WCEE, Vol II, 1977, pp.1028-1033

Table 1

$$\mathbf{e}_p^T = \frac{1}{\beta}\begin{Bmatrix} \psi_p - \frac{2}{2-\beta}\frac{X}{L} \\ -1 \\ 0 \end{Bmatrix}, \quad \mathbf{e}_1^T = \begin{Bmatrix} \psi_{1e} - \frac{6}{1+2\beta}\frac{X}{L} \\ -1 \\ \psi_{2e} + \frac{2(1-\beta)}{1+2\beta}\frac{X}{L} \end{Bmatrix}$$

$$\mathbf{e}_2^T = \begin{Bmatrix} \psi_{1e} - \frac{6}{1+2\beta}\frac{X}{L} + \frac{12}{(1+2\beta)(1-\beta)^2}\frac{Z-Z_B}{L}\frac{X}{L} \\ -1 \\ \psi_{2e} + \frac{2(1-\beta)}{1+2\beta}\frac{X}{L} - \frac{6}{(1+2\beta)(1-\beta)^2}\frac{Z-Z_B}{L}\frac{X}{L} \end{Bmatrix}$$

where

$$\psi_p = T_{10} + T_{11}\frac{\delta_{He}}{L} + T_{12}\frac{\delta_{Ve}}{L} + T_{13}\frac{\delta_{Hp}}{L}$$

$$\psi_{1e} = T_{20} + T_{21}\frac{\delta_{He}}{L} + T_{22}\frac{\delta_{Ve}}{L} + T_{23}\frac{\delta_{Hp}}{L}$$

$$\psi_{2e} = T_{30} + T_{31}\frac{\delta_{He}}{L} + T_{32}\frac{\delta_{Ve}}{L} + T_{33}\frac{\delta_{Hp}}{L}$$

$$T_{10} = 2R_1 - \frac{2}{3}\frac{1}{\beta(2-\beta)}R_2, \quad T_{11} = \frac{2(1+2\beta-\beta^2)}{(1+2\beta)(2-\beta)}$$

$$T_{12} = \frac{2\beta^2(1-\beta)}{3(1+2\beta)(2-\beta)}, \quad T_{13} = \frac{4}{3}\frac{3-2\beta}{(2-\beta)^2}$$

$$T_{20} = 2R_1 - \frac{2}{1+2\beta}R_2, \quad T_{21} = \frac{6}{5}\frac{1+4\beta+5\beta^2}{(1+2\beta)^2}$$

$$T_{22} = -\frac{1}{10}\frac{(1-\beta)(1+5\beta+10\beta^2)}{(1+2\beta)^2}, \quad T_{23} = T_{11}$$

$$T_{30} = \frac{2}{3}\frac{1-\beta}{1+2\beta}R_2, \quad T_{31} = T_{22}$$

$$T_{32} = \frac{1}{15}\frac{(1-\beta)(2+10\beta+15\beta^2)}{(1+2\beta)^2}, \quad T_{33} = T_{12}$$

$$R_1 = \sum_j \frac{1}{2L-Z_j}\delta_{HPj}, \quad R_2 = \sum_j \frac{Z_j^2}{(2L-Z_j)L^2}\delta_{HPj}$$

Z_j : j-th value of Z_B
δ_{HPj} : increment of δ_{HP} between j-th value and (j+1)-th value of Z_B

Table 2

$$\xi_1 = \frac{\lambda}{4}\beta(2-\beta)\frac{1-e}{e}\left\{\frac{6}{\lambda}\frac{1}{1+2\beta}A_1 - \psi_{1e}A_2\right\}$$

$$\xi_2 = \frac{\lambda}{4}\beta(2-\beta)\frac{1-e}{e}A_2$$

$$\xi_3 = \frac{\lambda}{4}\beta(2-\beta)\frac{1-e}{e}\left\{-\frac{2}{\lambda}\frac{1-\beta}{1+2\beta}A_1 - \psi_{2e}A_2\right\}$$

$$\xi_4 = \frac{\lambda}{4}\beta(2-\beta)\frac{1-e}{e}(-A_3+A_4)$$

$$\eta_1 = \frac{1-e}{e}\left[\left\{\frac{3}{2}\frac{\beta(2-\beta)}{1+2\beta}\psi_p - \frac{\beta}{2}\psi_{1e}\right\}A_1 + \left\{-\frac{\lambda}{4}\beta(2-\beta)\psi_p\psi_{1e} + \frac{3}{\lambda}\frac{\beta}{1+2\beta}\right\}A_2\right]$$

$$\eta_2 = \frac{1-e}{e}\left\{\frac{\beta}{2}A_1 + \frac{\lambda}{4}\beta(2-\beta)\psi_p A_2\right\}$$

$$\eta_3 = \frac{1-e}{e}\left[\left\{-\frac{1}{2}\frac{\beta(2-\beta)(1-\beta)}{1+2\beta}\psi_p - \frac{\beta}{2}\psi_{2e}\right\}A_1 + \left\{-\frac{\lambda}{4}\beta(2-\beta)\psi_p\psi_{2e} - \frac{1}{\lambda}\frac{\beta(1-\beta)}{1+2\beta}\right\}A_2\right]$$

$$\eta_4 = \frac{1-e}{e}\left\{\frac{\lambda}{4}\beta(2-\beta)\psi_p(-A_3+A_4) - \frac{\beta}{2}(A_3+A_4)\right\}$$

where

$$A_1 = {}_pm_3 + {}_mm_3 \quad A_2 = {}_pm_3 - {}_mm_3$$

$$A_3 = {}_pm_3\,{}_pm_1({}_pm_2\,\sigma_e + {}_p\sigma)\frac{1}{E}$$

$$A_4 = {}_mm_3\,{}_mm_1({}_mm_2\,\sigma_e + {}_m\sigma)\frac{1}{E}$$

${}_pm_1, {}_pm_2, {}_pm_3, {}_p\sigma$: m_1, m_2, m_3, σ where $X = i_Y$

${}_mm_1, {}_mm_2, {}_mm_3, {}_m\sigma$: m_1, m_2, m_3, σ where $X = -i_Y$

Table 3

$$k_1 = \begin{bmatrix} \frac{1}{\varepsilon_y}\psi_{1e}^2 + \frac{12}{\varepsilon_y\lambda^2}\frac{1+3\beta}{(1+2\beta)^2(1-\beta)} & -\frac{1}{\varepsilon_y}\psi_{1e} & \frac{1}{\varepsilon_y}\psi_{1e}\psi_{2e} - \frac{6}{\varepsilon_y\lambda^2}\frac{1+3\beta}{(1+2\beta)^2(1-\beta)} \\ & \frac{1}{\varepsilon_y} & -\frac{1}{\varepsilon_y}\psi_{2e} \\ \text{Symmetry} & & \frac{1}{\varepsilon_y}\psi_{2e}^2 + \frac{4}{\varepsilon_y\lambda^2}\frac{1+3\beta-\beta^3}{(1+2\beta)^2(1-\beta)} \end{bmatrix}$$

$$k_2 = \begin{bmatrix} \frac{1}{\varepsilon_y}\frac{1-e}{e}\left[\left\{\frac{18}{\lambda^2}\frac{\beta}{(1+2\beta)} + \frac{\beta}{2}\psi_{1e}^2\right\}A_1 - \frac{6}{\lambda}\frac{\beta}{1+2\beta}\psi_{1e}A_2\right] & \frac{1}{\varepsilon_y}\frac{1-e}{e}\left(-\frac{\beta}{2}\psi_{1e}A_1 + \frac{3}{\lambda}\frac{\beta}{1+2\beta}A_2\right) & \frac{1}{\varepsilon_y}\frac{1-e}{e}\left[\left\{\frac{\beta}{2}\psi_{1e}\psi_{2e} - \frac{6}{\lambda^2}\frac{\beta(1-\beta)}{(1+2\beta)^2}\right\}A_1 + \left\{\frac{1}{\lambda}\frac{\beta(1-\beta)}{1+2\beta}\psi_{1e} - \frac{3}{\lambda}\frac{\beta}{1+2\beta}\psi_{2e}\right\}A_2\right] \\ & \frac{1}{\varepsilon_y}\frac{1-e}{e}\frac{\beta}{2}A_1 & \frac{1}{\varepsilon_y}\frac{1-e}{e}\left\{-\frac{\beta}{2}\psi_{2e}A_1 - \frac{1}{\lambda}\frac{\beta(1-\beta)}{1+2\beta}A_2\right\} \\ \text{Symmetry} & & \frac{1}{\varepsilon_y}\frac{1-e}{e}\left[\left\{\frac{2}{\lambda^2}\frac{\beta(1-\beta)^2}{(1+2\beta)^2} + \frac{\beta}{2}\psi_{2e}^2\right\}A_1 + \frac{2}{\lambda}\frac{\beta(1-\beta)}{(1+2\beta)}\psi_{2e}A_2\right] \end{bmatrix}$$

$$k_3 = \begin{bmatrix} p(-\gamma_{21} - 2\gamma_{11}\xi_1 - \gamma_{13}\xi_1^2) & p(-\gamma_{13}\xi_1\xi_2 - \gamma_{23}\xi_2) & p(-\gamma_{22} - \gamma_{12}\xi_1 - \gamma_{23}\xi_3 - \gamma_{13}\xi_1\xi_3) \\ & p(-\gamma_{13}\xi_2^2) & p(-\gamma_{12}\xi_2 - \gamma_{13}\xi_2\xi_3) \\ \text{Symmetry} & & p(-\gamma_{32} - 2\gamma_{12}\xi_3 - \gamma_{13}\xi_3^2) \end{bmatrix}$$

$$k_{30} = \begin{Bmatrix} p(-\gamma_{23}\xi_4 - \gamma_{13}\xi_1\xi_4) \\ p(-\gamma_{13}\xi_2\xi_4) \\ p(-\gamma_{33}\xi_4 - \gamma_{13}\xi_3\xi_4) \end{Bmatrix}$$

where $p = \frac{P}{P_y}$, ε_y = yield strain

Table 4

5-stories , T=0.74 seconds							
stories	λ	i_Y (cm)	A (cm²)	B (cm)	d (cm)	m (t·sec²/cm)	e
3-5	13.9	12.9	120	800	50	0.04	0.02
1-2	10.8	16.7	250	800	60	0.04	0.02

2-stories , T=0.44 seconds							
stories	λ	i_Y (cm)	A (cm²)	B (cm)	d (cm)	m (t·sec²/cm)	e
1-2	13.9	12.9	120	800	50	0.04	0.02

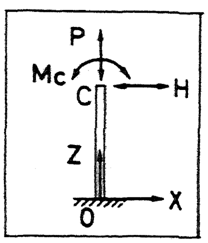


Fig.1 Loading Condition

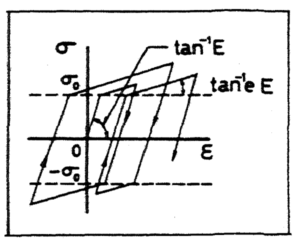


Fig.2 Hysteretic Stress-Strain Relation

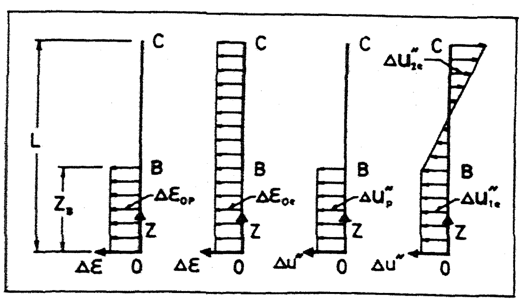


Fig.3 Incremental Curvature and Average Strain Distribution

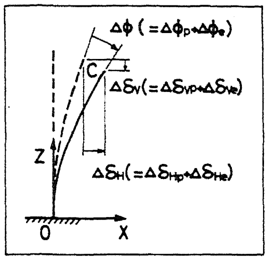


Fig.4
Incremental Displacement and Rotation at the End

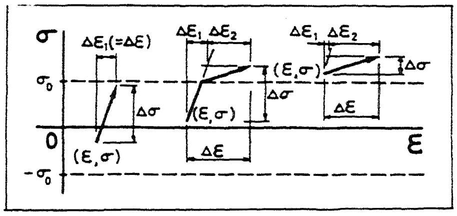


Fig.5
Incremental Stress-Strain Relation

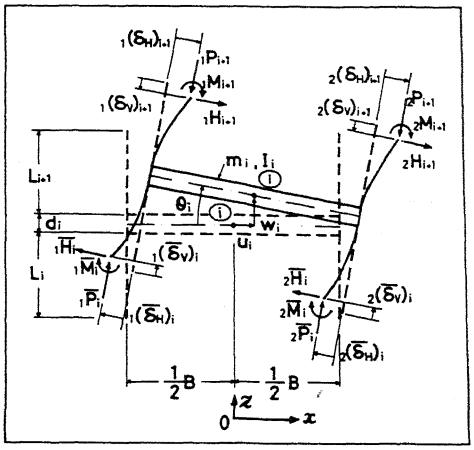


Fig.6
Loading Condition of i-th Floor and Half-Columns

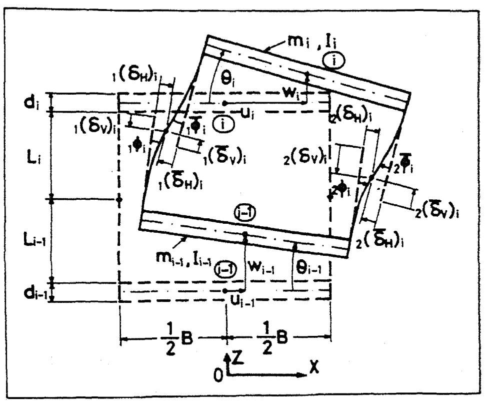


Fig.7 Deformation of Half-Columns of i-th Story

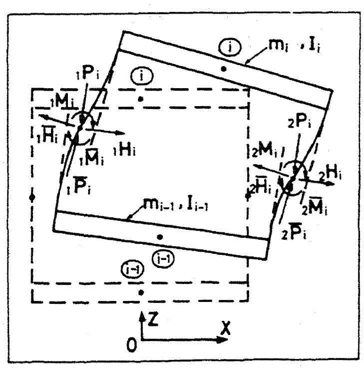
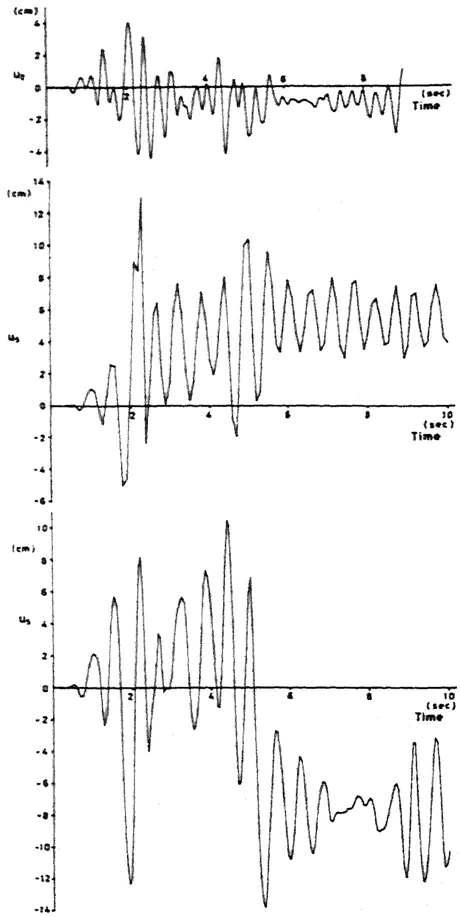


Fig.8 Forces at the Center of Columns of i-th Story

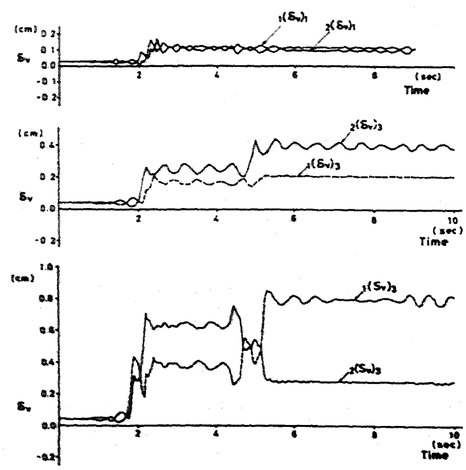


a. 2-Story Frame, $\alpha=2.0$

b. 5-Story Frame, $\alpha=1.0$

c. 5-Story Frame, $\alpha=2.0$

Fig.9 Horizontal Displacement Time History



a. 2-Story Frame, $\alpha=2.0$

b. 5-Story Frame, $\alpha=1.0$

c. 5-Story Frame, $\alpha=2.0$

Fig.10 Axial Deformation Time History