EQUIVALENT INTERACTION MODAL DAMPING

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SUMMARY

Half-space lumped spring and mass representation of soil structure interaction systems often results in coupled modes of vibration. The governing equations of motion can be solved using direct integration techniques, however, the engineer then loeses the physical insight that is offered by normal mode methods of analysis. Herein, a procedure is described that may be used to uncouple the damping forces of an interaction system. Equivalent modal damping values are obtained such that each exact and approximate modal transfer function matches at the corresponding frequency.

INTRODUCTION

In the analysis of the dynamics of soil-structure interaction, the effect of the foundation can be introduced into the system by the half space impedance approach; the real and imaginary parts of the impedance functions are interpreted as being the stiffness and damping coefficients of discrete springs representing the soil medium; in most practical analyses these coefficients are assumed to be independent of frequency (2). The normal mode methods of structural dynamics cannot be applied to a typical interaction problem because the damping of the foundation will be relatively high compared to that of the structure and the damping forces will tend to cause coupling between the undamped free-vibration modes.

Nevertheless, the normal mode method is desirable because of the physical insight that the engineer has in using modal analysis. Furthermore, response computations by the response spectrum technique is possible only if the damping in each mode is known.

There are various procedures to obtain the approximate modal damping values of the interaction system. The work done herein is an extension of Tsai's technique (3) to structures that may have up to 6 degrees-of-freedom (DOF) per node, three translations and three rotations.

FORMULATION

The general form of the equations of motion of an

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interaction system may be written as

$$M\ddot{x} + C\dot{x} + Kx = Mr\ddot{u} \qquad (1)$$

where M, C and K are generalized mass, damping and stiffness matrices x is the generalized displacement vector, $\ddot{u}$ is the prescribed base acceleration, and r is a vector whose i th entry is a one or a zero depending on the corresponding degree-of-freedom is in the direction of the ground motion or not. In Eq.(1) the mass and the stiffness matrices can be assembled directly by discrete element properties. Unfortunately, this is not the case for the damping matrix. The information available about the damping characteristics of the soil-structure system consists of the fixed base structure modal damping values (4) and the foundation damping coefficients, which are generally obtained from the elastic or visco-elastic half space theory. However, it has been shown that the problem of generating the damping matrix can be overcome by assembling it in a dummy coordinate system. The derivation of the necessary transformation matrix A, which is composed of the mode shapes of the fixed base structure and the coordinates of the discrete mass points, can be found in (1). Letting

$$x=Ay \qquad (2)$$

substituting Eq.(2) into Eq.(1), premultiplying by A^T then gives

$$A^T MA\ddot{y}+A^T CA\dot{y}+A^T KAy=-A^T Mr\ddot{u} \qquad (3)$$

where, it may be shown that

$$A^T CA=\mathrm{diag}(2\xi_1\omega_1,\ldots,cx,cy,cz,cxx,cyy,czz) \qquad (3a)$$

and ξ_i is the i th fixed-base structural mode percent critical damping, ω_i is the corresponding frequency and cx, cy etc are soil medium damping coefficients.

Solving the undamped eigenvalue problem defined in Eq. (3) for the mode shapes Q and the natural frequencies λ_i and letting

$$y=Qz \;; \qquad \chi=Q^T A^T Mr \qquad (4)$$

Eq.(3) can be put into the form

$$I\ddot{z}+C^0\dot{z}+\text{diag}(\lambda^2)z=-\chi\ddot{u} \tag{5}$$

In general the damping matrix of Eq. (5) will have off-diagonal terms causing coupling of modes. In this case the response can be obtained by direct integration. However, modal analysis is more attractive from the point of view of practical engineering. To achieve this end, define the following equivalent system

$$I\ddot{z}+\text{diag}(2\eta\lambda)\dot{z}+\text{diag}(\lambda^2)z=-\chi\ddot{u} \tag{6}$$

The aim is to obtain η_i over the frequency range of interest such that the solution of Eq.(6) will produce an approximate solution to Eq.(5). For systems where the superstructure has only one translational DOF per node point Tsai (3) has developed a method for computing η_i. The principle is to match the rigorous and normal mode solutions of the transfer function at a critical structural location simultaneously at the natural frequencies of interest. The structures considered herein are more general; they may be irregularly shaped and may have up to six DOF per mass point. Therefore, to identify the sensitive structural location may be a complex task. Hence, the procedure followed here is to compute the equivalent modal damping values such that the magnitude of each exact and approximate modal transfer function matches at the corresponding frequency. Letting

$$\ddot{u}=e^{ipt} \quad ; \quad z=H(p)e^{ipt} \tag{7}$$

the exact and approximate transfer functions $H^e(p)$ and $H^a(p)$ respectively may be obtained by substituting Eq.(7) into Eq's.(5) and (6) as

$$H^e(p)=-\left[-p^2I+ipC^0+\text{diag}(\lambda^2)\right]^{-1}\chi \tag{8}$$

$$H^a(p)=-\left[-p^2I+ip\,\text{diag}(2\eta\lambda)+\text{diag}(\lambda^2)\right]^{-1}\chi \tag{9}$$

For the k th mode the maximum value of $H^a(p)$ occurs at $p=\lambda_k$

$$|H_k^a(\lambda_k)| = |\chi_k|/2\eta_k\lambda_k^2 \tag{10}$$

Going back to Eq.(8), setting $p = \lambda_k$, evaluating the inversion and matrix multiplication gives the corresponding value of the k th exact modal transfer function. Equating this to the expression given in Eq. (10) and solving for η_k

$$\eta_k = |\chi_k| \;/ 2\, \lambda_k^2 \,|H_k^e(\lambda_k)| \tag{11}$$

then yields the equivalent interaction modal damping. Note that this procedure will produce the exact modal damping values if C^0 is diagonal.

EXAMPLES

Two models are analyzed. The first represents a simplified model of a typical containment structure and does not have any eccentricities. The fixed base dynamic properties of the structure are shown in Table I. The discrete soil stiffness and damping constants were computed using the formulae given in (2). Only in-plane rocking and translational soil springs were considered. Two cases; corresponding to soil medium shear wave velocity V_s of 1000 ft/sec. and 2000 ft/sec, were analyzed. The comparisons of the instructure response spectra of the top mass obtained by direct integration of the coupled equations of motion and by modal analysis, in which the equivalent modal damping values were employed, are depicted in Fig's 1 and 2.

The second model investigated in a hypothetical structure with large eccentricities as shown in Fig. 3; it is identical to the example problem of reference (1). The comparison of the instructure response spectra is plotted in Fig. 4.

As may be observed, modal analysis using equivalent interaction damping constants generally underestimates the response, however, the difference between the exact and approximate solutions is about 10%.

SUMMARY AND CONCLUSIONS

A technique has been presented for computing equivalent soil structure interaction modal damping values. The principle used is to match the magnitude of the exact and approximate modal transfer function at the corresponding modal frequency.

Example problems solved indicate that although responses may be underestimated, the differences are within acceptable engineering practice.

REFERENCES

1. Ibrahim, A.M. and Hadjian, A.H. "The Composite Damping Matrix for Three Dimensional Soil-structure Systems," 2nd ASCE Speciality conf. On Structural Design of Nuclear Plant Facilities, Dec. 1975, New Orleans, USA.

2. Richart, Jr., F.E., Hall, Jr., J. R. and Woods, R.D. Vibrations of Soils and Foundations, Prentice-Hall, Inc. New Jersey, 1970.

3. Tsai, N.C. "Modal Damping for Soil-Structure Interaction" Journal of Engineering Mechanics Division, ASCE, 100 (1974) 323-41.

4. USNRC Regulatory Guide 1.61 "Damping Values for Seismic Design of Nuclear Power Plants." October, 1973.

TABLE I. STRUCTURE 1 FIXED BASE DYNAMIC PROPERTIES

Mode N°	Frequency (cps)	Percent Damping	Mass 1	Mass 2	Mass 3
1	3.75	2	0.053	0.028	0.009
2	21.13	2	-0.039	0.027	0.028
3	52.64	2	0.017	-0.027	0.038

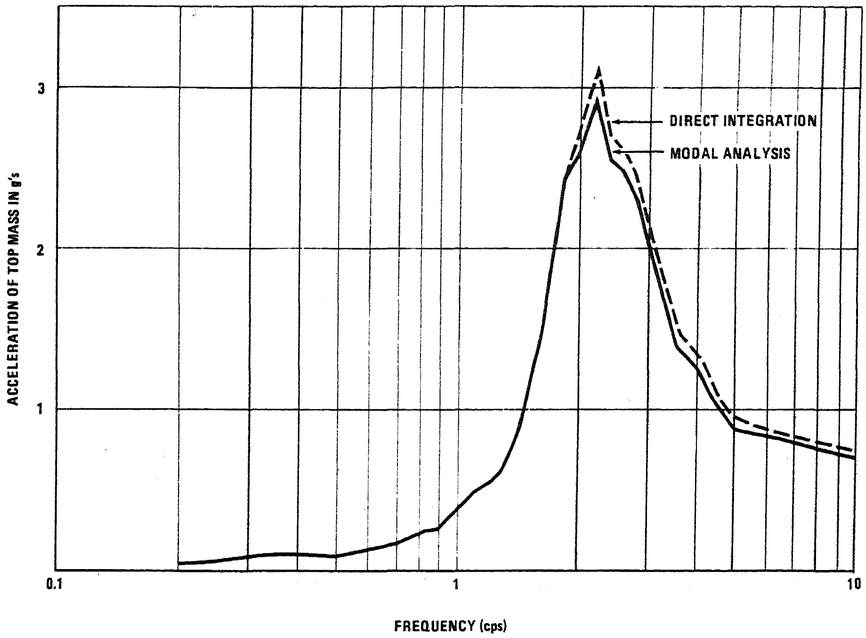

FIGURE 1. TOP MASS ACCELERATION RESPONSE SPECTRA, V_s = 1000, 2% DAMPING (MODEL I)

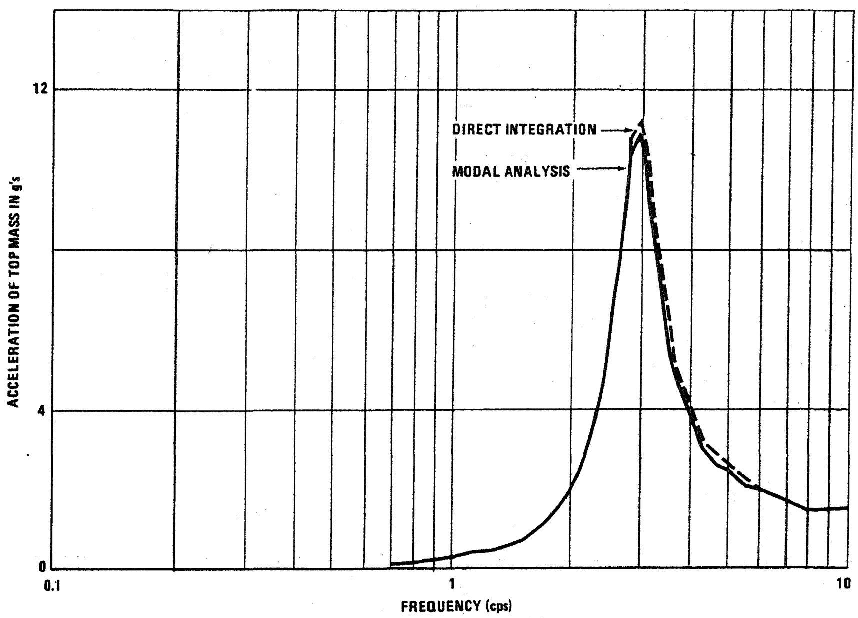

FIGURE 2. TOP MASS ACCELERATION RESPONSE SPECTRA, V_s = 2000, 2% DAMPING (MODEL I)

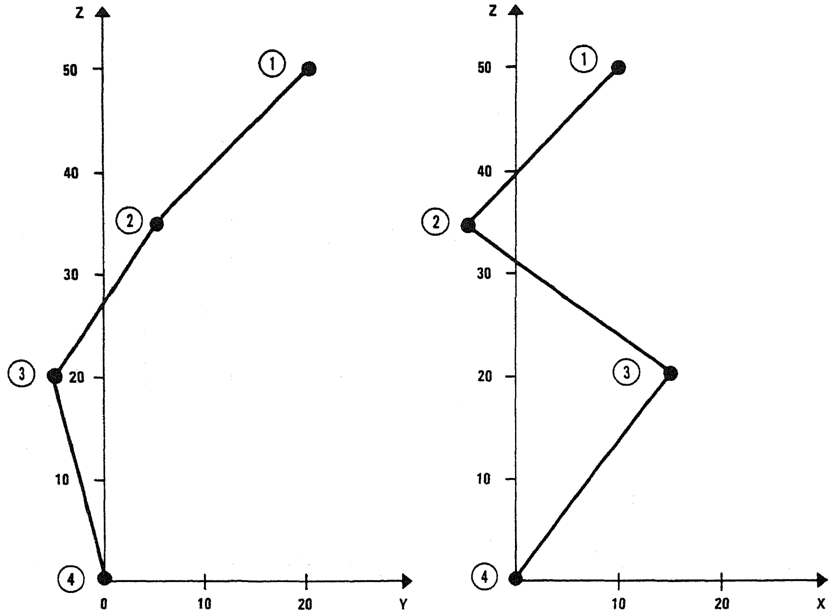

FIGURE 3. MODEL II ECCENTRICITIES

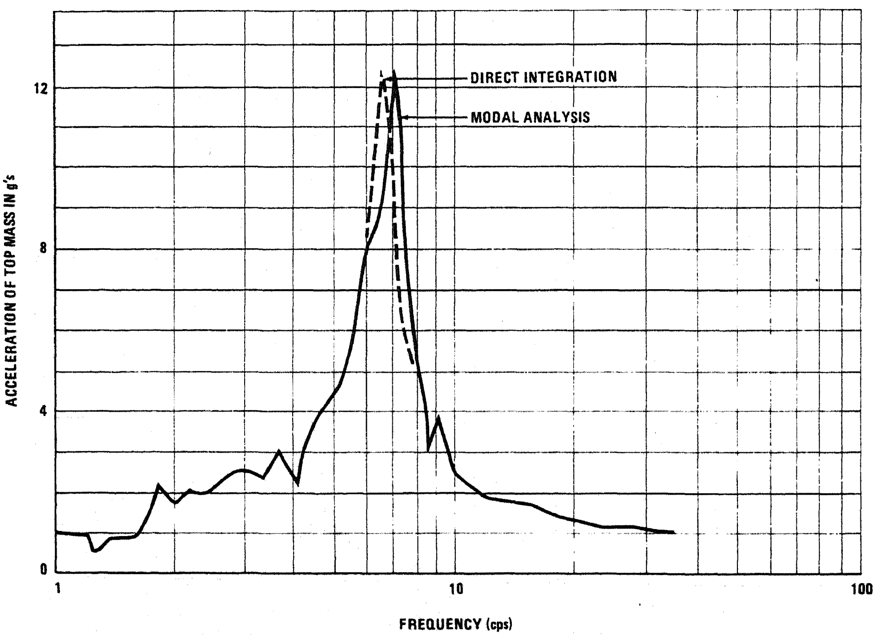

FIGURE 4. TOP MASS ACCELERATION RESPONSE SPECTRA, 1% DAMPING (MODEL II)