ANALYSIS OF SEISMIC RESPONSE OF CYLINDRICAL SHELLS WITH IMPERFECT CIRCULAR SECTION CONTAINING LIQUID

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SUMMARY

The seismic response of thin elastic cantilevered cylindrical shells partially filled with liquid related to horizontal earthquake excitations is studied. The circumferential section of the shell is not a perfect circle and the radius is represented by a Fourier cosine series. Displacements of a shell are expanded into the series with respect to the axial and circumferential directions based on the Rayleigh-Ritz method. The pressure of the internal liquid of a shell is divided into the convective pressure and impulsive pressure. The impulsive pressure is represented as the sum of pressures induced by rigid motion and elastic deformation of the shell. Numerical computations show the effects of the imperfect circular section to the seismic response of a shell.

1. INTRODUCTION

The analysis of seismic response of large cylindrical liquid storage tanks constructed in recent years is an ever-increasing problem. The methods of analysis frequently used assume the tank to be rigid under earthquake excitations and evaluate the dynamic responses of internal liquid to the tank (1). In the method proposed by Housner, the pressure of the internal liquid is divided into the convective and impulsive pressures. In the case of the analysis of the tank as a thin elastic circular cylindrical shell, this method is limitted in actual application because of the assumption that the tank is rigid. On the other hand, there are methods which assume the tank to be an elastic cantilevered beam due to the flexibility of tank wall (2). If the tank is considered a thin elastic shell, the flexibility of the tank is more effectively taken into the analysis. Therefor, the effects of flexibility of the tank, as an elastic shell-liquid system, to the seismic responses are also studied (ref. 3,4).

The objective herein is to show how the imperfect circular section of an elastic shell effects the dynamic response of the shell-liquid system. The case in which the circumferential section of a shell is not a perfect circle frequently becomes a problem in the production of large sized shells.

The pressure of internal liquid is separated into the convective pressure and impulsive pressure. Since the frequencies of the first few dominant modes of liquid sloshing are usually much smaller than the frequincies of the shell-liquid system, the elastic deformations of the shell are neglected for the convective pressure. The impulsive pressure is represented as the sum of pressures induced by the rigid motion and elastic deformation of the shell. Genaralized forces given by these pressure are obtained by the modal analysis method under earthquake excitations.

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2. VIBRATIONAL MODE OF SHELL

The coordinate system of a free-fixed cylindrical shell to be investigated is as shown in Fig.1. The shell is thin and elastic and contains liquid. The radius of the shell is represented by a Fourier cosine series because of the imperfect circular section. The fundamental frequency equation is derived based on the Rayleigh-Ritz method substituting the assumed displacements in the Lagrange Equation. The Lagrange Equation for the free vibration of shell-liquid system is given by

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_{r}}\right) - \frac{\partial T}{\partial q_{r}} + \frac{\partial S}{\partial q_{r}} = Q_{Aqr} \tag{2.1}$$

where T and S are the kinematic and strain energy of the shell, respectively. Q_{Aqr} is the generalized force resulting from the internal liquid pressure of the shell. q_r is the generalized coordinate and $(\dot{\ })$ is the derivative with respect to time. The deflection of a shell is assumed to be small and so the linear shell theory is used.

Assumed Displacement Modes of Shell. The shell displacements of the axial, circumferential and radial directions are defined in Fig. 1 as u, v and w, respectively. Using the method of separation of variables, the displacements can be represented by a series of functions with reference to flexural vibration modes of a cantilevered beam in generating line direction. With respect to the circumferential direction, the displacements are represented by a Fourier series. The resulting displacements are expressed as

$$\begin{aligned} u &= \sum_{j=1}^{M}\sum_{i=1}^{N} \cos j\theta \cdot f_i'(x) C_{ji}(t) \\ v &= \sum_{j=1}^{M}\sum_{i=1}^{N} \sin j\theta \cdot f_i(x) B_{ji}(t) \\ w &= \sum_{j=1}^{M}\sum_{i=1}^{N} \cos j\theta \cdot f_i(x) A_{ji}(t) \end{aligned} \tag{2.2.a-c}$$

where $C_{ji}(t)$, $B_{ji}(t)$, $A_{ji}(t)$ are independent variables and may be taken as the generalized coordinates in Eq. 2.1. If the problem is restricted to the shell whose cross section is a perfect circle, the displacements can be fixed to one circumferential wave mode because of the orthogonality with respect to the circumferential mode.

Energy Function. Kinematic energy T is given by

$$T = \frac{1}{2}\rho_s h_s \int_0^{2\pi}\int_0^{l} [\dot{u}^2 + \dot{v}^2 + \dot{w}^2] a \, d\theta dx \tag{2.3}$$

where ρ_s, h_s and l are mass density, thickness and length of the shell, respectively, and a is the radius of the shell with imperfect circular section. This radius is given by

$$a(\theta) = a_0 + \sum_{j=1}^{M} a_j \cos j\theta \tag{2.4}$$

where a_0 is the radius of the shell with perfect circular cross section and a_j, the coefficient of the initial imperfection. By the use of Flügge's shell theoly, strain energy S is expressed as

$$\begin{aligned} S = \frac{Eh_s}{2(1-\nu^2)}\int_0^{2\pi}\int_0^{l} \Big\{ & \left(\frac{\partial u}{\partial x}\right)^2 + \frac{1}{a^2}\left(\frac{\partial v}{\partial \theta} + w\right)^2 + \frac{2\nu}{a}\frac{\partial u}{\partial x}\left(\frac{\partial v}{\partial \theta} + w\right) + \frac{1-\nu}{2}\left(\frac{\partial v}{\partial x} + \frac{1}{a}\frac{\partial u}{\partial \theta}\right)^2 \\ & \frac{h_s^2}{12}\Big[\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + \frac{1}{a^4}\left(w + \frac{\partial^2 w}{\partial \theta^2}\right)^2 + \frac{1-\nu}{2a^2}\left(\frac{1}{a}\frac{\partial u}{\partial \theta} + \frac{\partial^2 w}{\partial x \partial \theta}\right)^2 + \frac{2\nu}{a^2}\frac{\partial^2 w}{\partial x^2}\left(\frac{\partial^2 w}{\partial \theta^2} - \frac{\partial v}{\partial \theta}\right) \\ & - \frac{2}{a}\frac{\partial u}{\partial x}\frac{\partial^2 w}{\partial x^2} + \frac{3(1-\nu)}{2a^2}\left(\frac{\partial v}{\partial x} - \frac{\partial^2 w}{\partial x \partial \theta}\right)^2\Big]\Big\} a \, d\theta dx \end{aligned} \tag{2.5}$$

where E and ν are modulus of elasticity and Poisson Ratio of the shell, respectively.

Characteristic Equation of Empty Shell. The characteristic equation of the empty circular cylindrical shell is obtained by neglecting the generalized force in the Lagrange Equation. Substitutions of the displacements in Eq. 2.2a—Eq.2.2c into the energy functions of Eq.2.1 and assumption of the harmonic vibration give the following equation

$$-\Delta_0\sum_{q=1}^{M}\sum_{j=1}^{M}\sum_{r=1}^{N}\sum_{i=1}^{N}\begin{bmatrix} m_{11qjri} & 0 & 0 \\ 0 & m_{22qjri} & 0 \\ 0 & 0 & m_{33qjri} \end{bmatrix}\begin{Bmatrix} C_{ji} \\ B_{ji} \\ A_{ji} \end{Bmatrix} + \sum_{q=1}^{M}\sum_{j=1}^{M}\sum_{r=1}^{N}\sum_{i=1}^{N}\begin{bmatrix} k_{11qjri} & k_{12qjri} & k_{13qjri} \\ k_{21qjri} & k_{22qjri} & k_{23qjri} \\ k_{31qjri} & k_{32qjri} & k_{33qjri} \end{bmatrix}\begin{Bmatrix} C_{ji} \\ B_{ji} \\ A_{ji} \end{Bmatrix}=0 \tag{2.6}$$

where $\Delta_0=(1-\nu^2)\cdot\rho_s\,\omega_0^2/E$, ω_0 is the circular frequency of the empty shell, m_{11qjri}, m_{22qjri} and m_{33qjri} are derived from T, and $k_{11qjri}\sim k_{33qjri}$ are derived from S. These are functions regarding the vibrational modes and radius of the shell as given by the Appendix.

Eq. 2.6 constitudes (3xMxN) related equations corresponding to the number of expansion of the vibrational modes and is expressed as the following matrix form

$$[[K]-\Delta_0[M_s]]\{Q\}=0 \tag{2.7}$$

where [K], [Ms] are the stiffness matrix and mass matrix of the shell, respectively and [Q] is given by

$$\{Q\}=[C_{11}\cdots C_{MN}B_{11}\cdots B_{MN}A_{11}\cdots A_{MN}]^T \tag{2.8}$$

3. GENERALIZED FORCE BY INTERNAL LIQUID

Velocity Potential for Free Vibration. Internal liquid of the shell is assumed to be nonviscous, irrotational and imcompressible. Based on the linear potential flow theory, the velocity potential induced by free vibration of the circular cylindrical shell-liquid system is obtained. This satisfies the boundary conditions on the shell wall and free surface of the liquid. If the effects of elastic deformation of the shell to the impulsive pressure are considered, the displacement of liquid on the free surface is neglected. Consequently, the solution of velocity potential is given (4) by

$$\phi=a_0\sum_{j=1}^{M}\cos j\theta\sum_{i=1}^{N}(\varphi_{Pji}+\varphi_{cji})\dot{A}_{ji} \tag{3.1}$$

where $\varphi_{Pji}(x,r)=\sum_{k=0}^{K}D_{jik}R_{jk}(r/a_0)\cos(\xi_k\frac{x}{a_0})$, $\varphi_{cji}(x,r)=-\sum_{s=0}^{S}\frac{E_{jis}}{\cosh(\lambda_{js}h/a_0)}$ $E_{jis}=\frac{2\lambda_{js}^2}{(\lambda_{js}^2-j^2)J_j(\lambda_{js})}\sum_{k}^{K}(-1)^k\frac{D_{jik}}{\xi_k^2+\lambda_{js}}$

$D_{jik}=\frac{2}{h(1+\delta_{0k})}\int_0^h f_i(x)\cos(\xi_k x/a_0)dx$, $R_{jk}(r/a_0)=I_j(\xi_k r/a_0)/[\xi_k\frac{\partial I_j(\xi_k)}{\partial r}]$,

and $\xi_k=k\pi a_0/h$, $\partial J_1(\lambda_{js})/\partial r=0$, r is coordinate of radial direction, h is height of liquid, φ_{pji} and φ_{cji} are particular and homogeneous solutions, respectively.

Added Mass on a Cylindrical Shell. Internal liquid pressure of the shell is given by the Bernoulli Equation

$$P=-\rho_L a_0\sum_{j=1}^{M}\cos j\theta\sum_{i=1}^{N}(\varphi_{pji}+\varphi_{cji})\ddot{A}_{ji} \tag{3.2}$$

where ρ_L is the mass density of the liquid. Expanding Eq. 3.2 into the series of $f_i(x)$ which is the mode of displacement w in the general line direction, the pressure at the shell wall is expressed by the added mass $m_{jii'}$

$$P = -\rho_s h_s \frac{1}{2} \sum_{j=1}^{M} \cos j\theta \sum_{i=1}^{N} \sum_{r=1}^{N} (m_{jri} + m_{jir}) f_r \ddot{A}_{ji} \tag{3.3}$$

where

$$m_{jri} = \rho_L a_0 / (\rho_s h_s) \left[\int_0^{\ell} \varphi_{pji} f_r \, dx + \int_0^{\ell} \varphi_{cji} f_r \, dx \right] \Big/ \int_0^{\ell} f_r^2 \, dx \tag{3.4}$$

Generalized Force. The generalized forces are derived from the work of external forces performed by the internal liquid of the shell. Since the liquid is assumed to be nonviscous, only the pressure in the radial direction is considered. By the definition that the virtual work of internal liquid pressure in a constrained coordinate is equivalent to the value in a generalized coordinate, the following equation is obtained

$$\delta W_{nc} = \int_0^{2\pi} \int_0^{\ell} P \delta w a \, d\theta \, dx = \sum_{q=1}^{M} \sum_{r=1}^{N} Q_{A_{qr}} \delta A_{qr} \tag{3.5}$$

where δW_{nc} is the virtual work and $\delta W = \sum_{q=1}^{M} \sum_{r=1}^{N} \partial w / \partial A_{qr} \, \delta A_{qr}$. Then, substituting Eq. 3.3 into Eq. 3.5, the generalized force $Q_{A_{qr}}$ is given by

$$Q_{A_{qr}} = -\rho_s h_s \frac{1}{2} \sum_{j=1}^{M} \sum_{i=1}^{N} (m_{jri} + m_{jir}) \int_0^{\ell} f_r^2 \, dx \int_0^{2\pi} \cos j\theta \cos q\theta \, a \, d\theta A_{ji} \tag{3.6}$$

4. EQUATION OF MOTION

Frequency Equation of Shell-Liquid System. Frequency equation of shell-liquid system is derived by Eq. 2.1, and is obtained in the same way as in the case of an empty shell, the following frequency equation is obtained.

$$[K] - \Delta [[M] + [M_L]] \{Q\} = 0 \tag{4.1}$$

where $\Delta = (1-\nu^2) \rho_s \omega^2 / E$, and ω is the circular frequency of the shell-liquid system. $[M_L]$ is the added mass matrix of internal liquid to the shell and is given by

$$[M_L] = \sum_{q=1}^{M} \sum_{j=1}^{M} \sum_{r=1}^{N} \sum_{i=1}^{N} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & m'_{33qjri} \end{bmatrix} \tag{4.2}$$

where $m'_{33qjri} = \frac{1}{2} (m_{jri} + m_{jir}) \int_0^{\ell} f_r^2 \, dx \int_0^{2\pi} \cos j\theta \cos q\theta \, a \, d\theta$

Solving Eq. 4.1 about Δ_{mn}, the characteristic vector $\{Q^{(mn)}\}$, the orthogonality between the fundamental modes is presented as

$$\{Q^{(mn)}\}[K]\{Q^{(m'n')}\} \begin{cases} = \{0\} & (mn \neq m'n') \\ = \Delta_{mn} [M_s^{(mn)} + M_L^{(mn)}] & (mn = m'n') \end{cases} \tag{4.3}$$

where $M_s^{(mn)}$ is the generalized mass of the elastic shell and $M_L^{(mn)}$ is the generalized added mass of impulsive pressure produced by the elastic displacement of the shell. They are given by

$$M_s^{(mn)} = \int_0^{2\pi} \int_0^{\ell} [u_{mn}^2(x,\theta) + v_{mn}^2(x,\theta) + w_{mn}^2(x,\theta)] a \, d\theta \, dx$$
$$M_L^{(mn)} = \int_0^{2\pi} \int_0^{\ell} m^{(mn)}(x,\theta) w_{mn}(x,\theta) a \, d\theta \, dx \tag{4.4.a-b}$$

where $u_{mn}(x,\theta)$, $v_{mn}(x,\theta)$, $w_{mn}(x,\theta)$ are the mn-th fundamental mode of shell-liquid system and $m^{(mn)}(x,\theta)$ is the equivalent added mass of the liquid about the mn-th fundamental mode of elastic displacement w. The equivalent added mass, $m^{(mn)}(x,\theta)$ is given by

$$m^{(mn)} = \frac{1}{2} \sum_{j=1}^{M} \sum_{i=1}^{N} \sum_{r=1}^{N} (m_{jri} + m_{jir}) f_r(x) \cos j\theta \, A_{ji}^{(mn)} \tag{4.5}$$

Equation of Forced Motion. The equation of forced motion is obtained by the Lagrange Equation which is given by

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\eta}_{qr}}\right)-\frac{\partial T}{\partial \eta_{qr}}+\frac{\partial S}{\partial \eta_{qr}}+\frac{\partial R}{\partial \dot{\eta}_{qr}}=N_{qr} \tag{4.6}$$

where R is the dissipation function of the shell dependent on the viscous damping, N_{qr} is the generalized force produced by liquid pressure and equivalent damping force, and η_{qr} is the orthogonal generalized coordinate.

The displacements of the shell, which is subjected to horizontal ground motion Z(t), are represented as follows

$$\begin{aligned} U&=\sum_{mn=1}^{M'\times N'} u_{mn}(x,\theta)\,\eta_{mn}(t)\\ V&=\sum_{mn=1}^{M'\times N'} v_{mn}(x,\theta)\,\eta_{mn}(t)-Z(t)\sin\theta\\ W&=\sum_{mn=1}^{M'\times N'} w_{mn}(x,\theta)\,\eta_{mn}(t)+Z(t)\cos\theta \end{aligned} \tag{4.7.a-c}$$

where $M' \leq M$, $N' \leq N$. The first term of the right hand side of the above equations are relative displacements of the shell to the ground.

The velocity potential which satisfies the boundary condition on the shell wall subjected to ground motion is expressed as

$$\Phi=\sum_{mn=1}^{M'\times N'}\Phi_{mn}(x,r,\theta)\,\dot{\eta}_{mn}(t)+\Phi_{10}(x,r,\theta)\,\dot{Z}(t) \tag{4.8}$$

where $\Phi_{mn}(x,r,\theta)$ and $\Phi_{10}(x,r,\theta)$ are the velocity potential corresponding to the elastic and rigid displacements of the shell, respectively. They are given (4) by

$$\begin{aligned} \Phi_{mn}&=\sum_{j=1}^{M}\cos j\theta\sum_{i=1}^{N}\varphi_{ji}(x,r)A_{ji}^{(mn)}\\ \Phi_{10}&=a_0\cos\theta\left[R_{10}(r/a_0)-\sum_{s=1}^{\ell}\frac{E_{10s}}{\cosh(\lambda_{1s}h/a_0)}J_1(\lambda_{1s}r/a_0)\cosh(\lambda_{1s}x/a_0)\right] \end{aligned} \tag{4.9.a,b}$$

Impulsive pressure of liquid is obtained from Eq. 4.8 as follows

$$P=\sum_{mn=1}^{M'\times N'}P_{mn}(x,r,t)+P_0(x,r,\theta,t) \tag{4.10}$$

where $P_{mn}=-\rho_L\Phi_{mn}\ddot{\eta}_{mn}$ and $P_{10}=-\rho_L\Phi_{10}\ddot{Z}$ corresponding to the impulsive pressures induced by elastic shell motion and rigid shell motion, respectively. Representing the liquid pressures on the shell wall by the equivalent added masses to the shell, Eq. 4.10 is expressed by

$$P=-\rho_s h_s\left[\sum_{mn=1}^{M'\times N'}m^{(mn)}(x,\theta)\,\ddot{\eta}_{mn}(t)+m^{(o)}(x,\theta)\,\ddot{Z}(t)\right] \tag{4.11}$$

where $m^{(o)}(x,\theta)=\rho_L a_0/\rho_s h_s\Phi_{10}$ is the equivalent added mass of P_{10} to the shell wall. The damping force produced by the liquid is assumed to be proportional to the equivalent added mass of the liquid and is given by

$$P_D=-2\alpha\rho_s h_s\sum_{mn=1}^{M'\times N'}m^{(mn)}(x,\theta)\,\dot{\eta}_{mn}(t) \tag{4.12}$$

where α is the viscous damping coefficient of the shell.

Using the above results, Eq. 4.6 can be written to express the qr-th uncoupled equation of forced motion of the liquid-shell system.

$$\ddot{\eta}_{qr}+2\xi_{qr}\omega_{qr}\dot{\eta}_{qr}+\omega_{qr}^2\eta_{qr}=-\beta_{qr}\ddot{Z} \tag{4.13}$$

where $\zeta_{qr}=\alpha/\omega_{qr}$ is the damping constant of the shell. β_{qr} is the participation factor of the r-th mode and is given by

$$\beta_{qr}=\frac{\Gamma_s^{(qr)}+\Gamma_L^{(qr)}}{M_s^{(qr)}+M_L^{(qr)}} \tag{4.14}$$

where $\Gamma_s^{(qr)}$ and $\Gamma_L^{(qr)}$ are the generalized loads of effective inertia force of the shell and the internal liquid, respectively, and are presented as

$$\begin{aligned}\Gamma_s^{(gr)} &= \int_0^{2\pi}\int_0^{\ell}\{-v_{gr}(x,\theta)+w_{gr}(x,\theta)\}a(\theta)d\theta dx \\ \Gamma_L^{(gr)} &= \int_0^{2\pi}\int_0^{\ell} m^{(o)}(x,\theta)\,w_{gr}(x,\theta)\,a(\theta)d\theta dx\end{aligned} \qquad (4.15.a,b)$$

Consequently, using the mode superposition method, the impulsive pressure of liquid is obtained from Eq. 4.10.

5. CONVECTIVE PRESSURE

The convective pressure is produced by the sloshing of liquid on the free surface and is given by the velocity potential which satisfies the boundary condition of the internal liquid with the rigid shell subjected to ground motion. This velocity potential is given (4) by

$$\begin{aligned}\Phi = a_o\cos\theta\{&\sum_{s=1}^{\hat{s}} J_1(\lambda_{1s} r/a_o)\cosh(\lambda_{1s} x/a_o)[\dot{q}_s(t)+\beta_s\dot{z}(t)] \\ &+\sum_{s=1}^{\hat{s}}\beta_s[\cosh(\lambda_{1s} h/a_o)-\cosh(\lambda_{1s} x/a_o)]\,J_1(\lambda_{1s} r/a_o)\dot{z}(t)\}\end{aligned} \qquad (5.1)$$

where q_s satisfies the sloshing equation of the liquid given by

$$\ddot{q}_s(t)+\omega_{1s}^2 q_s(t) = -\beta_s\ddot{z}(t) \qquad (5.2)$$

and ω_{1s} is the fundamental circular frequency of sloshing and

$$\beta_s = 2/[(\lambda_{1s}^2-1)J_1(\lambda_{1s})\cosh(\lambda_{1s}h/a_o)]$$

The second term in Eq. 5.1 coincides with $\Phi_{10}(x,r,\theta)$. Then, the liquid pressure derived from Eq. 5.1 is given by

$$p(x,r,\theta,t)=\sum_{s=1}^{\hat{s}} P_{cs}+P_{10} = -\rho_L\sum_{s=1}^{\hat{s}}\Phi_{cs}(\ddot{q}_s+\beta_s\ddot{z})+P_{10} \qquad (5.3)$$

where $\Phi_{cs}=a_o\cos\theta\, J_1(\lambda_{1s} r/a_o)\cosh(\lambda_{1s} x/a_o)$ and P_{cs} is the convective pressure induced by the s-th sloshing mode.

6. MAXIMUM RESPONSE VALUES

For the approximate values of maximum responses, the roots of square in regards to the maximum responses at each mode are considered. Assuming the pressure is divided into the sum of impulsive pressure and convective pressure, the maximum liquid pressure is given by

$$P=\sqrt{P_{10}^2+\sum_{mn=1}^{M'+N'}P_{mn}^2+\sum_{s=1}^{\hat{s}}P_{cs}^2} = -\rho_L\sqrt{\Phi_{10}\ddot{Z}_{max}+\sum_{mn=1}^{M'+N'}[\Phi_{mn}\beta_{mn}(S_{mn}-\ddot{Z}_{max})]^2+\sum_{mn=1}^{M'+N'}[\Phi_{cs}\beta_s S_{cs}]^2} \qquad (6.1)$$

where S_{mn} is the response acceleration spectre corresponding to the mn-th mode of the shell-liquid system, and S_{cs} is the response acceleration spectre corresponding to the s-th sloshing mode on the free liquid surface. The maximum shear force of the shell is given by two equations.

$$Q(x)=\rho_s h_s[Q_s(x_o)+Q_L(x_o)] \qquad (6.2)$$

where $Q_s(x_o)$ and $Q_L(x_o)$ correspond to the shell inertia force and the liquid pressure, respectively.

$$\begin{aligned}Q_s(x_o) &= \sqrt{\begin{aligned}&[\textstyle\int_0^{2\pi}[\sin^2\theta+\cos^2\theta]a\,d\theta(\ell-x_o)\ddot{Z}_{max}]^2 \\ &+\textstyle\sum_{mn=1}^{M'+N'}[\beta_{mn}(S_{mn}-\ddot{Z}_{max})\int_0^{2\pi}\int_0^{\ell}(-v_{mn}+w_{mn})a\,d\theta dx]^2\end{aligned}} \\ Q_L(x_o) &= \rho_L a_o/\rho_s h_s\sqrt{\begin{aligned}&[\textstyle\int_0^{2\pi}\int_0^{h}\Phi_{10}a\,d\theta dx\,\ddot{Z}_{max}]^2+\sum_{mn=1}^{M'+N'}[\beta_{mn}(S_{mn}-\ddot{Z}_{max})\int_0^{2\pi}\int_0^{h}\Phi_{mn}a\,d\theta dx]^2 \\ &+\textstyle\sum_{s=1}^{\hat{s}}[\beta_s S_{cs}\int_0^{2\pi}\int_0^{h}\Phi_{cs}a\,d\theta dx]^2\end{aligned}}\end{aligned} \qquad (6.3.a,b)$$

The maximum bending moment of the shell is obtained in the same way.

7. NUMERICAL RESULTS

Some of the results of present investigation, where the shell used herein has values a=1828cm, l=1219cm, hs=2.539cm, $E=2.1\times10^{6}$ Kg/cm^2, $\nu=0.3$, $\rho s=8.163\times10^{-6}$ Kg.sec^2/cm , are shown in Fig. 2 through Fig. 6.

In Table 1. the fundamental frequencies of an empty shell for various circumferential numbers are compared with the exact solutions obtained by iteration method. The comparisons indicate that the Rayleigh-Ritz method used herein gives good accuracy and the difference between the shells with perfect and imperfect circles almost does not appear.

In Fig. 2 and Fig. 3, the frequency response curves of acceleration of shell-liquid system are shown. The comparisons show that the imperfect circular section of the shell causes the coupling between the circumferential mode m=1 and m>1, but these effects to the response are small compared with the effect of first uncoupled circumferential mode m=1.

In Fig. 4, the response accelerations of the shell-liquid system are shown and in Fig. 5, the response pressures acting on the shell wall are shown. In this figure, Pc, Pr, Pe and P show the convective , rigid impulsive, elastic impulsive and root of square pressures, respectively.

8. CONCLUSIONS

The method presented shows the procedure considering the flexibility of the shell and the imperfectness of the circle section into the seismic response analysis of the shell-liquid system. Numerical results show that the effects of imperfect circular section to the seismic response of the shell are small.

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APPENDIX

$m_{11qjri}=\int_0^{2\pi} X_{11ri}\cos q\theta\cos j\theta\, a\,d\theta$, $m_{22qjri}=\int_0^{2\pi}\delta_{ri}X_{00ri}\sin q\theta\sin j\theta\, a\,d\theta$, $m_{33qjri}=\int_0^{2\pi}\delta_{ri}X_{00ri}\cos q\theta\cos j\theta\, a\,d\theta$,

$k_{11qjri}=\int_0^{2\pi}\{a\delta_{ri}X_{22}\cos j\theta\cos q\theta+\frac{1-\nu}{2}\frac{pj}{a}X_{11ri}\sin j\theta\sin q\theta+\frac{h^2}{12}\frac{1-\nu}{2a^2}jqX_{11ri}\sin j\theta\sin q\theta\}d\theta$, $k_{21qjri}=k_{12jqir}$,

$k_{13qjri}=\int_0^{2\pi}\{\nu X_{20ri}\cos j\theta\cos q\theta+\frac{h^2}{12}(\frac{1-\nu}{2a^2}jqX_{11ri}\sin j\theta\sin q\theta-\delta_{ri}X_{22ri}\cos j\theta\cos q\theta)\}d\theta$, $k_{12qjri}=\int_0^{2\pi}\{\nu jX_{20ri}\cos j\theta\cos q\theta-\frac{1-\nu}{2}qX_{11ri}\sin j\theta\sin q\theta\}d\theta$, $k_{22qjri}=\int_0^{2\pi}\{\frac{j}{a}j\delta_{ri}X_{00ri}\cos j\theta\cos q\theta+\frac{1-\nu}{2}aX_{11ri}\sin j\theta\sin q\theta+\frac{h^2}{12}\frac{3(1-\nu)}{2a}X_{11ri}\sin j\theta\sin q\theta\}d\theta$,

$k_{23qjri}=\int_0^{2\pi}\{\frac{q}{a}\delta_{ri}X_{00ri}\cos j\theta\cos q\theta+\frac{h^2}{12}(-\frac{\nu q}{a}X_{02ri}\cos j\theta\cos q\theta+\frac{3(1-\nu)}{2a}jX_{11ri}\sin j\theta\cos q\theta)\}d\theta$, $k_{31}=k_{13}$, $k_{32}=k_{23}$,

$k_{33qjri}=\int_0^{2\pi}[\frac{1}{a}\delta_{ri}X_{00ri}\cos j\theta\cos q\theta+\frac{h^2}{12}\{a\delta_{ri}X_{22ri}\cos j\theta\cos q\theta+\frac{1}{a^3}(1-j^2)(1-q^2)\delta_{ri}X_{00ri}\cos j\theta\cos q\theta+\frac{2(1-\nu)}{a}jqX_{11ri}\sin j\theta\sin q\theta-\frac{\nu q^2}{a}X_{02ri}\cos j\theta\cos q\theta-\frac{\nu j^2}{a}X_{20ri}\cos j\theta\cos q\theta\}]d\theta$,

$X_{00ri}=\int_0^l f_rf_i\,dx$, $X_{11ri}=\int_0^l f_r'f_i'\,dx$, $X_{22ri}=\int_0^l f_r''f_i''\,dx$, $X_{20ri}=\int_0^l f_r''f_i\,dx$, $X_{02ri}=\int_0^l f_rf_i''\,dx$.

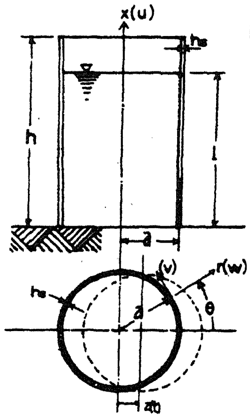

Fig.1 SHELL MODEL

n	m	SOLUTION BY RAYLEIGH-RITZ METHOD: WITH PERFECT CIRCULAR SECTION	SOLUTION BY RAYLEIGH-RITZ METHOD: WITH IMPERFECT CIRCULAR SECTION	EXACT SOLUTION: WITH PERFECT CIRCULAR SECTION
1	1	34.21 (1.8)	33.41 (0.6)	33.60
	2	23.70 (2.3)	23.71 (2.4)	23.16
	3	16.80 (2.1)	16.80 (2.1)	16.45
	4	12.29 (1.9)	12.29 (1.9)	12.06
2	1	43.45 (0.4)	42.58 (1.6)	43.27
	2	41.18 (1.0)	41.24 (1.2)	40.77
	3	37.22 (1.7)	37.09 (1.3)	36.61
	4	32.43 (2.2)	32.39 (2.1)	31.73
3	1	44.07 (0.4)	43.79 (0.2)	43.89
	2	43.20 (0.6)	43.45 (1.2)	42.93
	3	41.79 (3.4)	41.15 (1.9)	40.40
	4	39.91 (1.3)	39.66 (0.7)	39.40
4	1	44.66 (0.9)	44.70 (1.0)	44.27
	2	44.24 (0.9)	43.90 (0.1)	43.84
	3	43.52 (1.0)	42.28 (1.9)	43.09
	4	42.51 (1.1)	42.04 (0.0)	42.05

( ) : Percentage Error to Exact Solution

Table 1 COMPARISON OF FREQUENCIES

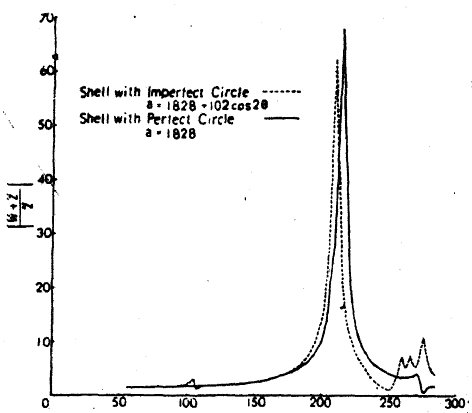

Fig. 2 FREQUENCY RESPONSES (h=0)

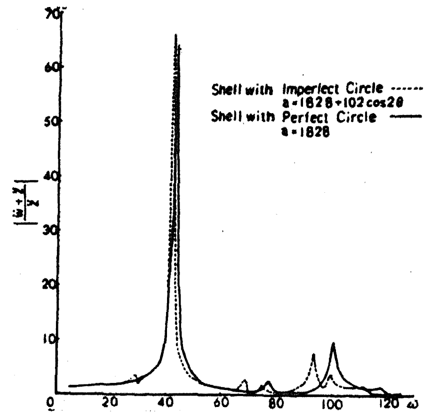

Fig. 3 FREQUENCY RESPONSES (h=1109cm)

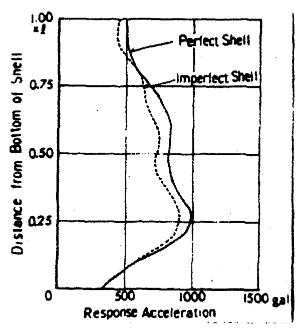

Fig. 4 RESPONSE ACCELERATIONS

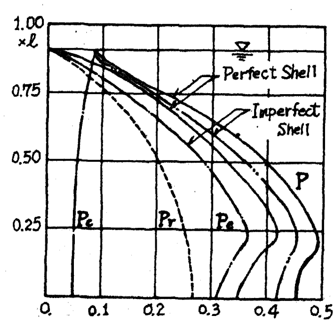

Fig. 5 RESPONSE PRESSURES (kg/cm^2)