ELASTO-PLASTIC EARTHQUAKE RESPONSE ANALYSIS FOR MULTI-STOREYED BUILDINGS TAKING ACCOUNT OF TORSION

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SYNOPSIS

Starting from a multi-storeyed shear-torsional type model, taking two dimensional horizontal earthquake wave as input and adopting degrading trilinear mode of restoring force characteristic curve, a method is presented in this paper for analysis of nonlinear earthquake response of structures with torsion and a computer program GANU is worked out using the presented method. A numerical example of three-storeyed factory building is given. Results indicate the detrimental influence torsion caused by obviously unsymmetrical mass or stiffness of structure should not be ignored.

INTRODUCTION

Recently, significant development has been made by using directly seismic records as input and by step-by-step integration method to solve problems regarding elasto-plastic earthquake response of multi-storeyed structures, which in turns put forward vitally theory and practice of designing structures to undergo earthquake shocks.

Some published papers (1-4) dealing with nonlinear earthquake response were devoted to plane structures. For structures having unsymmetrical mass distribution or stiffness allocation, problems still remain unsolved.

Starting from a multi-storeyed shear-torsional type model, taking two dimensional horizotal earthquake waves as input and adopting degrading trilinear mode restoring force characteristics curve, an approach is developed in this paper for analysis of nonlinear earthquake response of multi-storeyed structures with torsion and a corresponding computer program GANU has been worked out.

DUSCRIPTION AND BASICAL ASSUMPTION

Fig. 1 shows a multi-storeyed shear-torsional type model made up of rigid floor diaphragms and story structures between them.

Mass of the structure is considered to be concentrated at

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various levers of floors, center of mass on each floor may be located not on a same vertical axis, so does center of stiffness of each floor. A suitable vertical axis is chosed as axis of rotation, namely center of rotation. Under action of earthquake shaking, horizontal displacements along x and y direction and angular rotation response around axis of rotation will occur at every story.

Members for bearing story shears might be frames, infilled frames or wall systems, or certain composite types of these three.

STIFFNESS MARTRIX FOR STORY STRUCTURES

Let U_j , V_j , Φ_j , and U_{j-1} , V_{j-1} , Φ_{j-1} denote respectively horizontal displacements along x and y direction and angular rotation response of j and j-1 floors. (positive is shown in Fig.2)

Expression in matrix form for end forces of story structures is as follows:

$$\{Q_j\} = [\tilde{K}_j]\{U_j\} \tag{1}$$

where,

$$\{Q_j\} = \{Q_{xj-1,j}\ Q_{yj-1,j}\ M_{j-1,j}\ Q_{xj,j-1}\ Q_{yj,j-1}\ M_{j,j-1}\}^T$$
$$\{U_j\} = \{U_{j-1}\ V_{j-1}\ \Phi_{j-1}\ U_j\ V_j\ \Phi_j\}^T \tag{2}$$

$$[\tilde{K}_j] = \begin{bmatrix} K_{xj} & 0 & K^x_{\Phi j} & -K_{xj} & 0 & -K^x_{\Phi j} \\ 0 & K_{yj} & K^y_{\Phi j} & 0 & -K_{yj} & -K^y_{\Phi j} \\ K^x_{\Phi j} & K^y_{\Phi j} & K_{\Phi j} & -K^x_{\Phi j} & -K^y_{\Phi j} & -K_{\Phi j} \\ -K_{xj} & 0 & -K^x_{\Phi j} & K_{xj} & 0 & K^x_{\Phi j} \\ 0 & -K_{yj} & -K^y_{\Phi j} & 0 & K_{yj} & K^y_{\Phi j} \\ -K^x_{\Phi j} & -K^y_{\Phi j} & -K_{\Phi j} & K^x_{\Phi j} & K^y_{\Phi j} & K_{\Phi j} \end{bmatrix} \tag{3}$$

$[\tilde{K}_j]$ —stiffness matrix of j story structure

In above formulae, $Q_{xj-1,j}$, $Q_{yj-1,j}$, $M_{j-1,j}$ and $Q_{xj,j-1}$, $Q_{yj,j-1}$, $M_{j,j-1}$ are shears and torsions along x and y direction at the end of the j-th level and $(j-1)$th level of j storey, respectively.

DIFFERENTIAL EQUATION OF MOTION AND ITS SOLUTION

The differential equation of motion for the structure in matrix form may be expressed as follows:

$$[\tilde{M}]\{\ddot{U}\} + \{F_d\} + [K]\{U\} = -[M]\{\ddot{U}_0\} \tag{4}$$

where,

$$
[\tilde{M}] = \begin{bmatrix}
M_1 & 0 & M_1a_1 & & & & & & & & \\
0 & M_1 & M_1b_1 & & & & & & & & \\
M_1a_1 & M_1b_1 & M_1^{\phi} & & & & & & & & \\
 & & & M_2 & 0 & M_2a_2 & & & & & \\
 & & & 0 & M_2 & M_2b_2 & & & & & \\
 & & & M_2a_2 & M_2b_2 & M_2^{\phi} & & & & & \\
 & & & & & & \ddots & & & & \\
 & & & & & & & M_n & 0 & M_na_n \\
 & & & & & & & 0 & M_n & M_nb_n \\
 & & & & & & & M_na_n & M_nb_n & M_n^{\phi}
\end{bmatrix} \qquad (5)
$$

$$\{\ddot{U}\} = \{\ddot{U}_1\ \ddot{V}_1\ \ddot{\Phi}_1 \cdots\cdots \ddot{U}_n\ \ddot{V}_n\ \ddot{\Phi}_n\}^T \qquad (6)$$

$$\{U\} = \{U_1\ V_1\ \Phi_1 \cdots\cdots U_n\ V_n\ \Phi_n\}^T \qquad (7)$$

$$
[M] = \begin{bmatrix}
M_1 & & & & & \\
 & M_1 & & & & \\
 & & M_1a_1 & & & \\
 & & & \ddots & & \\
 & & & & M_n & & \\
 & & & & & M_n & \\
 & & & & & & M_na_n
\end{bmatrix} \qquad (8)
$$

$$\{\ddot{U}_0\} = \{\ddot{U}_0\ \ddot{V}_0\ \ddot{U}_0+\eta_1\ddot{V}_0 \cdots\cdots \ddot{U}_0\ \ddot{V}_0\ \ddot{U}_0+\eta_n\ddot{V}_0\}^T \qquad (9)$$

$$\{F_d\} = [C]\{\dot{U}\} \qquad (10)$$

[K] — stiffness matrix of whole structure

[C] — „ damping matrix

$\ddot{U}_0(t)$, $\ddot{V}_0(t)$ — acceleration of ground movement along x and y direction

$$\eta_j = b_j/a_j \qquad (11)$$

M_j — total mass of j floor

a_j — distance between mass center and center of rotation along y direction of j storey

b_j — distance between mass center and center of rotation along x direction of j storey

The degrading trilinear mode restoring force characteristics curve shown in Fig, 3 is used. For solving the problem, we take up the following semi-incremental type of differential equation

$$[\tilde{M}]\{\ddot{x}\}_{i+1} + [C]_i^{i+1}\{\Delta\dot{x}\}_i^{i+1} + [K]_i^{i+1}\{\Delta x\}_i^{i+1} + \{Q\}_i = -[M]\{\ddot{U}_0\}_{i+1} \qquad (12)$$

where,

$$\{Q\}_i = \{Q\}_{i-1} + [K]_{i-1}^{i}\{\Delta x\}_{i-1}^{i} + [C]_{i-1}^{i}\{\Delta\dot{x}\}_{i-1}^{i}$$

$$\{Q\}_0 = 0 \qquad (13)$$

The above formulae can be transformed to a quasi-static form

$$[K^*]_i^{i+1}\{\Delta x\}^{i+1} = \{\Delta P^*\}_i^{i+1} \tag{14}$$

where,

$$[K^*]_i^{i+1} = [K]_i^{i+1} + \frac{4}{\Delta t^2}[\tilde{M}] + \frac{2}{\Delta t}[C]_i^{i+1} \tag{15}$$

$$\{\Delta P^*\}_i^{i+1} = -\{Q\}_i - [M]\{\ddot{U}_o\}_{i+1} + [\tilde{M}]\left(\frac{4}{\Delta t}\{\dot{x}\}_i + \{\ddot{x}\}_i\right) + 2[C]_i^{i+1}\{\dot{x}\}_i \tag{16}$$

$\Delta t = t_{i+1} - t_i$ —— time increment

Solving equation (14) provides the solution of $\{\Delta \ddot{x}\}_i^{i+1}$ which leads to the further determination of $\{\Delta \dot{x}\}_i^{i+1}$ and $\{\Delta x\}_i^{i+1}$ according to known formulas. Once the calculation in one step is completed. The next step should begin taking the results of X, $\dot{X}$, $\ddot{X}$ vectors obtained at the end of the previous step as the initial conditions at the beginning of the new step. This procedure can be carried out step by step from the initial instant to any desired time to give the time history of responses.

NUMERICAL EXAMPLE

Fig.4 shows a factory building of three-span three-storey framed construction, Its eccentricity in x direction is caused by unsymmetrical arrangement of equipments on it. Elasto-plastic earthquake response of the structure was calculated by the method stated above on a digital computer, using the program GANU. Degrading trilinear mode of restoring force characteristic curve is adopted for each structural member bearing shears which composes story structures. Data for calculation are listed in table 1.

Due to symmetry of stiffness and mass in the y direction only one dimensional horizontal seismic wave is taken as input and record of the horizontal component (N-S) of El-Centro Earthquake is applied for such purpose with a time duration of 8 seconds and a peak value of 200 gal.

Time history of displacements at center of rotation and torsional angular change of each floor are shown in Fig.5 (a-f).

For comparing the unfavourable influence by torsion, results of response are to be compared with those obtained without taking the eccentricity of mass into consideration, that is, assuming the mass of structure being symmetrized.

The comparison is shown in Fig.6, curves are drawn for considering and without considering the effect owing to eccentricity of mass. In the figures, the solid line denotes the case not accounting mass eccentricity, while the dotted line the response of the most adverse planary frame in case of taking account of torsion.

From the comparison made above, it can be seen that the detrimental influence of torsion due to obviously unsymmetrical mass or stiffness of the structure should not be ignored.

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Table 1. Basical data of the frame

floor i	No. of plane frames	K	F_c	F_y	X_y	A_3	M_{ij}
top floor i=3	1, 2	8.140	6.59	9.4	1.42	0	0.12096
	3, 4	8.148	6.59	9.4	1.42	0	0.05184
second floor i = 2	1, 2	8.782	10.45	16.9	2.28	0	0.13384
	3, 4	8.782	10.45	16.9	2.28	0	0.05735
ground floor i = 1	1, 2	9.664	14.4	25.7	3.73	0	0.13104
	3, 4	9.664	14.4	25.7	3.73	0	0.15616

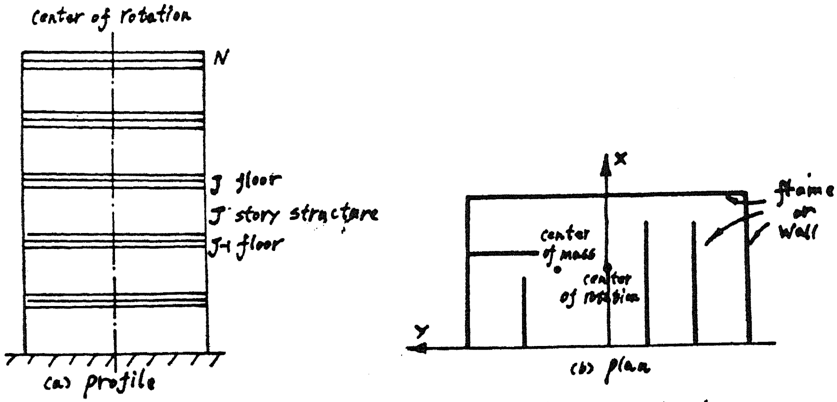


Fig.1. schematic view of the multi-storeyed structure

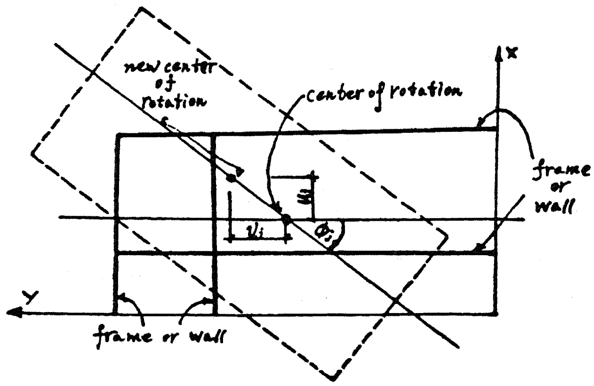
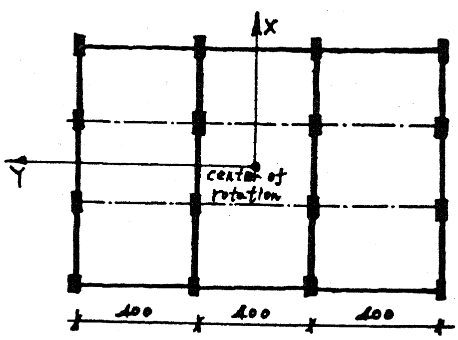


Fig. 2 displacement of J floor



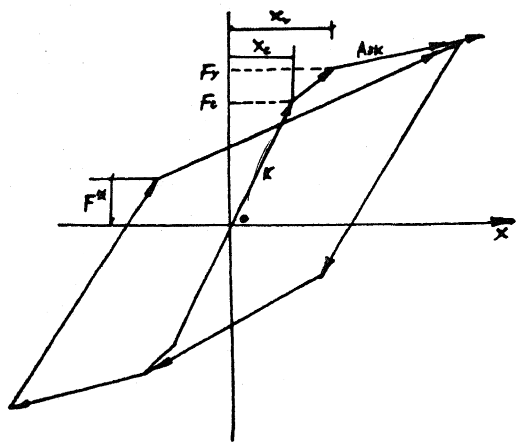


Fig. 4 plan of frame

Fig. 3

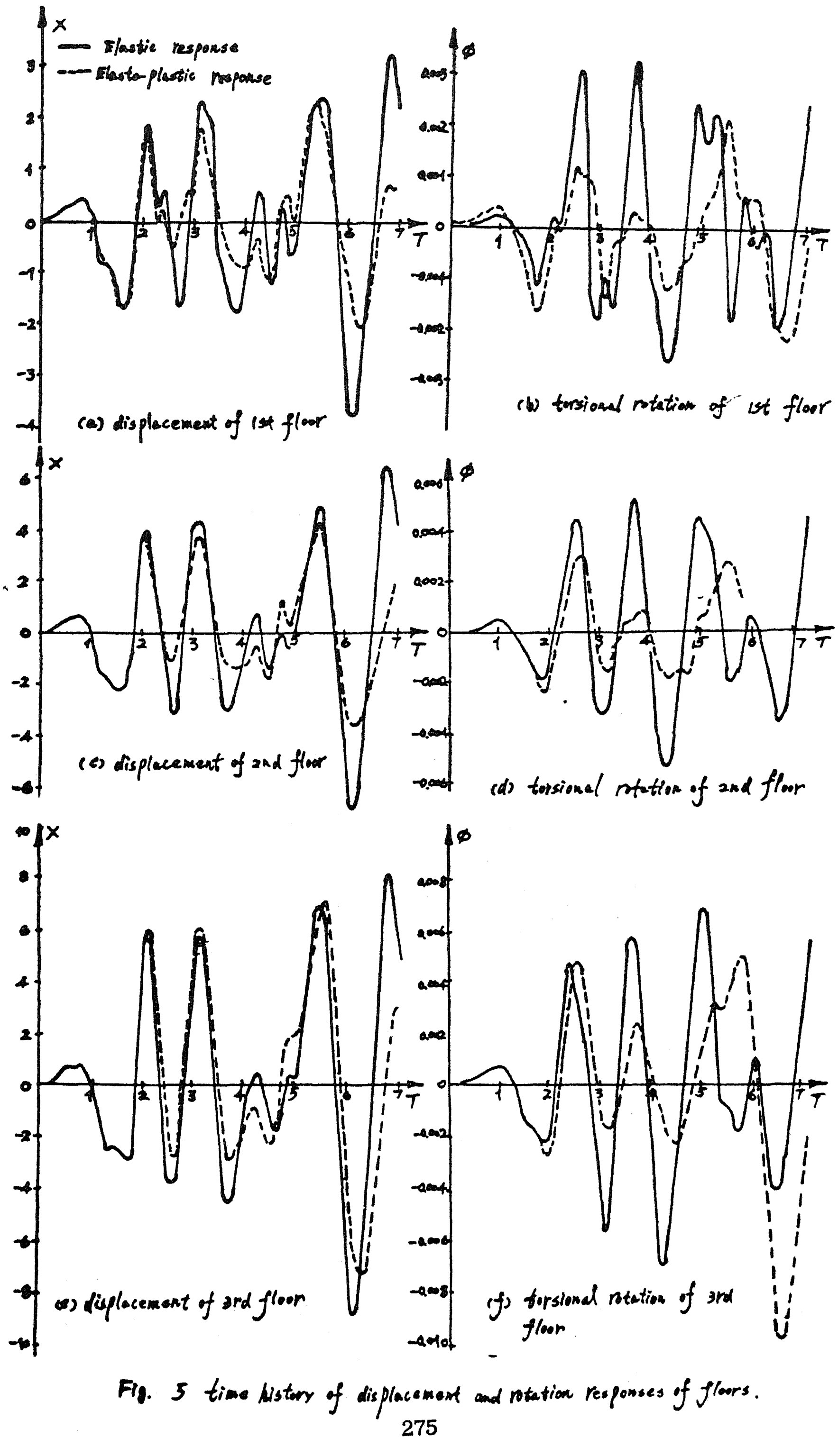


Fig. 5 time history of displacement and rotation responses of floors.

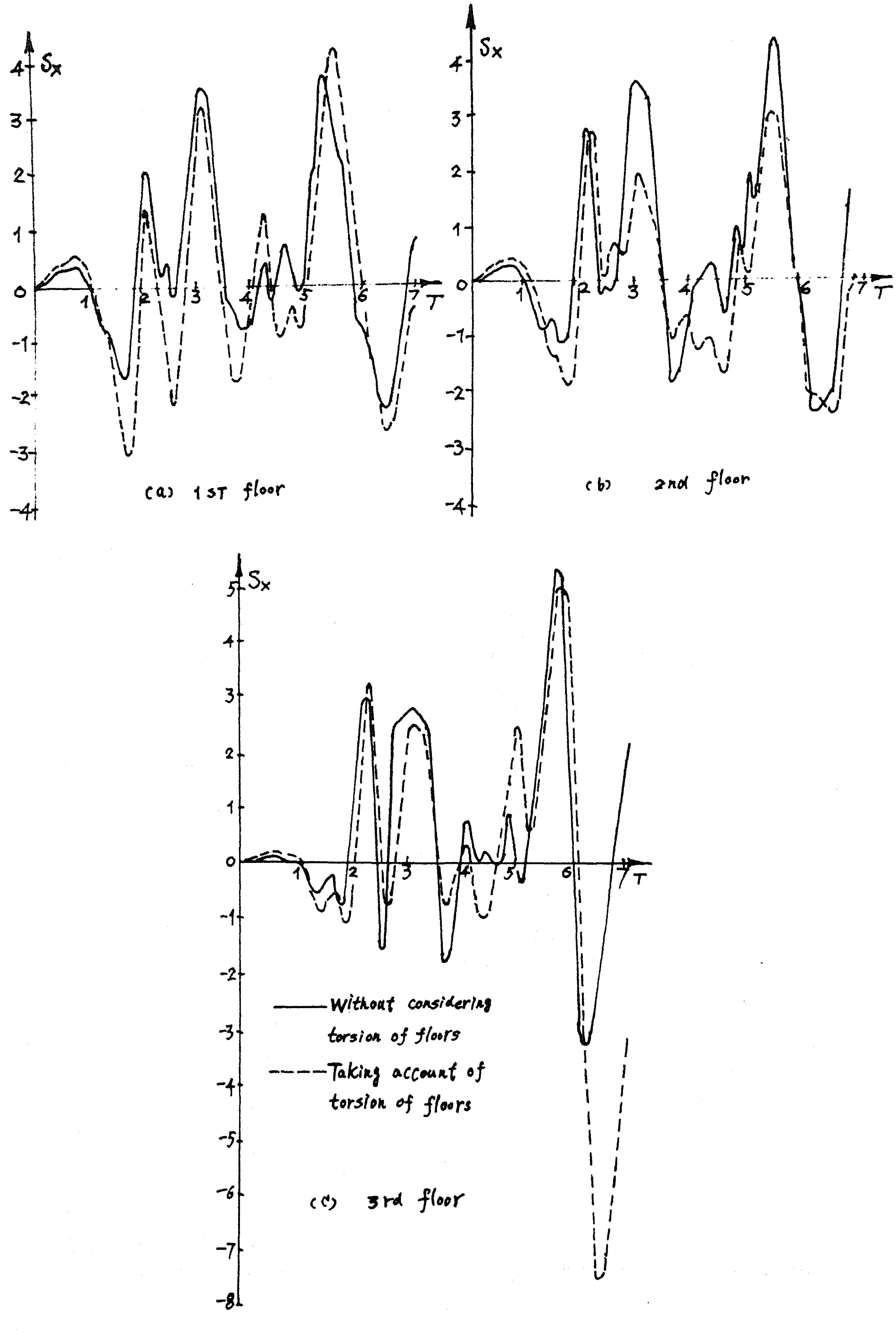


Fig. 6 Comparison of story displacement responses of frames