EARTHQUAKE ANALYSIS OF 3D STRUCTURES WITH FLEXIBLE FLOORS

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SUMMARY

A simplified force method is presented for the lateral load analysis of "slab type" three dimensional structures for which the inplane slab deformation has to be considered. In the proposed method shear walls, slabs and frames are employed as statically determinate and indeterminate substructures respectively. The necessary preliminary analysis of substructures can easily be carried out by means of two techniques developed for this purpose. Not only to increase the efficiency of the proposed method but also to eliminate the possible ill-conditioning in the set of compatibility equations, some groups of interaction forces between slabs and vertical elements are selected as the unknowns of the problem.

INTRODUCTION

It is generally assumed that the floor systems of 3D structures are rigid in their own planes. Even though this assumption makes a big reduction in the amount of unknowns, it is no longer valid for some type of structures [1], and for some sort of floor slabs [2]. As far as the inplane deformations of slabs are concerned the degrees of freedom which have to be considered increase rapidly. Accordingly ordinary computer programs can not be utilized. Because of limited capacities of core memories specially developed programs based on substructuring techniques both for static and dynamic analysis are needed. There is no other way of handling the problem in general. However, for so-called "slab type" structures, lateral load analysis can be simplified so that it can be carried out by a desk calculator with a very limited core memory. For this purpose an iterative solution based on displacement method and a direct solution based on force method have been developed. The displacement method developed can be used for 2D planar systems which are the substructures needed by the force method. The details of this method takes place elsewhere [3]. The proposed force method is being outlined and exemplified in the preceeding paragraphs.

ASSUMPTIONS

The following major assumptions have been made in this study; (i) Nodal rotations due to lateral loads are equal on the same story level, (ii) Torsional rigidity of slabs are small enough to be ignored in comparision to the flexural rigidity of shearwalls and frames in their own planes, and the torsional rigidity of shear walls and frame elements can be neglected when they are considered together with inplane flexural rigidity of slabs, (iii) Slab elements can be handled as deep beams as far as the inplane deformation of these elements are concerned, (iv) Axial deformations due to lateral loads can be neglected and foundation slab is infinitely rigid.

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The first assumption given above is commonly used in simplified lateral load analyses as well as the second and the third one. However the assumptions (ii) and (iii) have already been checked parametrically [3] and experimentally [6]. And it is concluded that both of them are acceptable especially from practical point of view. The restrictions due to the last assumption can easily be removed from the following formulation.

THE PROPOSED METHOD

The lateral loads transmitted to the lateral load resistant frames and to the structural walls or to the braced frames by the diaphragm action of slabs have been selected as the primary unknowns of 3D slab type buildings. In this procedure deformable slabs become continous beams elastically supported by the lateral load resistant vertical elements which are considered as isolated substructures, "Fig. 1b". Instead of using the individual releases as unknowns, self-equilibrating sets of them have been choosen as the unknowns of the problem. There are several advantages of this preference; (i)- to localize the internal force diagrams due to the different generalized forces which makes the hand calculations simple and results narrower bandwidths, (ii) to have relatively bigger elements on the main diagonal of the compatibility matrices than the off diagonal elements which may cause ill-conditioning when they get bigger than the corresponding diagonal elements.

If the three of the interaction forces are put together in a group as it is shown in "Fig. 1c", $\bar{X}$ the internal force diagrams will be localized within the two spans of the structure. If the nine of them in balance are considered in a group and are taken as a unique unknown, $\bar{\bar{X}}$,then the related diagrams will no longer have any branches beyond the adjacent two stories either, "Fig. 1d". In the following outline the unknowns will consist of three interaction forces. Whenever it is needed one can simply transform the set of equations based on $\bar{X}$ to the set of equations based on $\bar{\bar{X}}$,[3] .

Using the first three interaction forces as the first generalized force, $\bar{X}$ and considering the load-deformation state due to this redundants one can easily get to the "Eq.1",

$$[\delta_{11}]_{nxn} = \frac{1}{\ell_1}[F^{ad}_{w1}]_{nxn} + \left(\frac{1}{\ell_1}+\frac{1}{\ell_2}\right)[F^{ad}_{f_1}]_{nxn} + \frac{1}{\ell_2}[F^{ad}_{f2}]_{nxn} + [F^{s}_{11}]_{nxn} \tag{1}$$

by the virtual work theorem. The first column of $[\delta_{11}]$ is the algebric summation of the lateral displacements at each story, corresponding to the each force which takes part in the generalized force on the (n) th storey of the structure. The second column of the same matrix consist of the displacements due to the similar set of $\bar{X}_1$ acting on the (n-1)th storey, etc. In "Eq.1" $[F^{ad}_{w1}]$, $[F^{ad}_{f1}]$, $[F^{ad}_{f2}]$ are respectively the lateral flexibility matrices of walls and frames based on absolute displacements. In the same equation $[F^{s}_{11}]$ indicates the contribution of slabs to the lateral displacements,

$$[F^{s}_{11}] = (\ell_1+\ell_2)(1/(3EI_s) + 1/(\ell_1\ell_2GF'_s))\ [I] \tag{2}$$

where EI_s and GF'_s show respectively the inplane flexural and shear rigidities of slabs. $[I]$ indicates the unit matrix.

Since there will be common parts of internal forces due to the set of loadings $\overline{X}_1$, $\overline{X}_2$ and $\overline{X}_3$ virtual work theorem yields the following equations,

$$[\delta_{12}] = -\frac{1}{\ell_2}\left(\frac{1}{\ell_1}+\frac{1}{\ell_2}\right)[F^{ad}_{f1}] - \frac{1}{\ell_2}\left(\frac{1}{\ell_1}+\frac{1}{\ell_3}\right)[F^{ad}_{f2}] - \left(\frac{\ell_2}{6EI_s}-\frac{1}{\ell_2 GF_s}\right)[I] \tag{3}$$

$$[\delta_{13}] = \frac{1}{\ell_2}[F^{ad}_{f2}] \qquad [\delta_{14}] = [\delta_{15}] = \dots = [0]$$

The summation of lateral displacements at the cuts due to external loading and in the direction of releases take place in $\overline{X}_1$ can be calculated by virtual work teorem as follows,

$$[\delta_{1o}] = \frac{1}{\ell_1}[F^{ad}_{w1}][P^{w1}_o] - \left(\frac{1}{\ell_1}+\frac{1}{\ell_2}\right)[F^{ad}_{f1}][P^{f1}_o] + \frac{1}{\ell_2}[P^{ad}_f][P^{f2}_o] \tag{4}$$

where $[P^{w1}_o]$, $[P^{f1}_o]$ and $[P^{f2}_o]$ show the external loads concentrated on common nodes of slabs and walls, and slabs and frames. Similar equations to "Eq. 4" can be achieved for $\overline{X}_2 \dots \overline{X}_{m-1}$ without any difficulty.

The compatibility conditions can be expressed now in terms of "Eqs.1-4" as follows,

$$\begin{bmatrix} [\delta_{11}] & [\delta_{12}] & [\delta_{13}] & 0 & 0 \\ & [\delta_{22}] & [\delta_{23}] & [\delta_{24}] & 0 \\ & & & & \\ & \text{Sim.} & & & [\delta_{(m-1)(m-1)}] \end{bmatrix} \begin{bmatrix} [X_1] \\ [X_2] \\ \vdots \\ [X_{m-1}] \end{bmatrix} = \begin{bmatrix} [\delta_{1o}] \\ [\delta_{2o}] \\ \vdots \\ [\delta_{m-1}] \end{bmatrix} \tag{5}$$

$$[\delta]\,[X] = [\delta_o]$$

After having solved the "Eq.5" lateral loads distributed among the walls and frames can be determined by superposition. And then the internal forces will be achieved by the analysis of substructures which are supposed to be solved for lateral unit loads during the early steps of calculations. For this purpose multibay multistory structures are first replaced by a multistorey single bay frame then the following two recurrence formulae can be referred,

$$\theta_f^1 = - (Q_f^1 h^1/2 + Q_f^2 h^2/2 - k_{12}^{10} \theta_f^o) /(k_{12}^{10} + 2\bar{k}_{11}^b) \quad (6)$$

$$D_f^1 = D_f^o + Q_f^1 /(2k_{44}^o) - h^1/(2(\theta_f^1 + \theta_f^o)) = D_f^o + \delta \quad (7)$$

where in k_{11}, k_{12} are rotational stiffnesses and k_{44} is the stiffness associated with relative displacement of the column elements of substitude frame. Superscripts 1,2,0 indicate the storey dealt with and the two stories above and belove that one. Superscript (b) denote the equivalent beam. Q, h, D, θ_f are respectively the story shear force, story height, absolute displacement and nodal rotations of frame. "Eq.6" and "Eq.7" are successively applied from the first storey for which the boundary conditions are known, to the top.

In order to obtain the "Eq. 6 and 7" one more assumption has been used in addition to the assumptions listed above

$$\theta_f^1 \cong \theta_f^2 \quad (8)$$

These formulae have originally been derived for a 2D structure with nonprismatic members, but they are given here just for structures which consist of prismatic elements. Similar simple formulation can be given for braced frames [3]. 2D analysis will be accomplished after the end forces of each member are calculated by means of the end displacements evaluated through the "Eqs. 6 and 7".

The lateral displacements of abrubtly tapered shear walls which are the second type of substructures can manually be calculated by the algorithm prepared for a quick compilation of the preparatory calculations needed by the proposed method. In this algorithm the rotation differences between the two story levels of shear walls are obtained by the application of virtual work theorem and then "Eq.7" is refered for displacement. Shear deformation which has to be considered is easily taken into account by this procedure through k_{44} .

The proposed method can easily be adopted for the analysis of 3D structures with rigid floors, just omitting the terms associated with the inplane deformation of slabs given in "Eq.2" and taken place in "Eq.3". If this is the case, the ammount of unknowns is reduced to the amount of stories, n, putting all the frames together. The same technique can be employed for the case of nonrigid slabs in several ways. If the relative displacement between frames are neglected and only the relative displacements between the frames and walls are kept in the analysis, then the amount of unknowns is reduced to n. If it is assumed that the deformed configuration of slabs are parabolic, again, the unknowns are reduced to n. But the best of those is the first one with a modification. In this way, a group of frames are put together and the terms which express the inplane deformation of slabs between these frames are considered,[3].

EXAMPLES

A multibay multistory structure shown in "Fig.2a" has been selected as an example to have an idea about the accuracy and the adequacy of the proposed method. A typical lateraly loaded steel frame of the selected 3D structure has been analyzed first by "Eqs. 6 and 7". For this purpose a substitude one bay equivalent frame defined in "Fig. 2b" has been used. All necesarry quantities in the manual process as well as the results are gethered in "Tab.1". Nodal rotations and lateral displacements differ only 1.48% and 0.40% from the corresponding avarage values obtained by SAPIV, [5] respectively. Secondly lateral load analysis of the individual shear wall taken from the same 3D structure and shown in "Fig.2b" has been manually completed by filling the 7 th column in "Tab.2" and using the "Eq.7" . k_{44} used in "Eq.7" may be determined so that the shear deformation can be imparted to the analysis.

If the middle five frames are considered together, the substitute 3D system shown in "Fig.2d" is reached. This system has only 2n unknowns. The results presented in the columns 1-3 of "Tab.3" are achieved by the proposed method. The lateral flexibility matrices of substructures used in this analysis have been prepared by the aid of SAPIV just for avoiding the additional error accumulation due to the approximate analysis of substructures. The solution found in literature is also enclosed to the same table [4]. The avarage weighted differences are only 1.57% and 0.38% for the frame and wall displacements respestively. Omitting the terms related to the inplane deformation of slab,it has been reached to the solution based on rigid slab assumption which are listed in column [7]. In this example each unknown consists of three self-equilibrating releases.

CONCLUSIONS

The following conclusions can be drawn from this investigation which essentially aims to simplify the lateral load analysis of 3D structures; (i) Whether the inplane slab deformations are considered or not the analysis of 3D "slab type" structures can manually be done in the same way. The amount of unknowns can be reduced to the amount of stories. The possible ill-conditioning can be eliminated by the use of suggested group of interaction forces as the unknown of the problem, (ii) The results based on rigid floor assumption are not on the safe side especially for the lower middle and the upper exterior lateral load resisting vertical elements as well as the upper story slabs, (iii) The lateral load analysis of 2D frames with non-prismatic members and with any type of boundary conditions can be carried out by means of two recurrence formulae.

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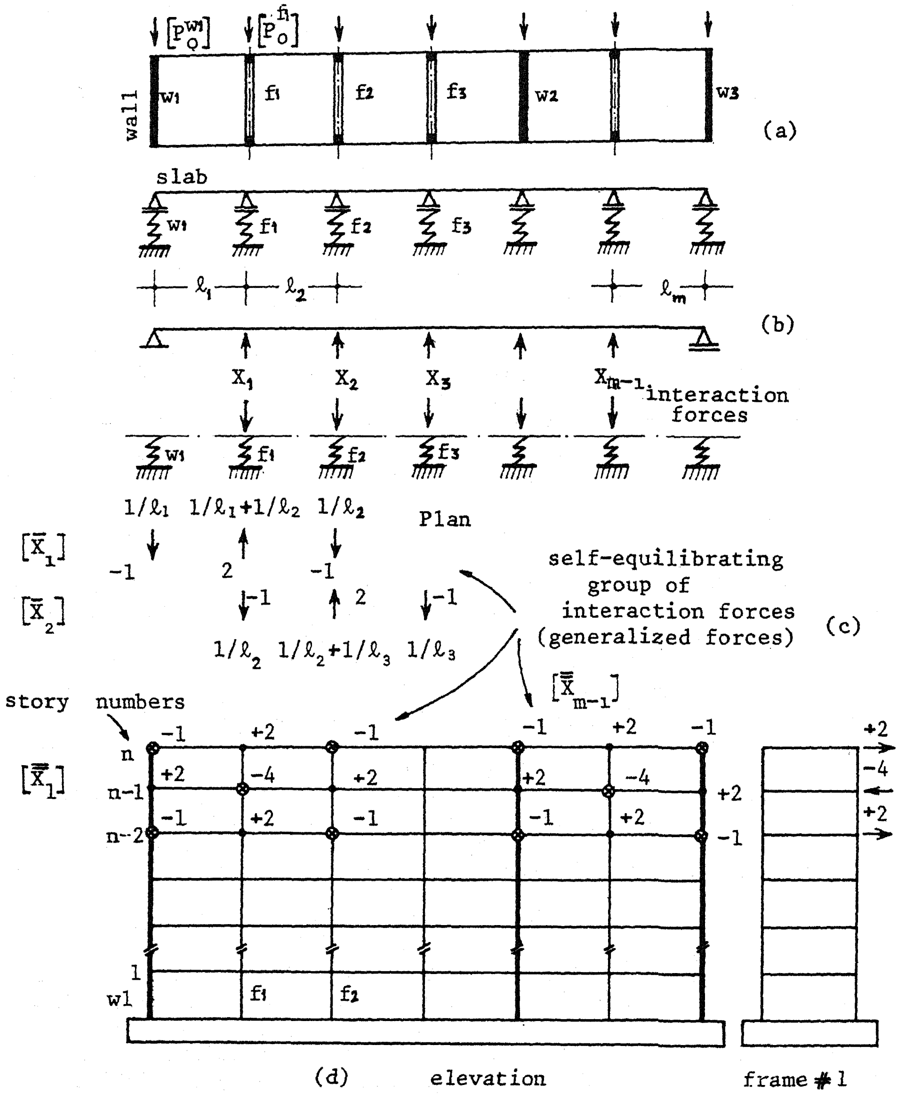
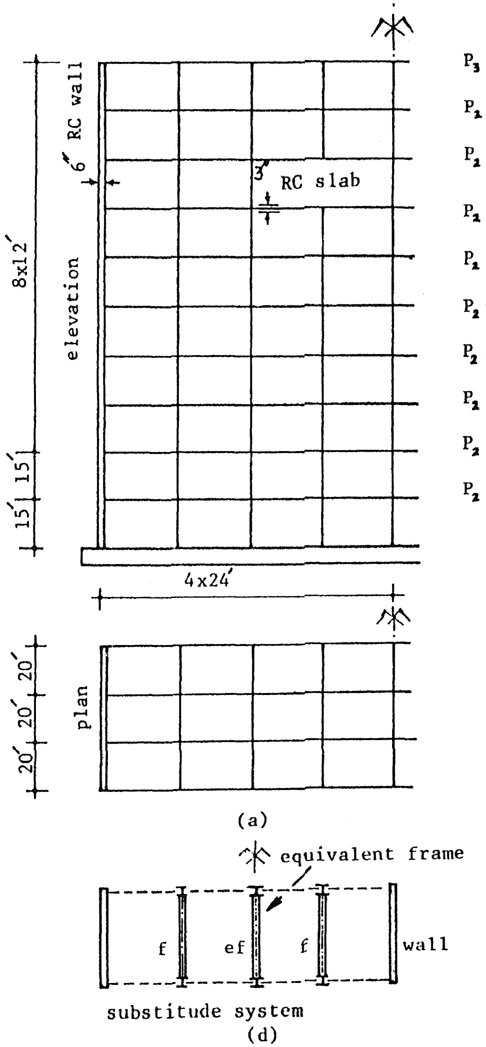
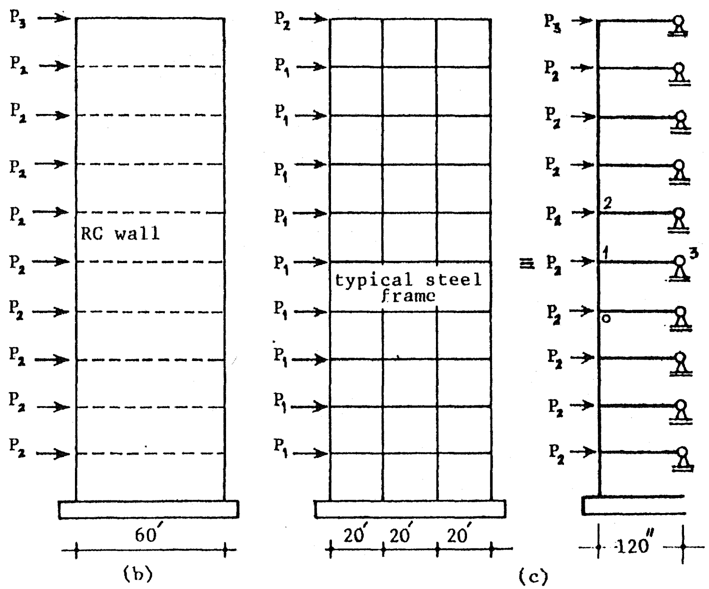
FIGURES



"Fig. 1"





$$k_{12}^{13} = 3\left(\frac{6EI_b}{20'}\right)$$

$$\theta_f^1 \cong \theta_f^2$$

story	$I_{col.}$	I_{beam}
10	254. in^4	1380.
9	443.	"
8	739.	"
7	927.	"
6	1152.	"
5	1414.	"
4	1708.	"
3	2168.	"
2	2477.	"
1	2627.	"

P_1 = 7200 lb
P_2 = 3600 lb
P_3 = 1800 lb
ν = 0.1
k = 0.833
E_c = $3x10^6$ psi
E_s = $30x10^6$ psi

"Fig. 2"

TABLES

Table-1

Story	$\frac{P}{2}$ [k]	$\frac{Q}{2}$ [k]	h [in]	$\frac{Qh}{2} \cdot 10^{-3}$	Column			Beam $\cdot 10^{-6}$		$10^{-6}(k^{10} + 2 k^{13}_{12})$	$\theta \cdot 10^{-4}$	δ [in]	D [in]
					$EI \cdot 10^{-6}$	$k \cdot 10^{-3}$	$k_{44} \cdot 10^{-3}$	EI_b	$\bar{k}_{12} \cdot 10^{-13}$				
10	1.8	1.8	144	259.2	12 240.	211.7	61.2			6421.7	-0.465	0.0462	1.8355
9	3.6	5.4	"	777.6	26 580.	369.2	106.8			6579.2	-1.869	0.0893	1.7893
8	"	9.0	"	1296.0	44 430.	615.8	178.2			6825.8	-3.509	0.1134	1.7000
7	"	12.6	"	1814.4	55 620.	772.5	223.5			6982.5	-5.223	0.1440	1.5866
6	"	16.2	"	2332.8	69 120.	960.0	227.8	41400.	3105.	7170.0	-6.947	0.1709	1.4426
5	"	19.8	"	2851.2	84 840.	1178.3	340.9			7388.3	-8.686	0.1960	1.2717
4	"	23.4	"	3369.6	102 480.	1423.3	411.8			7633.3	-10.469	0.2218	1.0757
3	"	27.0	"	3888.0	130 080.	1806.7	522.8			8016.7	-12.440	0.2494	0.8539
2	"	30.6	180	5508.0	148 620.	1651.3	305.8			7861.3	-15.028	0.3672	0.6045
1	"	34.2	"	6156.0	157 620.	1751.3	324.3			7961.3	-14.651	0.2373	0.2373

Table-2

1	2	3	4	5	6	7	8	9	10	11	12
Story	P [lb]	Q [lb]	h [in]	Qh	M	$\frac{M^i+M^{i+1}}{2} \cdot h^i/(EI)^i$	$\theta \cdot 10^6$	$EI \cdot 10^{-12}$	$\bar{k}_{44} \cdot 10^{-6}$	$\delta \cdot 10^6$ [in]	$D \cdot 10^6$ [in]
					0.						
10	1800	1800	144	259200	259200.	0.033	25.779	559.872	33.58	3763.40	34926.9
9	3600	5400	"	777600	1036800.	0.167	25.746	"	"	3856.21	31163.5
8	"	9000	"	1296000	2332800.	0.433	25.579	"	"	3920.22	27307.2
7	"	12600	"	1814400	4147200.	0.833	25.146	"	"	3936.27	23387.0
6	"	16200	"	2332800	6480000.	1.367	24.313	"	"	3885.08	19450.8
5	"	19800	"	2851200	9331200.	2.033	22.946	"	"	3747.48	15565.7
4	"	23400	"	3369600	12700800.	2.833	20.913	"	"	3504.34	11818.2
3	"	27000	"	3888000	16588800.	3.767	18.080	"	"	3136.35	8313.9
2	"	30600	180	5508000	22096800.	6.219	14.313	"	26.64	3165.30	5177.5
1	"	34200	"	6156000	28252800.	8.094	8.094	"	"	2012.20	2012.2

Table- 3

Story	PROPOSED METHOD			[4]			Rigid Floor Assumpt	Weighted Avrg.Diff.	
	D_w	D_{f1}	D_{ef}	D_w	D_{f1}	D_{f4}		3-6	1-4
	1	2	3	4	5	6	7	8	9
10	0.2444	0.2512	0.2570	0.2434	0.2487	0.2570	0.2445		
9	0.2184	0.2327	0.2491	0.2175	0.2290	0.2464	0.2183		
8	0.1918	0.2081	0.2219	0.1910	0.2041	0.2241	0.1924		
7	0.1647	0.1823	0.1972	0.1641	0.1783	0.1998	0.1655		
6	0.1374	0.1556	0.1710	0.1369	0.1516	0.1740	0.1383	%	%
5	0.1104	0.1290	0.1448	0.1100	0.1251	0.1481	0.1114	1.57	0.38
4	0.0841	0.1028	0.1193	0.0838	0.0993	0.1229	0.0851		
3	0.0593	0.0785	0.0948	0.0591	0.0747	0.0985	0.0604		
2	0.0370	0.0548	0.0700	0.0369	0.0514	0.0731	0.0381		
1	0.0142	0.0235	0.0312	0.0142	0.0214	0.0319	0.0149		