STUDY ON SOIL-PILE INTERACTION OF PILE GROUPS IN VERTICAL VIBRATION

Yoshihisa GYŌTEN[I], Tadahiro FUKUSUMI[II], Takahito INOUE[III], Toshiki MIZUNO[III]

SUMMARY

This paper deals with the theoretical study concerning soil-pile interaction of pile groups in vertical vibration in and out of phase among piles. The analysis is performed by application of the elastic wave theory to the visco-elastic layer overlying on the bedrock. Besides, the pile responses to the harmonic excitation of the displacement and the complex stiffness at the pile head are obtained for various dimensionless parameters.

INTRODUCTION

In previous studies, the solutions for the cases of the end bearing pile[1], the floating pile and the pile group in phase[2] were solved. In this paper, the theoretical solutions have been obtained for pile groups where the rigid body is fixed to the pile heads and the harmonic excitation with the phase lag is applied at the respective pile heads. In this analysis, the following assumptions were made:

(1) The soil overlying on the rigid bedrock is a visco-elastic, homogeneous and isotropic layer with the hysteretic damping.

(2) The horizontal displacement is negligibly small.

(3) There are neither normal nor shear stress on the free surface of the soil layer and no displacement at the bottom of the soil layer.

(4) The pile is perfectly elastic and vertical. Its cross-section is circular and its surface contacts with the soil layer perfectly.

(5) The resistance between the rigid body and the surface of the soil layer is neglected. The displacements of the respective pile heads are equal and the rigid body doesn't rotate.

DYNAMIC RESISTANCE FACTOR OF A SOIL LAYER

In the case of pile groups, as the pile is influenced by the other piles through the medium of the soil layer, the resistance factor including this effect must be obtained. For this purpose, the solution of the forced vibration of the soil layer must be solved. Herein, the following approximation is applied: The soil layer is split into [I] and [II] as shown in Fig. 1. The soil layer [II] is regarded as three-dimensional visco-elastic system. The equation of vertical motion for [II] can be written as

$$\mu\{\eta^2+i[Dv(\eta^2-2)+2Ds]\}\frac{\partial^2}{\partial z^2}w_{II}e^{i\omega t}+\mu(1+iDs)\left(\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial r^2}\right)w_{II}e^{i\omega t}=\rho\frac{\partial^2}{\partial t^2}w_{II}e^{i\omega t} \quad (1)$$

where λ,μ=the real part of Lamé's constants, λ',μ'=the viscosity coefficient associated with λ and μ respectively, ρ=soil density, $\eta=V_l/Vs=\sqrt{[(\lambda+2\mu)/\mu]}$ in which V_l, Vs=the longitudinal and shear wave velocities of soil, $i=\sqrt{(-1)}$, ω= excitation frequency, $Dv=\lambda'/\lambda$, $Ds=\mu'/\mu$ are hysteretic damping ratios. The soil layer [I] is considered as one-dimensional visco-elastic rod. Its equation of vibration can be written as

$$-\pi r_0^2\mu\{\eta^2+i[Dv(\eta^2-2)+2Ds]\}\frac{\partial^2}{\partial z^2}w_Ie^{i\omega t}+\pi r_0^2\rho\frac{\partial^2}{\partial t^2}w_Ie^{i\omega t}=P(z)e^{i\omega t}+P_f(z)e^{i\omega t} \quad (2)$$

where P(z)=the excitation force P_0 acting on the surface of [I] expanded in

I, II, III Professor and Research Associate of Struct. Eng. and Graduate Student, Kobe University, Kobe, Japan

Fourier series and $P_f(z)$=the resistance force from [II]. By solving Eqs. 1 and 2 and equaling the displacement at the boundary of [I] and [II], the displacement and the shear stress of the soil layer due to the forced vibration are obtained as

$$w=\frac{2P_0}{\pi\mu H}\sum_{n=1}^{\infty} Bn\frac{K_0(\bar{q}_n\bar{r})}{K_0(\bar{q}_n\bar{r}_0)}\sin(\bar{h}_n\bar{z}) \quad (3) \qquad \tau_{rz}=-\frac{2P_0(1+iDs)}{\pi H^2}\sum_{n=1}^{\infty} Bn\bar{q}_n\frac{K_1(\bar{q}_n\bar{r})}{K_0(\bar{q}_n\bar{r}_0)}\sin(\bar{h}_n\bar{z}) \quad (4)$$

where $\bar{h}_n=h_nH=\pi(2n-1)/2$, $\bar{q}_n^2=q_n^2H^2=[\{\eta^2+i[Dv(\eta^2-2)+2Ds]\}\bar{h}_n^2-a_0^2]/(1+iDs)$, $\bar{r}_0=r_0/H$, $\bar{z}=z/H$, $\bar{r}=r/H$, $a_0=H\omega/Vs$, K_0, K_1 are the modified Bessel functions of the zero order and the first order of the second kind, respectively.

Let's consider that the harmonic excitation $P_je^{i(\omega t-\phi_j)}$ is applied at any point j (j=1..i..m) on the surface of the soil layer, as shown in Fig. 2. The displacement and the resistance force at the point corresponding to the circumference of pile i can be written as

$$w_i=\frac{2}{\pi\mu H}\sum_{j=1}^{m}\sum_{n=1}^{\infty} Bn\frac{P_jK_0(\bar{q}_n\bar{L}_{ij})}{K_0(\bar{q}_n\bar{r}_0)}e^{-i\phi_j}\sin(\bar{h}_n\bar{z}) \quad (5)$$

$$P_{fi}=-\frac{4\bar{r}_0(1+iDs)}{H}\sum_{j=1}^{m}\sum_{n=1}^{\infty} Bn\bar{q}_n\frac{K_1(\bar{q}_n\bar{L}_{ij})}{K_0(\bar{q}_n\bar{r}_0)}e^{-i\phi_j}\sin(\bar{h}_n\bar{z}) \quad (6)$$

where the displacement and the resistance force due to $P_je^{i(\omega t-\phi_j)}(j\neq i)$ are approximated by the values at the center of the soil rod replacing pile i, ϕ=the phase lag of the harmonic excitation applied at the respective pile heads, $\bar{L}_{ij}=L_{ij}/H$, L_{ij}=the distance between points i and j, $L_{ij}=r_0$ for i=j. The resistance factor of the soil layer α_i is defined as Eq. 7 and the dimensionless resistance factor in the n-th wave mode $\bar{\alpha}_{ni}$ is written as Eq. 8.

$$P_{fi}=-\alpha_iw_i \quad (7) \qquad \bar{\alpha}_{ni}=\alpha_{ni}/2\pi\mu=\bar{r}_0(1+iDs)\bar{q}_n\frac{\sum_{j=1}P_jK_1(\bar{q}_n\bar{L}_{ij})\,e^{-i\phi_j}}{\sum_{j=1}P_jK_0(\bar{q}_n\bar{L}_{ij})\,e^{-i\phi_j}} \quad (8)$$

EQUATION OF MOTION OF SOIL-PILE SYSTEM

Let's consider that the piles of the same material properties and diameter are forced to vibrate vertically due to the harmonic excitation $P_je^{i(\omega t-\phi_j)}$, respectively, as shown in Fig. 3. Since the resistance factor including the effect of the other piles is obtained, the equation of motion for the respective piles can be written as the similar equation to the case of the single pile. Thus, in the case of pile i

$$-Mp\frac{\partial^2}{\partial t^2}w_ie^{i(\omega t-\phi_i)}+EpS\frac{\partial^2}{\partial z^2}w_ie^{i(\omega t-\phi_i)}+P_{fi}(z)e^{i(\omega t-\phi_i)}=0 \quad (9)$$

where Mp=mass of the unit length of the pile, Ep=Young's modulus of the pile, S=cross-sectional area of the pile. By the following boundary conditions, $w_i(z)=w_i(r_0,z)$, $P_i(H)=P_i$ and $w_i(0)=0$, the displacement is obtained as

$$w_i(z)=[P_i/EpS\kappa_i\cos(\kappa_iH)]\sin(\kappa_iz) \quad (10)$$

where $\kappa_i^2=(Mp\omega^2-\alpha_i)/EpS$, $w_i(z)$=the displacement of pile i, $w_i(r_0,z)$=the displacement of the soil layer on the circumference of the pile. By expanding Eq. 10 in Fourier series in the range from 0 to H, the displacement of pile i is written as $w_i(z)=(P_ir_0/EpS)\bar{w}_i$, where

$$\bar{w}_i=(2/\bar{r}_0)\sum_{n=1}^{\infty}[(-1)^{n-1}/(\bar{h}_n^2-\bar{\lambda}^2+\gamma\bar{\alpha}_{ni})]\sin(\bar{h}_n\bar{z}) \quad (11)$$

In Eq. 11, the following non-dimensional quantities are introduced, $\bar{\rho}=\rho/\rho_p$, $\kappa_i^2H^2=\bar{\lambda}^2-\gamma\bar{\alpha}_{ni}$, $\bar{\lambda}^2=MpH^2\omega^2/EpS=\bar{V}^2a_0^2$, $\gamma=2\pi\mu H^2/EpS=2\bar{V}\bar{\rho}/\bar{r}_0^2$, $\bar{V}=Vs/Vp=\sqrt{(\mu\rho_p/\rho Ep)}$ where ρ_p=mass density of the pile, Vp=the longitudinal wave velocity of the pile. The complex stiffness of the soil-pile system at the pile head is written as $k_i=(EpS/r_0)\bar{k}_i$---(12), where the dimensionless stiffness $\bar{k}_i$ is the reciprocal of $\bar{w}_i$ at $\bar{z}$=1.0.

Let's assume that the harmonic excitation $P_0e^{i\omega t}$ is applied on the rigid body fixed to the pile heads and $P_je^{i\omega t}$ is transmitted to the respective pile heads, as shown in Fig. 4. Then, the equations of motion of pile i and the rigid body can be written as follows

$$-Mp\frac{\partial^2}{\partial t^2}w_ie^{i\omega t}+EpS\frac{\partial^2}{\partial z^2}w_ie^{i\omega t}+P_{fi}(z)e^{i\omega t}=0 \quad (13) \qquad M_0\frac{\partial^2}{\partial t^2}w_0e^{i\omega t}+\sum_{j=1}^{m}P_je^{i\omega t}=P_0e^{i\omega t} \quad (14)$$

where M_0=mass of the rigid body. By solving Eqs. 13 and 14 and making $w_i(H)$ equal to w_0, the displacement of pile i is obtained as $w_i(z)=(P_0r_0/EpS)\xi_i\bar{w}_i$, where $\xi_i=P_i/P_0$, $\bar{w}_i$ is Eq. 11. Especially, when all P_j are equal, $\xi_i=1/[m-2\bar{M}\bar{\lambda}^2\sum_{n=1}^{\infty}\{1/(\bar{h}_n{}^2-\bar{\lambda}^2+\gamma\bar{\alpha}_{ni})\}]$ where m=the number of piles, $\bar{M}=M_0/MpH$. The complex stiffness at the pile head is obtained from Eq. 12.

NUMERICAL RESULTS AND DISCUSSIONS

Some of the numerical results are shown in from Fig. 7 to Fig. 14. In the numerical calculation, the responses of the displacement w' and the complex stiffness k' at the pile head normalized by their static values are obtained. In these figures the values of the dimensionless parameters used are shown. The displacement includes the amplitude and the phase lag and the complex stiffness represents the stiffness and the damping for the real and the imaginary parts, respectively.

Discussion on the case of two piles with the phase lag (Fig. 5): The variations w' with $\bar{V}$, L/r_0 and ϕ are shown in Figs. 7, 9 and 11, respectively. Similarly the variations k' with $\bar{V}$, L/r_0 and ϕ are shown in Figs. 8, 10 and 12, respectively. From these figures, it is mentioned that these curves are influenced by $\bar{V}$, L/r_0 and ϕ and that when the harmonic excitation with the phase lag is applied at the respective pile heads, the responses of the pile group differ from those of the pile group without the phase lag. Especially, when ϕ is equal to π, the responses don't have the steep peaks at the natural frequency of the soil layer.

Discussion on the case of two piles with the rigid body (Fig. 6): The variations of w' and k' with $\bar{M}$ are shown in Figs. 13 and 14, respectively. From these figures, it is recognized that in the range of the frequency higher than resonance, these curves are influenced by $\bar{M}$, but $\bar{M}$ has little effect on the damping of k' and that the larger $\bar{M}$ is, the lower the first natural frequency of the soil-pile system becomes.

CONCLUSIONS

The above analysis leads to the following conclusions:

(1) The responses of the displacement and the complex stiffness of pile groups are much affected by the dimensionless distance among piles, the wave velocity ratio of the soil layer to the pile, the phase lag of the harmonic excitation applied at the respective pile heads and the mass ratio of the rigid body on the pile heads to the pile.

(2) When the harmonic excitation is applied with the phase lag at the respective pile heads of the pile group, the response differs from that of the pile group without the phase lag.

(3) The larger the mass ratio of the rigid body on the pile heads to the pile, the lower the natural frequency of the soil-pile system becomes.

ACKNOWLEDGEMENT

The authors wish to express their sincere gratitude to Professor Koji Mizuhata for contributing to this study as much as the authors. The authors are indebted to the members of the Structural Dynamics Laboratory at Kobe University for thier helpful cooperation to this work.

REFERENCES

1) Nogami, T. and Novak, M., (1976), "Soil-Pile Interaction in Vertical Vibration", Earthquake Engineering and Structural Dynamics, Vol. 4, pp. 277-293.

2) Gyoten, Y., Mizuhata, K., Fujimoto, Y., Okamoto, T. and Inoue, T., (1978), "Study on Soil-Pile Interaction in Vertical Vibration", Proc. of the Fifth Japan Earthquake Engineering Symposium, pp. 377-384.

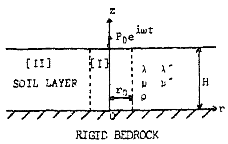

Fig. 1 Model of soil layer (1)

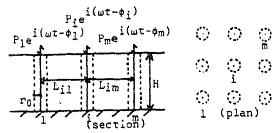

Fig. 2 Model of soil layer (2)

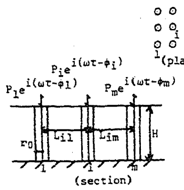

Fig. 3 Model of soil-pile system

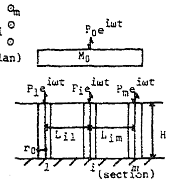

Fig. 4 Model of soil-pile-rigid body system

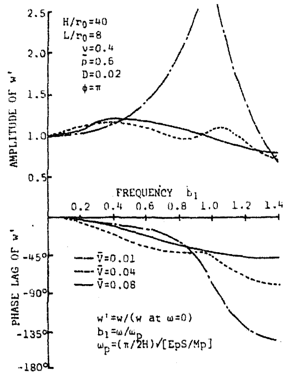

Fig. 7 Variations of frequency response of amplitude and phase lag of displacement with $\bar{V}$

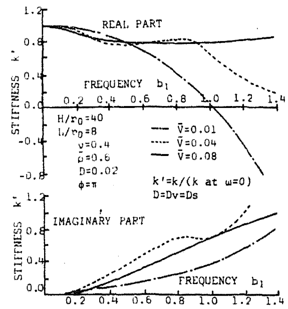

Fig. 8 Variations of pile stiffness vs. frequency with $\bar{V}$

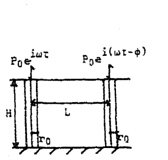

Fig. 5 Soil-pile system for numerical analysis

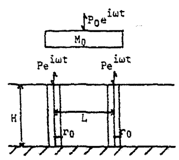

Fig. 6 Soil-pile-rigid body system for numerical analysis

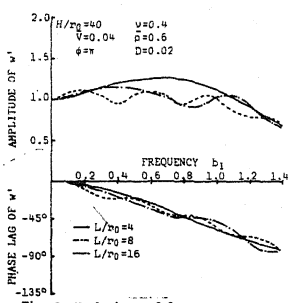

Fig. 9 Variations of frequency response of amplitude and phase lag of displacement with L/r_0

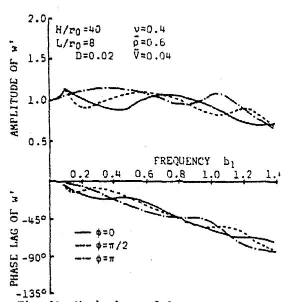

Fig. 11 Variations of frequency response of amplitude and phase lag of displacement with ϕ

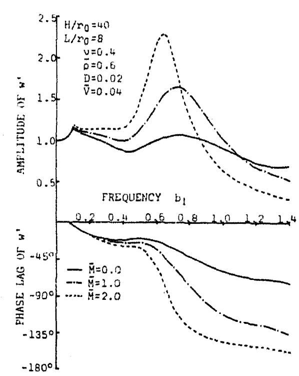

Fig. 13 Variations of frequency response of amplitude and phase lag of displacement with $\bar{M}$

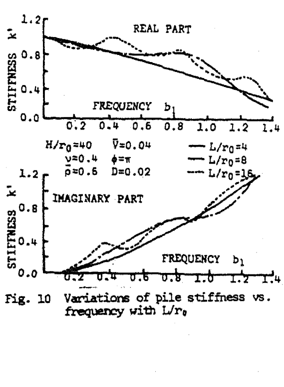

Fig. 10 Variations of pile stiffness vs. frequency with L/r_0

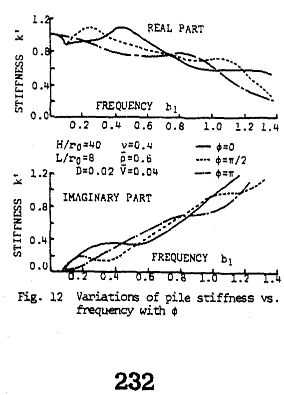

Fig. 12 Variations of pile stiffness vs. frequency with ϕ

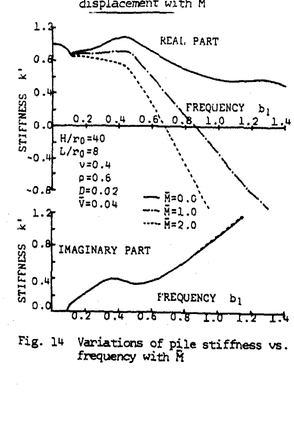

Fig. 14 Variations of pile stiffness vs. frequency with $\bar{M}$