SEISMIC ANALYSIS OF STRUCTURES WITH RECTANGULAR BASES ON PORUS ELASTIC SOIL MEDIUM

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SUMMARY

An analytical study is made of the soil-structure interaction effects on the dynamic response of structures due to seismic excitation. The system considered consists of a semi-infinite soil medium, a rigid base slab resting on the soil and a lumped mass structural model attached to the base slab. The soil-structure system is generalized to a three-dimensional mathematical model and the concept of a porous elastic soil medium is introduced into the analysis. The method is extended to obtain solutions for perfectly elastic and visco-elastic media. Numerical results show good correlation with the work of previous investigators.

INTRODUCTION

The determination of the dynamic response of structures supported on a porous elastic, viscoelastic or perfectly elastic soil medium has received increased attention in the last two decades primarily due to special requirements in the design of nuclear power plants. The analysis of the problem presents certain difficulties, mainly due to the soil-structure interaction effects.

The interaction effects tend to decrease fixed base natural frequencies, and may increase or decrease spectrum responses for equipment and piping stress analyses. Stresses in the structure during an earthquake depend on the magnitude of excitation and the subsoil characteristics. It is therefore necessary to know the foundation impedance, which depends on the soil properties, the frequency of the disturbing force, and the shape of the loading area.

There have been many studies reported in the past by other investigators. In most of the previous investigations the soil is usually modeled as a two-dimensional half space, and the structure is represented by a lumped mass system. In the low frequency range some investigators represented the soil properties with equivalent springs and dampers.

Most buildings have a rectangular foundation base and their solution is extremely difficult. In general, for seismic analysis the earthquake excitation should be considered to occur in three directions. Furthermore, the ground water sometimes reaches the grade elevation. In such cases the soil must be treated as a two phase complex consisting of an elastic skeleton of a solid phase filled with the water of the liquid phase.

In order to perform a more realistic study, in this paper the

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soil-structure system is generalized to a three-dimensional mathematical model involving a rectangular foundation slab. In addition the concept of a porous elastic soil medium is introduced into the analysis. Applying Fourier analysis, Fourier series and Fourier transform methods, coupled with Dirac delta functions, closed-form solutions are obtained for the static displacements and spring constants of a single base and multiple base foundations of structures.

The solution to the problem of the porous elastic medium is then extended to obtain solutions for perfectly elastic and viscoelastic media.

STRUCTURAL SYSTEM

In the present analysis the model of the structure is confined to a lumped mass system with distributed flexibility, supported by a rigid rectangular base founded on a semi-infinite soil medium, as shown in Fig. 1. The structure-foundation-soil system has a three-dimensional configuration referred to a fixed Cartesian system of axes. The origin of the reference X^1, X^2, and X^3 axes of the superstructure is located at the center of the interface with the slab, so that the X^1-X^2 plane coincides with the half-space surface and the positive X^3-axis points upwards. In the case of the soil medium, an x,y,z coordinate system is used with origin at the same point, but with the positive z-axis pointing downward, as indicated in Fig. 2.

In the analysis several assumptions are made as follows: (i) All masses of each part of the structure are lumped at the center of the rigid floor or base slab, (ii) the base slab is considered to be rigid, so that it does not undergo any elastic deformations, (iii) the input ground earthquake motions are at the surface of the soil medium and parallel to the three reference axes, (iv) the center of gravity and the center of rigidity of each mass may not coincide, giving rise to torsional vibrations, and (v) the dynamic force-displacement relation of the soil medium can be characterized by a set of frequency dependent impedance functions.

With reference to the last of the above assumptions, in order to perform the interaction analysis, interaction forces and moments are applied at the interface between the soil and the structure base, which has a finite dimension, 2b in width and 2c in length as shown in Fig. 2. The derivation of the equations of motion of the soil-structure interaction system including coupling effects represented in terms of interaction translational force and rotational moment vectors has been given by Kao (1), therefore, it will not be repeated here.

Porous elastic half space

Saturated soil is a two phase material consisting of a solid skeleton and water filled voids, referred to as a porous elastic soil medium with pore water flow. In the case of earthquake excitation, the wave travels through the porous elastic medium in the frequency range of 1.0-50.0 Hz. In formulating the behavior of the soil the usual assumptions are made, including the assumptions that the material is homogeneous and isotropic, the strains are small, the soil skeleton is completely saturated and water is incompressible, the soil and water move with the same ground acceleration, body forces are negligible, and the medium obeys Darcy's Law.

Based on the above assumptions the strain-displacement and stress-strain relations are

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$$
$$\sigma_{ij} = 2\mu e_{ij} + (\lambda e - p)\delta_{ij} \quad (1)$$

in which e_{ij} and σ_{ij} $(i,j=1,2,3)$ are the components of the strain and stress tensors, respectively, u_i is the displacement component along the i-th direction, λ and μ are Lamé's constants, p represents the pore water pressure, and δ_{ij} is the Kronecker delta. The equations of motion can in turn be expressed as

$$(\lambda+2\mu)\nabla^2 e - \nabla^2 p = \rho\ddot{e} \quad (2)$$

where ∇^2 denotes the Laplacian operator, ρ is the mass density of soil and water, and e represents the dilatation. Based on Darcy's Law and considering the water to be incompressible, the following relation can be obtained

$$\nabla^2 p = \frac{1}{K}\dot{e} \quad (3)$$

where K is the permeability coefficient per unit weight of water. Combining Eqs. (2) and (3), the result is

$$\{\nabla^2 - \frac{1}{K(\lambda+2\mu)}\frac{\partial}{\partial t} - \frac{1}{C_1^2}\frac{\partial^2}{\partial t^2}\}\, e = 0 \quad (4)$$

in which C_1 is the dilatational wave velocity given by

$$C_1 = \sqrt{\frac{\lambda+2\mu}{\rho}} \quad (5)$$

Solution of the wave equations

Taking the triple Fourier transform with respect to x,y, and t, Eq. (4) is transformed to

$$\nabla^2\bar{e} - \frac{1}{K(\lambda+2\mu)}\bar{\dot{e}} - \frac{1}{C_1^2}\bar{\ddot{e}} = 0 \quad (6)$$

where

$$\nabla^2\bar{e} = (-\alpha^2-\beta^2+\frac{d^2}{dz^2})\,\bar{e} \quad (7)$$

$$\bar{\dot{e}} = i\omega\bar{e} \quad (8)$$

$$\ddot{\bar{e}} = -\omega^2\bar{e} \tag{9}$$

and

$$\bar{e} = \left(\frac{1}{2\pi}\right)^{3/2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e(x,y,z,t)e^{i(\alpha x+\beta y+\omega t)}\, dxdydt \tag{10}$$

Substituting Eqs. (7) through (10) into Eq. (6), the solution for $\bar{e}$ is obtained as

$$\bar{e} = A_1 e^{-r_1 z} + B_1 e^{r_1 z} \tag{11}$$

in which

$$r_1^2 = \alpha^2 + \beta^2 - \frac{\omega^2}{C_1^2} + \frac{i\omega}{K(\lambda+2\mu)} \quad , \qquad Re(r_1) > 0 \tag{12}$$

As the depth of the soil medium $z\to\infty$, $\bar{e}$ must be finite, so that $B_1=0$, therefore

$$\bar{e} = A_1 e^{-r_1 z} \tag{13}$$

where, A_1 is an arbitrary function to be determined from boundary conditions. The inverse triple Fourier transform of $\bar{e}$ is

$$e(x,y,z,t) = \left(\frac{1}{2\pi}\right)^{3/2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A_1 e^{-r_1 z+i(x\alpha+y\beta+t\omega)}\, d\alpha d\beta d\omega \tag{14}$$

The solution for the pore water pressure is obtained in a similar manner, so that the transformed pore pressure is expressed as

$$\bar{p} = \frac{iA_1\omega}{K(r_1^2-r_2^2)} \left[e^{-r_1 z} - \frac{r_1}{r_2} e^{-r_2 z}\right] \tag{15}$$

in which

$$r_2^2 = \alpha^2 + \beta^2 \quad , \qquad r_2 > 0 \tag{16}$$

The transformed displacement components can be found in a similar manner (1). Likewise the stress components can be obtained following a similar procedure, and can then be evaluated at z=0, the surface of the soil medium, so that

$$- \frac{h^2 r_3}{\mu} \bar{\sigma}_{xz}(0) = \frac{2\alpha r_1 r_3}{r_1^2 - r_3^2} \{-i(j^2-h^2) + (r_1^2-r_3^2-j^2) \frac{N^2K_1}{\mu}\} A_1 + h^2(r_3^2+\alpha^2) A_2$$

$$+ h^2\alpha\beta A_3$$

$$- \frac{h^2 r_3}{\mu} \bar{\sigma}_{yz}(0) = \frac{2\beta r_1 r_3}{r_1^2 - r_3^2} \{-i(j^2-h^2) + (r_1^2-r_3^2-j^2) \frac{N^2K_1}{\mu}\} A_1 + h^2\alpha\beta A_2 \qquad (17)$$

$$+ h^2(r_3^2+\beta^2)A_3$$

$$- \frac{h^2}{\mu} \bar{\sigma}_{zz}(0) = \frac{1}{r_1^2-r_3^2} \{j^2r_1^2 + (j^2-2h^2)r_3^2 + 2i[(r_1^2-r_3^2)r_1r_2 - j^2r_1^2] \cdot \frac{N^2K_1}{\mu}\} A_1$$

$$+ 2ih^2\alpha A_2 + 2ih^2\beta A_3$$

where, N is the ratio of the shear wave velocity to the compression wave velocity of the medium, and

$$C_2 = \sqrt{\frac{\mu}{\rho}}, \quad r_3^2 = \alpha^2 + \beta^2 - \frac{\omega^2}{C_2^2}, \quad r_3 > 0, \quad h^2 = \frac{\omega^2}{C_1^2}, \quad j^2 = \frac{\omega^2}{C_2^2} \qquad (18)$$

The three arbitrary functions A_1, A_2, and A_3 can be determined from Eqs. (17) in terms of the specified contact stresses at the interface of the loading area. This leads to an eigenvalue problem which yields the frequency equation for the Rayleigh surface wave.

The displacement components of the ground surface can now be found. Then taking the inverse triple Fourier transform, the solutions for the displacement components at the ground surface subject to various types of motion are obtained in the form of integrals (1). Based on the expressions for the displacements, the dynamic ground compliances are obtained as the ratio of the foundation displacement in the direction of excitation to harmonic disturbing force or moment acting on the rigid foundation.

Finally, the solution for the reduced problem of a rectangular slab on a perfectly elastic half space can be obtained from the above expressions by setting the pore water pressure equal to zero, as shown in Ref. (1). Based on this solution and the correspondence principle, the solution for a rectangular slab on a viscoelastic half-space is obtained in a straight-forward manner.

NUMERICAL SOLUTIONS

In obtaining numerical results, initially static elastic problems pertaining to deflections and spring constants of the foundation medium are investigated and compared with those of existing results as given previously by Love (2) for vertical displacements and by Barkan (3) and Gorbunov-Passadov (4) for spring constants. Such comparisons indicate close correlation, as shown in Table 1, pertaining to the spring constants.

Table 1. Comparison of Spring Constants

Loading Condition	Spring Constant	Other Investigators	Present Study
Vertical	K_z	5.73 Gc Barkan [3]	5.01 Gc
Horizontal	K_x	4.75 Gc Barkan [3]	4.28 Gc
Rocking	K_θ	5.76 Gc^3 Gorbunov-Possadov [3]	6.48 Gc^3

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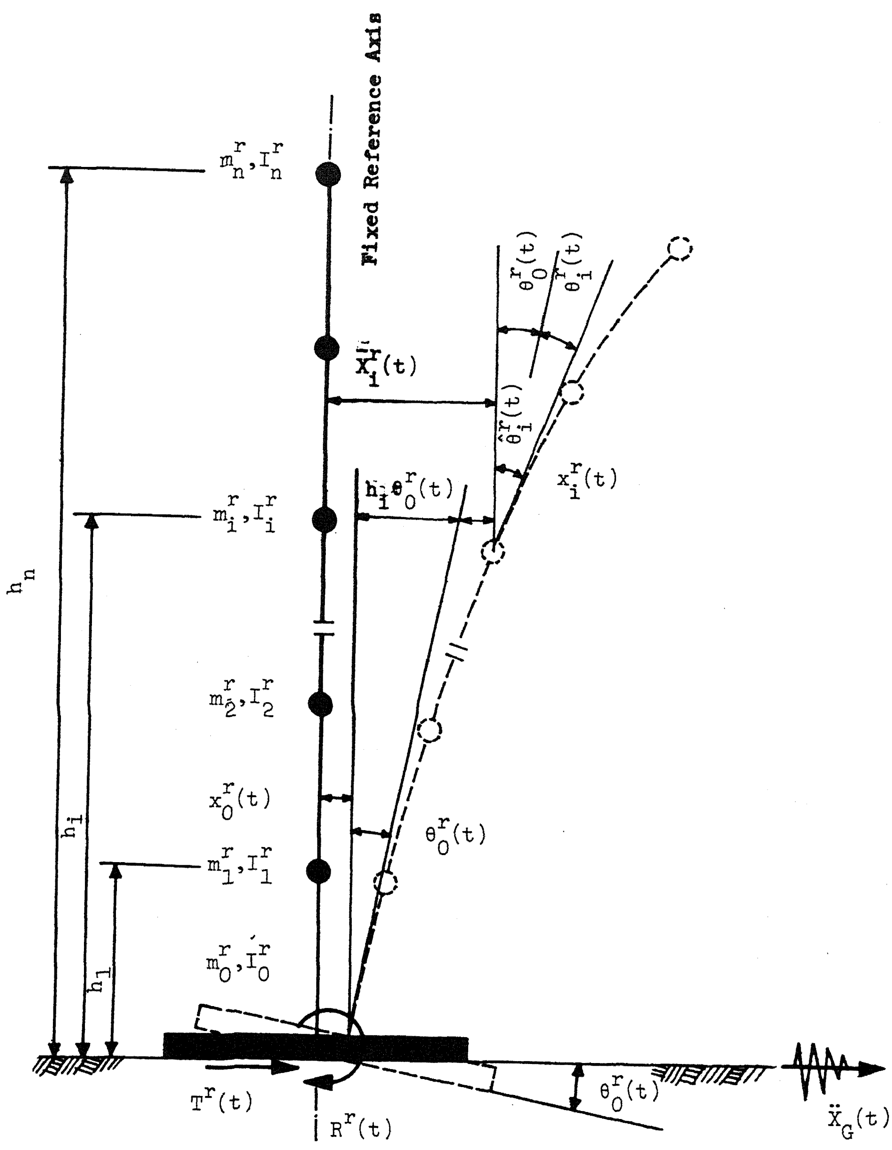

Fig. 1 - Structure-Foundation-Soil System

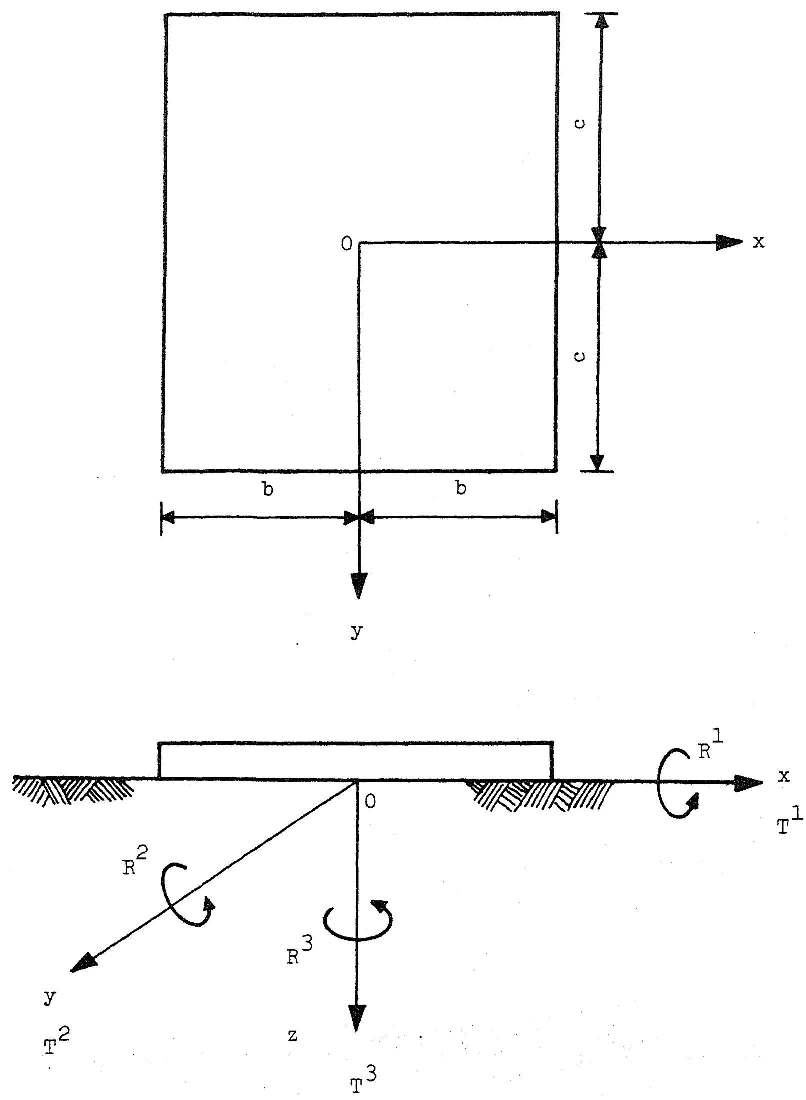

Fig. 2 - Foundation Base Slab