DYNAMIC SOIL-STRUCTURE INTERACTION UNDER A TOPOGRAPHICAL SITE CONDITION

by

Takuji KOBORI[I] and Yuzo SHINOZAKI[II]

SUMMARY

A study is made of the dynamic response of an elastic structure in the semi-cylindrical alluvial valley and subjected to the action of obliquely incident SH waves. Two kinds of soil-footing-structure interaction systems are considered. It is shown that the more closely the two-dimensional soil-structure system is located at the edge of the alluvial valley, the more pronounced the effect of incidence angle of SH waves on the displacement amplitude of the structure becomes and that the torsional vibration of the three-dimensional soil-structure system has predominant effect on the relative response of the structure.

INTRODUCTION

It has been recognized that the dynamic behavior of a building structure during an earthquake is considerably affected by the geological formation and the property of the soil medium under the structure. In particular, the geological and topographical irregularities in the soil medium seem to have a tremendous effect on the characteristics of the earthquake ground motion and also on the associated structural damage of the building. Although there have been many investigations about the elastic wave propagation in or around such geological and topographical irregularities of the soil medium, we can however point out only a few investigations about the effects of the amplification and focussing properties of elastic wave motions resulting from the local geological and subsurface soil conditions on the earthquake response of the structure considering the soil-structure interaction.[1~3]

In this paper, therefore, we investigate the effects of topographical site conditions on the earthquake response of a structure. Two kinds of soil-footing-structure interaction systems are considered as follows; (a) Two-dimensional antiplane soil-structure interaction system with an embedded rigid semi-cylindrical footing welded to a semi-cylindrical alluvial valley. (b) Three-dimensional soil-structure interaction system with a rigid circular footing placed on a semi-cylindrical alluvial valley.

MODEL AND METHOD OF SOLUTION

Two-dimensional antiplane soil-structure interaction.[2]

The two-dimensional soil-structure interaction system considered in this study is shown in Fig. 1. The structure is supposed to be a one-dimensional shear wall and to be erected upon a rigid semi-cylindrical footing with radius a_2 located on a semi-cylindrical alluvial valley with radius a_1 which is characterized by the mass density, ρ_1, and the shear wave velocity, c_1. We could define two polar coordinates, (r_2, ϕ_2) and (r_1, ϕ_1) with their

I Professor, Faculty of Engineering, Kyoto University, Kyoto Japan.
II Research Assistant, Faculty of Engineering, Kyoto Univ., Kyoto Japan.

origins located at the centers of the footing and the alluvial valley, respectively.

The incident plane SH-wave u_2^i is

$$u_2^i = u_0 \exp(-i(\omega t - \kappa_2 x_1 \cos\theta + \kappa_2 y_1 \sin\theta)) \qquad \cdots (1)$$

in which $\kappa_2 = \omega/c_2$: the wave number in soil medium II. Since the reflected wave u_2^r from the half-space boundary is the same form as u_2^i except that y is replaced by -y , the total free-field motion $u_2^i + u_2^r$ is expressed in terms of Bessel and harmonic functions convenient to match the boundary conditions.

When the soil-structure interaction system is subjected to the obliquely incident SH waves, three kinds of scattered waves should be considered in order to match the boundary conditions. They are the scattered wave u_1^F in the alluvial valley radiated from the footing, the scattered wave u_1^V in the alluvial valley and the scattered wave u_2^R in the surrounding elastic half-space radiated from the alluvial valley. We assume that the scattered waves u_1^F , u_1^V , and u_2^R have the forms.

$$u_1^F = \sum_{n=0}^{\infty} b_n H_n^{(1)}(\kappa_1 r_2) \cos n\phi_2 \qquad \cdots (2a)$$

$$u_1^V = \sum_{n=0}^{\infty} c_n J_n(\kappa_1 r_1) \cos n\phi_1 \qquad \cdots (2b)$$

$$u_2^R = \sum_{n=0}^{\infty} d_n H_n^{(1)}(\kappa_2 r_1) \cos n\phi_1 \qquad \cdots (2c)$$

in which b_n , c_n and d_n are the unknown coefficients. The scattered waves u_1^F , u_1^V and u_2^R in Eqs. (2) satisfy the scalar wave equation in polar coordinates in each domain of the soil media. We could express the scattered wave u_1^F in (r_1, ϕ_1) coordinates and the scattered wave u_1^V in (r_2, ϕ_2) coordinates convenient to satisfy the boundary conditions. Applying the Graf's Addition Theorem, we can transform u_1^V to the (r_2, ϕ_2) coordinates

$$u_1^V = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} (-1)^m c_n \epsilon_m L_m^n(\kappa_1 D) J_m(\kappa_1 r_2) \cos m\phi_2/2 \qquad \cdots (3)$$

in which $L_m^n(\kappa_1 D) = J_{n+m}(\kappa_1 D) + (-1)^m J_{n-m}(\kappa_1 D)$.

And the scattered wave u_1^F at the boundary $r_1 = a_1$ can be expanded into the Fourier series

$$u_1^F \big|_{r_1=a_1} = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \epsilon_m b_n Q_{nm} \cos m\phi_1 \qquad \cdots (4)$$

in which $Q_{nm} = \int_0^{\pi} [H_n^{(1)}(\kappa_1 r_2) \cos n\phi_2] \big|_{r_1=a_1} \cos m\phi_1 \, d\phi_1/\pi$.

In order to determine the displacement amplitude Δ of the footing subjected to seismic SH waves, we could divide the problem effectively into the problems: (A) The first step in the analysis corresponds to the evaluation of the force-displacement relation (the impedance function) for the soil in contact with the rigid massless semi-cylindrical footing embedded in the alluvial valley. (B) Secondly, we evaluate the "driving force", the resulting force which is exerted on the rigid massless semi-cylindrical footing by the soil, while the rigid footing is fixed to be immovable when it is subjected to the obliquely incident SH waves.

Imposing the boundary conditions upon $u_2^i + u_2^r$, u_2^R, u_1^V, and u_1^F, we get the infinite simultaneous equations for the unknown coefficients b_n^j, c_n^j, and d_n^j (j = A, B. n = 0, 1, 2, ---) in which the inhomogeneous terms are modified according to the problem (A) or the problem (B) expressed by

$$\epsilon_m \sum_{n=0}^{\infty} Q_{nm} b_n^j + J_m(X a_0)\, c_m^j - H_m^{(1)}(X a_0/c)\, d_m^j = {}_1 f_m^j \qquad \cdots (5a)$$

$$\epsilon_m \sum_{n=0}^{\infty} R_{nm} b_n^j + [\, J_m(X a_0)\,]' c_m^j - \mu [\, H_m^{(1)}(X a_0/c)\,]' d_m^j = {}_2 f_m^j \qquad \cdots (5b)$$

$$2 H_m^{(1)}(a_0)\, b_m^j + \epsilon_m J_m(a_0)(-1)^m \sum_{n=0}^{\infty} L_m^n(X D a_0)\, c_n^j = {}_3 f_m^j \qquad \cdots (5c)$$

in which a prime superscript denotes differentiation with respect to the argument. And R_{nm} is represented by the differential form of Q_{nm}. In Eqs. (5), the following dimensionless parameters are introduced

$$X = a_1/a_2 \;,\quad XD = D/a_2 \;,\quad a_0 = \kappa_1 a_2 = \omega a_2/c_1$$
$$c = c_2/c_1 \;,\quad \rho = \rho_2/\rho_1 \;,\quad \mu = \mu_2/\mu_1$$

The infinite simultaneous equations, Eqs. (5), can be solved approximately by considering the finite number of terms which closely approximate them. Once the coefficients b_n^j, c_n^j, and d_n^j, (j = A, B) are determined, the impedance functions and the driving forces of the rigid massless semi-cylindrical footing are easily evaluated.

Three-dimensional soil-structure interaction

Three-dimensional soil-footing-structure interaction system is shown in Fig. 2. The superstructure modelled consists of a top mass supported by four columns on a rigid circular footing with radius a_2 placed upon the surface of the semi-cylindrical alluvial valley which is subjected to the obliquely incident plane SH waves expressed in Eq. (1). Since it may be expected that the variation of the surface displacement amplitudes of the alluvial valley changes rapidly according as the frequency of the incident SH waves increases, it should be necessary to consider not only the translational vibration but also the torsional one of the three-dimensional soil-structure interaction system even though the superstructure may be idealized to have no eccentric mass and rigidity distributions.

It is much difficult to evaluate both torsional and translational vibration of the structure subjected to the obliquely incident SH waves considering strictly that the energy radiated from the footing is scattered by the boundary between the alluvial valley and the surrounding elastic half-space. Therefore, we have adopted the impedance functions of both torsional and translational vibration of the rigid massless circular footing on an elastic half-space which has the same characteristics as the alluvial valley. They were already evaluated in the previous works.[4,5] It may be easily supposed that the bigger the alluvial valley becomes compared with the footing, the more valid this approximation on the impedance functions will become.

In addition to those impedance functions of the rigid circular massless footing, it is necessary to evaluate the driving forces of the rigid circular massless footing on the surface of the alluvial valley subjected to the obliquely incident plane SH waves to determine the displacement amplitudes of the structure as described in the preceding section. These driving forces of the rigid circular massless footing are also approximately evaluated as follows.

First of all, the surface displacement of the alluvial valley subjected to incident SH waves is evaluated by Eq. (2b) and the surface displacement in the vicinity of the footing is expanded into the Fourier series by

$$u_1^V|_{y_1=0} = \sum_{n=0}^{\infty} c_n \, J_n(a_0(x+XD)) = F(x) \qquad \cdots (6a)$$

$$F(x) = \sum_{m=-\infty}^{\infty} f_m \exp(i\,m\,x) \qquad \cdots (6b)$$

in which $$f_m = \int_{-\pi}^{\pi} F(x) \exp(-i\,m\,x)\,dx \,/\, 2\pi \qquad \cdots (6c)$$

Then the displacement components on the surface of the alluvial valley in cylindrical coordinates, which are indispensable to evaluate the driving forces of the rigid circular massless footing, can be written as

$$u_r = \sum_{m=0}^{\infty} {}_c f_m \, (J_0(mr) + J_2(mr)) \sin\theta \qquad \cdots (7a)$$

$$u_\theta = i \sum_{m=0}^{\infty} {}_s f_m \, J_1(mr) + \sum_{m=0}^{\infty} {}_c f_m \, (J_0(mr) - J_2(mr)) \cos\theta \qquad \cdots (7b)$$

in which ${}_c f_m$ and ${}_s f_m$ are the Fourier cosine and sine integrations of $F(x)$, respectively, evaluated by Eq. (6c).

We have adopted these displacement components as the inhomogeneous terms of dual integral equations governing the dynamic mixed boundary value problems at the surface of the alluvial valley. By appropriate manipulation and substitution, the dual integral equations are reduced to Fredholm integral equations of the second kind which may be solved numerically. Thus, we have evaluated the driving forces for both torsional and translational vibrations of the rigid circular massless footing on the surface of the alluvial valley subjected to the obliquely incident plane SH waves.

NUMERICAL RESULTS AND DISCUSSIONS

We need the impedance functions as well as the driving forces of the rigid massless footing in order to investigate the effects of topographical site conditions on the dynamic responses of the soil-structure interaction systems shown in Figs. 1 and 2. Though both the impedance functions and the driving forces are frequency dependent, once they are numerically determined for the dimensionless frequency $a_0 = \omega a_2 / c_1$, the characteristics of responses of structures can be found simply by algebraic calculations.

Two-dimensional antiplane soil-structure interaction[2]

We assume, for simplicity, that the building is represented by a linear shear wall, whose rigidity is μ_b and shear wave velocity and mass density are c_b

and ρ_b , respectively.

Figs. 3 show the displacement amplitudes $|\Delta|$ of the footing normalized by the amplitude u_0 of the incident SH waves over the dimensionless frequency a_0 for the different values of the incidence angle, "THETA", Θ of SH waves for $\rho = 1.2$ and $c = 1.2$ compared with the amplitude response in the case of the uniform half-space which is represented by the dash-dot line. In the figures, the following parameters are introduced: MO and MB correspond to the masses per unit length of the footing and the shear wall, respectively. And $Ms = \pi\rho_1 a_2^2 / 2$ is the mass of the alluvial valley replaced by semi-circular cross section footing. ε describes the flexibility and the relative height of the shear wall and is defined by $\varepsilon = c_1 H / c_b a_2$.

The zeros of Δ at $a_\theta = [(2n + 1)/2](\pi / \varepsilon)$ for $n = 0, 1, 2,$ --- correspond to the natural frequencies of the shear wall. At these frequencies the base shear force per unit length is cancelled by the input driving forces and the footing does not move. When the center of the footing coincides with that of the alluvial valley, the displacement amplitude $|\Delta|$ of the footing is independent of the incidence angle Θ of SH waves, since the incident free-field motion as well as the soil-structure interaction model is symmetry with respect to the y axis. And the amplitude responses for $XD = 0.0$ show numerous sharp peaks at the resonant frequencies of the alluvial valley with the boundary at $r_1 = a_1$ fixed. Even when the structure is located at a distance from the center of the alluvial valley, the amplitude responses except when Θ is equal to be zero show similar numerous local peaks and troughs to the amplitude response for $XD = 0.0$. The amplitude response for $\Theta = 0$ increases with the increase of the distance XD , and reaches twice as much as the amplitude response for the uniform half-space when the footing is located most closely at the edge of the valley.

Figs. 4 show the relative responses of displacement amplitude between the footing and the top of the structure normalized by the amplitude u_0 of incident SH waves over the dimensionless frequency a_0 These figures show that the topographical site conditions of the alluvial valley have little influence on the relative responses of the displacement of the structure for the low frequencies from 0 to 0.3 for which both the top of the structure and the footing are supposed to vibrate in phase. However, as also shown in Fig. 3, the relative responses of the structure located closely at the edge of the alluvial valley for the higher frequencies show more pronounced amplification than the relative response of the structure for the uniform half-space.

Three-dimensional soil-structure interaction

In the following figures, the following parameters are introduced,

$$\lambda_T = \omega_T a_2/c_1 \quad , \quad \lambda_H = \omega_H a_2/c_1$$

in which ω_T : the natural frequency of torsional vibration of the top mass with the fixed base; ω_H : the natural frequency of translational vibration of the top mass with the fixed base; IF/IS and IT/IS are the dimensionless moments of inertia about the vertical axis of the footing and the top mass, respectively, and MF/MS and MT/MS are the dimensionless masses of the footing and the top mass, respectively. The Poisson's ratio of the alluvial valley is assumed to be 0.25.

Figs. 5 show the amplitude characteristics of the torsional displacements of the footing and the top mass normalized by the amplitude u_0 of the incident SH waves over the dimensionless frequency parameter a_0 / λ_T for several values of the incident angle Θ of SH waves. When the structure is located at the center of the alluvial valley and is subjected to the vertically propagating SH waves, no torsional vibration of the structure is generated. It is when the structure is located at the position for $D = 6a_2$ and subjected to the horizontally propagating SH waves that the torsional response of the structure is most amplified.

Figs. 6 show the amplitude characteristics of the translational displacements of the top mass normalized by the amplitude u_0 of the incident SH waves over the dimensionless frequency parameter a_0 / λ_H for several values of the incident angle Θ of SH waves. They are compared with the response of the top mass of a structure on an elastic half-space subjected to the vertically propagating SH waves which is represented by the dash-dot line. It is shown that the located position of the structure has as much influence on the translational responses of the three-dimensional soil-structure system as on the translational responses of the two-dimensional antiplane soil-structure system.

Figs. 7 show the relative displacement amplitudes, normalized by the amplitude u_0 of the incident SH waves, between the footing and the top mass of the structure considering both torsional and translational responses over dimensionless frequency a_0 for several values of the incident angle Θ of SH waves, which are compared with the response of the structure on an elastic half-space subjected to the vertically propagating SH waves. It should be noted that the relative responses between the footing and the top mass of the structure located at for $D = 6a_2$ subjected to the horizontally propagating SH waves are greatly amplified in the neighborhood of the resonant frequencies of torsional vibration of the top mass.

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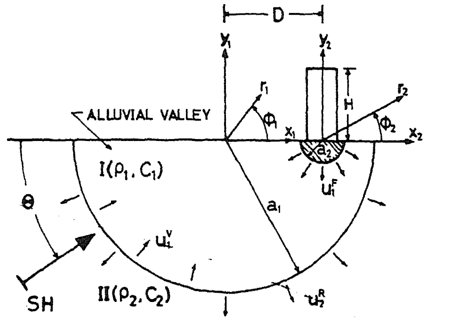


Fig. 1. Two-dimensional antiplane soil-structure interaction system.

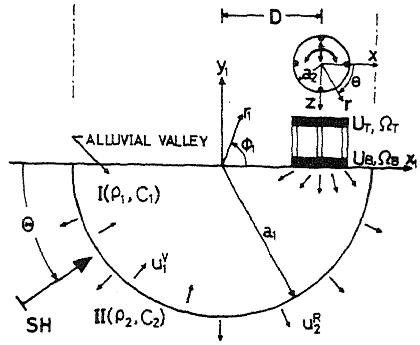


Fig. 2. Three-dimensional soil-structure interaction system.

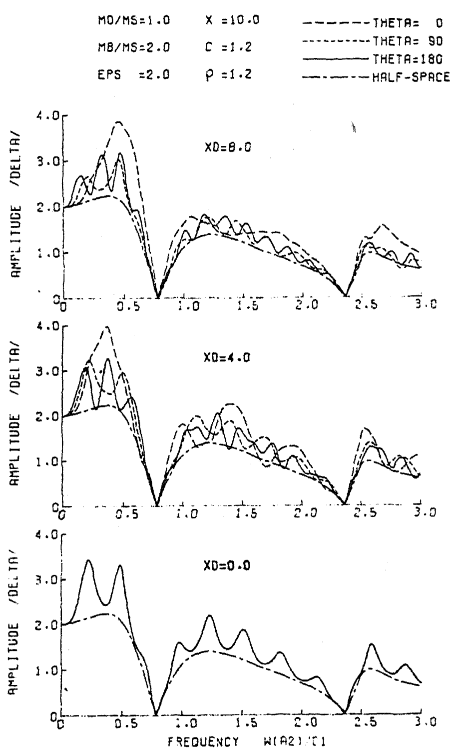


Fig. 3. Displacement amplitudes of the footing considering two-dimensional soil-structure interaction.

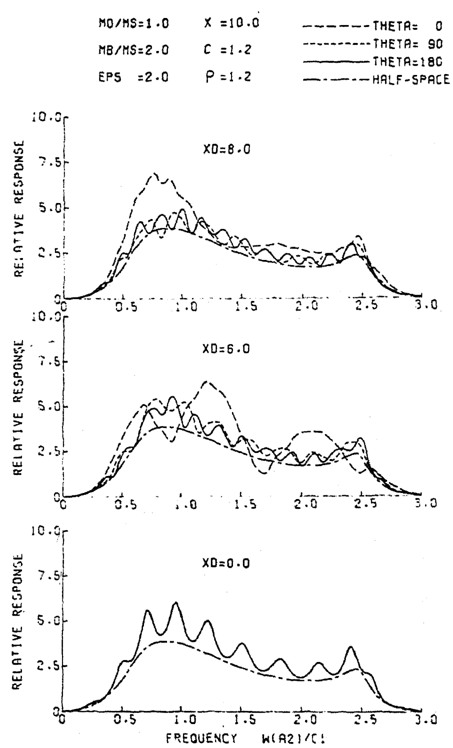
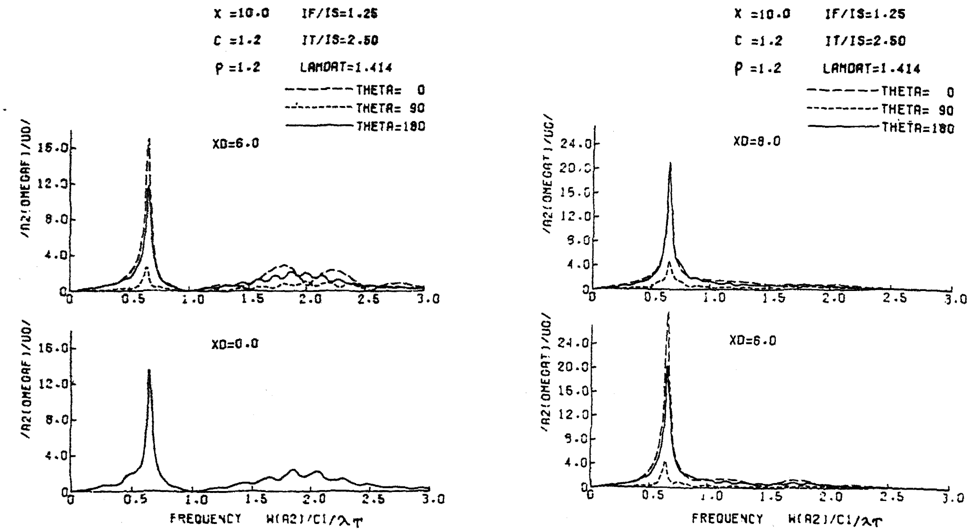


Fig. 4. Relative responses between the footing and top of the structure considering two-dimensional soil-structure interaction.



(A) The footing. (B) The top mass.

Fig. 5. Torsional displacement amplitudes of the structure considering three-dimensional soil-structure interaction.

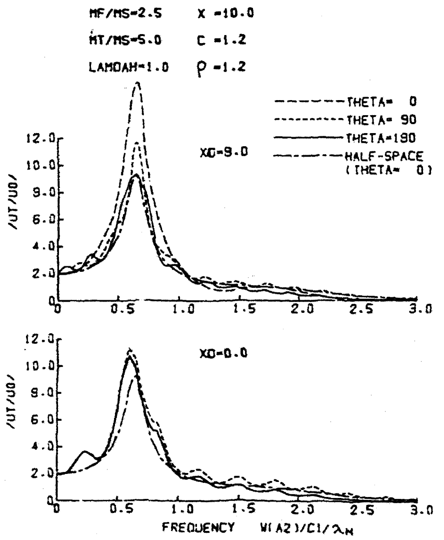


Fig. 6. Translational displacement amplitudes of the top mass of the structure considering three-dimensional soil-structure interaction.

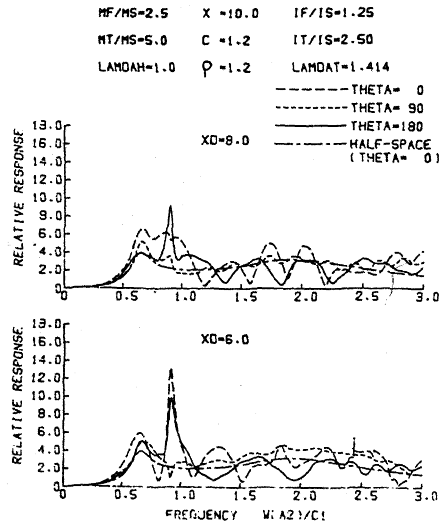


Fig. 7. Relative responses between the footing and top mass of the structure considering three-dimensional soil-structure interaction.