DYNAMIC BEHAVIOR OF FLEXIBLE STRUCTURES WITH VIBRATION ABSORBER

by

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SYNOPSIS

The dynamic response characteristics of flexible beams installed with dynamic vibration absorbers which are made up of mass, spring, and dashpot are dicussed in this paper. Not being modeled on the usual two-freedom-system, this vibration system is analysed exactly by the Laplace Transformation Method. The logarithmic decrements and the steady state response are calculated for simply supported beams and cantilever beams. Furthermore, by applying the absorbers to a suspension bridge, the effects are investigated experimentally and theoretically.

INTRODUCTION

In the design of long-span bridges and high-rise buildings, the vibration caused by earthquake motions or wind forces is one of the severe problems to be settled. Some attempts to reduce the dynamic motions by them have been done for the structural safety or amenity by adopting a dynamic vibration absorber called TMD (tuned mass damper) or dynamic damper. For example, in U.S.A., the application of a large scall TMD to tower structure is reported. In Japan,a dynamic damper is equiped to a pedestrian bridge to reduce the uncomfortable vibrations induced by passers-by.

But, examples of practical application of the dynamic vibration absorbers are generally found in mechanical engineering systems. Therefore, in the analysis of the vibrational characteristics of the system, the effect of flexibility of structures is not required to be taken into account and the mathematical model of this system is usually a two-degree-of-freedom one. However, this approach has deficiencies in its application to flexible structures. Namely, neither the vibrational modes of high degree or phase lags nor the behaviors of the vibrating absorbers can be analysed accurately. Consequently, it is also impossible to make the capacity of the vibration absorber optimum by the conventional method.

A simply supported beam and a cantilever beam are taken up here as typical flexible structures considering the application to pedestrian bridges, towers or high rise buildings.

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Working of the dynamic vibration absorber to the reduction of vibrations is expressed by the logarithmic decrements in each damped vibration mode. The response of cantilever beams exited by sinusoidal displacements at their fixed ends is calculated.
Finally, as an experimental study, a model test of a single spanned suspension bridge installed with dynamic vibration absorbers on its stiffened girder is presented.

BASIC EQUATIONS OF MOTION

Consider a uniform beam with constant flexural rigidity EI. A number of vibration absorbers are installed at arbitarary locations. The absorbers are assumed to consist of mass M_j, spring K_j and dashpot C_j as shown in Fig.(1). The equations of motion of transverse vibrations for the beam and the absorbers are coupled and given using the Diracs derta function as follows.

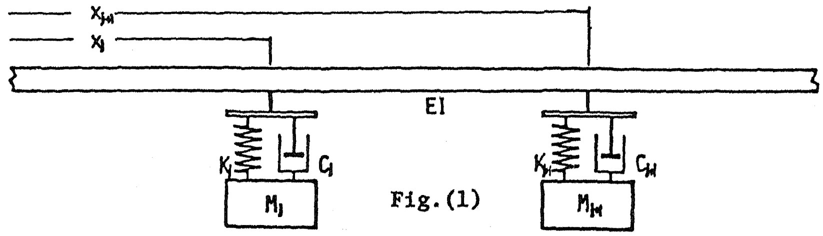

Fig.(1)

$$EI\frac{\partial y}{\partial x} - m\frac{\partial y}{\partial t} - \sum C_j\left[\frac{\partial y}{\partial t}-\frac{\partial y_j}{\partial t}\right]\delta(x-x_j) - \sum K_j(y-y_j)\delta(x-x_j) = \sum P_{k(x,t)} \quad \text{.................(1)}$$

$$M_j\frac{d^2y_j}{dt^2} + C_j\left[\frac{dy}{dt}-\frac{dy_{xj}}{dt}\right] + K_j(y_j-y_{xj}) = 0 \quad \text{.............................(2)}$$

where $P_{k(x,t)}$ is external forces acting on the point at x.
By dividing by m and M, and noting

$$\beta^4 = -s^2/b^2 \quad b^2 = EI/m \quad n_j = C_j/m \quad \kappa_j = K_j/m \quad n_{jo} = C_j/M_j \quad \kappa_{jo} = K_j/M_j$$

the Laplace transform of Eq.(2) with respect to t is given by

$$y_{jt} = \frac{1}{s^2+n_{jo}s+\kappa_{jo}}\left[s\,y_{jo} + y_{j1} + (n_{jo}s+\kappa_{jo})Y_{tx_j} - n_{jo}s\,y_{ox_j}\right] \quad \text{.............................(3)}$$

and the Laplace transform of Eq.(1) with respect to t and x is

$$Y_{tx} = \frac{1}{u^4-\beta^4}\Big[u^3\phi_0 + u^2\phi_1 + u\phi_2 + \phi_3 - \frac{1}{b^2}\Big[s y_{ox} + y_{1x} - \sum(n_j s+\kappa_j)Y_{tx_j}e^{-x_j u}$$
$$- \sum(n_j s+\kappa_j)y_{jt}e^{-x_j u} - \sum(n_j y_{ox_j} - y_{jo})e^{-x_j u} - \sum P_{tu}\Big]\Big] \quad \text{..........(4)}$$

in which the following initial conditions and boundarly conditions are adopted.

$t=0$: $y_{(x,0)} = y_0$ $\quad \partial y_{(x,0)}/\partial t = y_1$ $\quad y_{j(0)} = y_{jo}$ $\quad \partial y_{j(0)}/\partial t = y_{j1}$
$x=0$: $y_{(0,t)} = \varphi_0$ $\quad \partial y_{(0,t)}/\partial x = \varphi_1$ $\quad \partial^2 y_{(0,t)}/\partial x^2 = \varphi_2$ $\quad \partial^3 y_{(0,t)}/\partial x^3 = \varphi_3$

and $\quad L_t[y_{(x,t)}] = Y_{t(x,s)} = Y_t, \quad L_t[y_{i(t)}] = y_{it(s)} = y_{it}, \quad L_x[Y_{t(x,s)}] = Y_{tx(u,s)} = Y_{tx}$

$L_t[\varphi_0] = \Phi_0, \quad L_t[\varphi_1] = \Phi_1, \quad L_t[\varphi_2] = \Phi_2, \quad L_t[\varphi_3] = \Phi_3$

$Y_{t(x_i,s)} = Y_{tx_i}, \quad L_x[y_0] = y_{0x} \quad L_x[y_1] = y_{1x}, \quad L_{tx}[P_{(x,t)}] = P_{xt}$

From the inverse transform of the right hand terms of Eq.(4) with respect to u and s, the solution is obtained.

CANTILEVER BEAM

To investigate the effectiveness of dynamic vibration absorber, the transverse vibration of a uniform cantilever beam which is driven at its fixed end by a sinusoidally varying displacement in the form of $A \sin \omega t$ is analysed herein. A vibration absorber is attached to a arbitarary point of the beam shown in Fig.(2).
The equations of motion of the beam and the absorber are written as follows respectively.

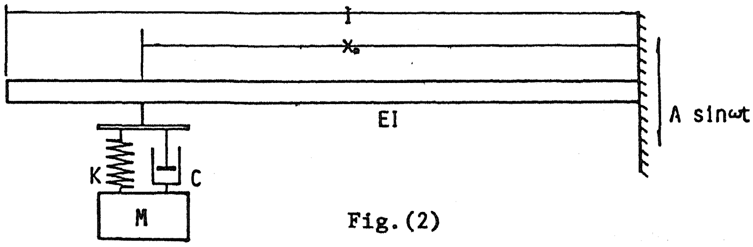

Fig.(2)

$$EI\frac{\partial^4 y}{\partial x^4} - m\frac{\partial^2 y}{\partial t^2} - C\left[\frac{\partial y}{\partial t} - \frac{\partial y}{\partial t}\right]\delta(x-x_0) - K(y-y)\,\delta(x-x_0) = 0 \quad \text{.....................(5)}$$

$$M\frac{d^2 y}{dt^2} - C\left[\frac{dy}{dt} - \frac{dy_{x_0}}{dt}\right] - K(y - y_{x_0}) = 0 \quad \text{...................................(6)}$$

Applying here also the Laplace Transform Method to Eq.(5) and Eq.(6), the following transform equation is derived.

$$Y_{tx} = \frac{1}{u^4-\beta^4}\left[u^3\Phi_0 - u^2\Phi_1 - [f_2(f_4-f_3)/b^2 f_4]\, Y_{tx_0}\, e^{-x_0 u}\right] \quad \text{.............................(7)}$$

In these equations, the initial conditions and the boundary conditions are

$t = 0 \;:\; y_{(x,0)} = 0 \quad \partial y_{(x,0)}/\partial t = 0. \quad y_{(0)} = 0. \quad dy_{(0)}/dt = 0.$

$x = 0 \;:\; y_{(0,t)} = \varphi_0 \quad \partial y_{(0,t)}/\partial x = \varphi_1 \quad \partial^2 y_{(0,t)}/\partial x^2 = 0. \quad \partial^3 y_{(0,t)}/\partial x^3 = 0$

$x = l \;:\; y_{(l,t)} = A \sin \omega t - F_{(t)}. \quad \partial y_{(l,t)}/\partial x = 0.$

and $\quad f_2 = ns + K. \quad f_3 = n_0 s + K_0. \quad f_4 = s^2 + n_0 s + K_0.$

From Eq.(7), the inverse transform of Y_{tx} with respect to u becomes

$$Y_t = [L_t[F_{(t)}]/D]\,[D_0(\cosh\beta x + \cos\beta x) - D_1(\sinh\beta x + \sin\beta x)]$$
$$- [f_2(f_4 - f_3)/2\beta^3 b^2 f_4]\,[D_0(\cosh\beta x_0 + \cos\beta x_0) - D_1(\sinh\beta x_0 + \sin\beta x_0)]\,[\sinh\beta(x-x_0) - \sin\beta(x-x_0)] \quad \text{..(8)}$$

$$\mathbb{D} = 2(1+\cosh\beta x\cdot\cos\beta x) - [f_1(f_4 - f_3)/2\beta^3 b^2 f_4][(\sinh\beta x_0 + \sin\beta x_0)(\cosh\beta l + \cos\beta l)[\cosh\beta(l - x_0) - \cos\beta(l - x_0)) + (\cosh\beta x_0 + \cos\beta x_0)[\sinh\beta(l - x_0) - \sin\beta(l - x_0)](\cosh\beta l + \cos\beta l) - (\cosh\beta x_0 + \cos\beta x_0)(\sinh\beta l + \sin\beta l)[\cosh\beta(l - x_0) - \cos\beta(l - x_0)] - (\sinh\beta x_0 + \sin\beta x_0)[\sinh\beta(l - x_0) - \sin\beta(l - x_0)](\sinh\beta l - \sin\beta l)] \quad \ldots\ldots (9)$$

$$D_0 = \cosh\beta l + \cos\beta l - [f_2(f_4 - f_3)/2\beta^3 b^2 f_4](\sinh\beta x_0 + \sin\beta x_0)[\cosh\beta(l - x_0) - \cos\beta(l - x_0)]$$

$$D_1 = \sinh\beta l - \sin\beta l - [f_2(f_4 - f_3)/2\beta^3 b^2 f_4](\cosh\beta x_0 + \cos\beta x_0)[\cosh\beta(l - x_0) - \cos\beta(l - x_0)]$$

Letting $\mathbb{D}=0$, the characteristic equation should be obtained. Assuming that the roots of the characteristic equation is $r = R_n \pm iI_n$, the complex number s becomes

$$s = -2pR_nI_n \pm ip(I_n^2 - R_n^2). \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

From Eq.(9), the damped natural frequency ω_{nd} and the logarithmic decrement δ_n are calculated as follows respectively.

$$\omega_{nd} = [(I_n^2 - R_n^2)/l^2]\sqrt{EI/m} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

$$\delta_n = 4\pi R_n I_n/(I_n^2 - R_n^2) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

And the inverse transform of Y_t is equal to the sum of the residues at the all singular points of Y_t.

$$y_{(x,t)} = [A\omega D e^{st}/\{d(s^2+\omega^2)/ds.\ \mathbb{D}\}]_{s=\pm i\omega} + [\Sigma A\omega D e^{st}/(s^2+\omega^2)\cdot d\mathbb{D}/ds)]_{s=-2pR_nI_n \pm ip(I_n^2 - R_n^2)} \quad . (13)$$

Fig.(3) and Fig.(4) show the relationship between the logarithmic decrements and the damping coefficient of vibration absorbers in the 4th and the 3rd damped vibration. Dimensionless ratio $\mu_c = C_c l/\sqrt{mEI}$ could be conveniently written in terms of the value of the coefficient of viscosity $C_c = 2\sqrt{MK}$ which is required for the critical damp of the vibration absorber. By these figures, it may be seen that there is a appropriate damping coefficient of absorbers which makes absorber most effective and even though the absorber mass is small, a respectable logarithmic decrement could be expected in any vibration mode.

Representative results of the steady state response at the free end of cantilever beam is illustrated in Fig.(5). This figure shows that even with a very small vibration absorber (mass ratio $\bar{m}$ =100, μ_c =0.1), the maximum resonant deflection of the free end is only 10 times of the fixed end displacement if the absorber is tuned in good condition.

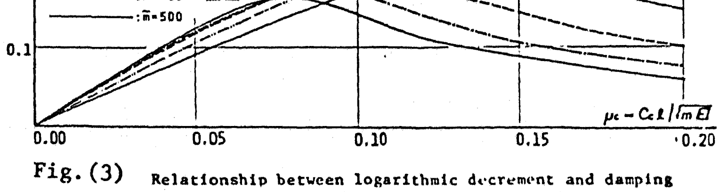

Fig.(3) Relationship between logarithmic decrement and damping coefficient of absorbers in the 4th mode vibration