# LAYERED SOIL-PILE-STRUCTURE DYNAMIC INTERACTION

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## SYNOPSIS

This paper investigates the soil-structure interaction problem for a multiple span continuous bridge supported by high piers with deep pile foundations in layered soil medium. Analysis is carried out on both an isolated pier-footing system with the girder effect on its top and a complete structure, checking the structural modeling in connection with the analysis. The substructure method is applied to advantage. Importance of the soil effect, which is quite different from a caisson type foundation, is pointed out for the present structure.

## INTRODUCTION

Soil-structure dynamic interaction has gained much interest in the earthquake engineering field, and has been recognized as an important factor in designing structures in certain situations. The work involved is to find the soil impedance functions associated with the foundation motion to be encountered and the subsequent structural response analysis based on these. Continuum mechanics approach or finte element methods are available for the former purpose, and the so-called dynamic substructure method is used to advantage to combine the soil effect with the superstructure dynamics [1-3].

Often the soil medium is represented for convenience of analysis by an elastic or visco-elastic homogeneous half-space. The real soil profile, however, is rather layered in such a way that a soft surface layer overlies a stiffer one. Another common modeling is to assume layers on a rigid base rock. The dynamic features may be quite different in getting impedance functions for a half-space from when consider an input motion at base rock. The surface layer plays a substantial role in structural response in certain situations.

Herein a three-span continuous bridge with high pier, which is supported by a deep group-pile foundation (Figs. 1) is analyzed with the above things in mind. The behavior in the direction parpendicular to the bridge axis is of interest. The formulation consists of (i) Evaluating a pile-head impedance and a driving force from soil-pile interaction; (ii) Establishing the governing equations of motion for a group pile - footing-pier system and for a rigidly fixed girder system; (iii) Integrating these sub- and superstructures with use of a force equilibrium and displacement compatibility at their interface. Classical normal modes decomposition is assumed for the constraint superstructure as well as substructure if necessary. This process is described in Ref. 3.

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In modeling the structure, the following three cases are considered according to the phase of analysis: Case I-Isolated pier structure with girder's mass of half of adjacent spans: Case II- Isolated pier structure with a single-degree-of-freedom girder; and Case III-Complete structure.

FORMULATION

Soil-Pile Interaction- Dynamics of a pile embedded deep in soil medium are analyzed from a continuum mechanics approach, solving the Navier's wave equation and beam equation. For a layered soil medium an extension of the preceeding work by Tajimi[4] and Novak et al.[5] is made with use of transfer matrix method [6]. In Ref. 6 simplified idealization is made such that: (i) Soil profile is layered within which the material properties are constant; whose bottom is rigid base at which earthquake motion is imposed. Soil internal damping, representing the hysteretic energy absorption, is introduced by assuming complex Láme constants; (ii) Pile-footing system comprises a group of point bearing piles which reach down to the rigid base and built into it, and a rigid footing placed on them. Their connection is rigid. (iii) Soil restoring force is only counted from the soil surrounding piles, neglecting the one directly in contact with footing, since the former is primary while the latter secondary for a deep pile foundation in soft soil medium. (iv) Pile vibration is analyzed for lateral and longitudinal directions separately.

Soil-pile dynamics, when subjected to base input motion, is solved by breaking it into two phenomena: one is the free field soil layer vibration with piles absent due to an incident wave at the rigid base. This concerns the shear wave (SH) propagation; and the other relates the pile vibration due to the loading at its head because of the superstructure and to the soil reaction along its length because of the scattering wave from piles. The perfect bond assumption between soil and pile determines the latter quantity. Evaluation of the soil reaction is carried out by both rigorous and approximate (plane strain assumption) methods.

The pile head impedance matrix, which relates force and displacement at this section, is expressed as

$$[K_{pile}] = \begin{bmatrix} (E_pI_p/H^3)\ F_{xx} & (E_pI_p/H^2)\ F_{x\theta} & 0 \\ (E_pI_p/H^2)\ F_{\theta x} & (E_pI_p/H)\ F_{\theta\theta} & 0 \\ 0 & 0 & (E_pA_p/H)\ F_z \end{bmatrix} \qquad (1)$$

Substructure(Pile-Footing-Pier System)-Footing is assumed as a two-degree-of-freedom rigid body of horizontal translation and rotation. The governing equation, when subjected to an input motion $u_g$ at the rigid base, is

$$[M]_F\{\ddot{x}\}_F + [C]_F\{\dot{u}\}_F + [K]_F\{u\}_F = [\alpha]^T\{F\}_s + [\beta]_F^T\{R\} \qquad (2)$$

in which $\{x\}_F$=absolute displacement at the gravity center; $\{u\}_F$=relative displacement; $[M]_F$, $[C]_F$ and $[K]_F$=mass, damping and stiffness matrices, respectively; $[\beta]_F$=displacement influence matrix from footing gravity center to its top; $\{R\}$=internal forces acting at footing top. The first term in the forcing function $\{F\}_s$ is due to the soil layer vibration; and $[\alpha]$=displacement influence matrix from footing gravity center to pile head location.

In evaluating $[C]_F$ and $[K]_F$ the individual pile head impedance is summed up by accounting for the geometric constraints.

For the Case I and II analyses, the equation of motion for an isolated pier with girder's effect is involved, which is given from the matrix formulation by

$$[M]_P\{\ddot{x}\}_P + [C]_P\{\dot{u}\}_P + [K]_P\{u\}_P = \{0\} \qquad (3)$$

in which a proportional damping matrix $[C]_P$ is presumed to yield a constant damping factor from mass and stiffness matrices, $[M]_P$ and $[K]_P$. The inertia forces relate the internal forces

$$\{R\} = -[\beta]_P^T [M]_P \{\ddot{x}\}_P \qquad (4)$$

in which $[\beta]_P$= displacement influence matrix given from the rigid body kinematics.

<u>Superstructure(Girder System)</u>- The matrix formulation yields the governing equation for this part as

$$[M]_G\{\ddot{x}\}_G + [C]_G\{\dot{x}\}_G + [K]_G\{x\}_G = \{F\}_G \qquad (5)$$

in which $[M]_G$, $[C]_G$ and $[K]_G$= mass, damping and stiffness matrices, respectively, $\{F\}_G$= external force vector, $\{x\}_G$= absolute displacement vector which includes free nodal displacement $\{x_i\}$ together with the given displacement at pier top $\{x_j\}$. The former is evaluated as the sum of quasi-static deformation due to $\{x_j\}$ and dynamic effect with constraint boundary, i.e.

$$\{x_i\}_G = [\beta]_G\{x_j\}_G + \{x_i^c\}_G \qquad (6)$$

in which $[\beta]_G$= displacement influence matrix. Since the continuous multi-span girder makes a statically indeterminate system, it is computed by

$$[\beta]_G = -[K_{ii}]_G^{-1}[K_{ij}]_G \qquad (7)$$

in which $[K_{ij}]_G$= partitioned stiffness matrix corresponding the above variable separation. Furthermore, for constraint deformation normal modes decomposition is executed.

<u>Integrated Structure(Complete System)</u>- Multiple piers and continuous girder motions are coupled in order to investigate the complete system behavior-Case III analysis. The respective substructure equation is obtained as already formulated but exerting the internal forces $\{F_j\}$ at pier tops. The continuity between sub- and superstructures is required as

$$\{x_j\}_G = \{x_j\}_P \quad (8) \qquad \{F_j\}_G + \{F_j\}_P = \{0\} \qquad (9)$$

For convenience of analysis, normal modes are imposed on the substructure by neglecting the off-diagonal elements in the damping matrix in those coordinates, which is not an improper assumption for soil-structure interaction [2]. Hence, the integrated system equation can be expressed in terms of the respective subsystem normal modes only.

$$[M]_{GPF}\{\ddot{q}\}_{GPF} + [C]_{GPF}\{\dot{q}\}_{GPF} + [K]_{GPF}\{q\} = \{F\}_{GPF} \quad (10)$$

The Fast Fourier Transform algorithm is used to advantage for response analysis since the stiffness and forcing functions are frequency dependent due to the soil-structure interaction. First taking the Fourier transform of Eq. 10

$$[Z(\omega)]_{GPF}\{\hat{q}(\omega)\}_{GPF} = \{\hat{F}(\omega)\}_{GPF} \quad (11)$$

and computing for the frequency response function to obtain the response in frequency domain; and then one finds the time history as its inverse Fourier transform.

## NUMERICAL RESULTS AND DISCUSSION

Some numerical computations were carried out for the Case I, II and III analyses on structural models illustrated in Figs. 1 and Tables 1. Through the example case studies the following is discussed:

Pile Head Impedance- The complex impedance, when expressed in the form of equivalent stiffness and viscous damping, are shown for each pile head movement in Figs. 2 versus a dimensionless frequency which is defined as $a_o=r_o\omega/V_s$ where $r_o$ is the pile radius, $\omega$ is a driving frequency and $V_s$ denotes the shear velocity of top soil layer. The internal soil damping D, which is the so-called loss factor, is assumed as 20 percent. This value is likely measured during strong earthquake motions. Fig. 2.a is computed for a single uniform surface layer of $V_s$=200 m/s. The rigorous solution, being frequency dependent in the very low range of $a_o$, for instance less than 0.2 for the present case, indicates some irregularities at the natural frequencies of the surface soil layer; and beyond that beomes rather flat. Surprisingly, the plane strain solution with no internal soil damping can give a close approximation to the rigorous one. According to other analyses, this approximation becomes much better as the internal soil damping grows but poor as it decreases. In Fig. 2.b are shown the case of a multi-layered soil profile at Bannosu Area, Kagawa, Japan, which is constructed after nonlinear analysis by the SHAKE program for Miyagiken-oki earthquake (June 1978) record but modified as 100 gal maximum acceleration.

Structural Dynamic Characteristics- These are investigated from the classical normal mode analysis for Eqs. 3 and 5 . In Case I and II analyses on isolated piers, a remarkable change of the fundamental mode is noted due to the soil-structure interaction from when rigidly supported. Mode shapes are omitted herein but one can observe the change in natural frequencies in Figs. 4. The fundamental natural frequency $\omega_1$ of the rigid support system is decreased to that of the interaction system represented by the peak of the footing rotation. Figs. 3 and Table 2 show the vibration modes of the complete system. For the interaction systems the soil impedance at flat frequency range is used. Modal synthesis is made such that the first three lateral normal modes are picked up from each substructure and the first three lateral and torsional modes are from each girder span. Comparison with other substructuring [7] gurantees the reproduction of the complete system modes in low frequency range. An interesting fact is noted, that the girder modes are not so affected by the soil effect as the pier modes are, where sway motion is exerted. One may give an explanation that the substructures are softened as to yield footing sway but the superstructure are excluded from being amplified in response, compared with when rigidly supported.

Frequency Response Function- Figs. 4 show the frequency response amplification for the isolated substructure - Case I and II analyses. The free field soil amplification is also depicted. As aforementioned, the soil-structure interaction is significant. Note that the footing sway is almost governed by the free surface soil layer vibration whose natural frequencies are denoted by $h_i$, but at the girder or pier top the interaction mode dominates the response. Case III is for the complete system in which soil profile from the Miyagi earthquake is used. Note that at the footing top the Model I has two peaks the first of which concerns the soil-structure interaction modes while the second of which represents the predominant soil layer vibration mode. Compared with the isolated pier analysis, where the reverse order of vibration modes is observed, the complete system analysis is emphasized. The same trend is noted in soil profile from Izu-oki earthquake (May 1974) record. At the girder part, the soil layer vibration amplifies the close soil-structure interaction modes.

Earthquake Response- Figs. 5 show the maximum relative response due to earthquake motion input at rigid base level. The half-reduced pile stiffness at the internal substructures are also analyzed. These classifications are made by the solid and dashed lines. First note that an isolated pier analysis gives a smaller response value compared with the complete system analysis. Second, reducing stiffness at internal foundations increases response in Model I but not necessary for Model II. Third, decreasing the stiffness and increasing the mass of one internal pier yield greater response due to its soil-pier interaction, which leads adverse effects for the girder part connected to it. The trend is more remarkable for the Izu earthquake motion than for the Miyagi earthquake motion. In Figs. 5 are also shown the response by classical normal modes analysis (the thin lines). This approximation is comparable for Model I but underestimates for Model II.

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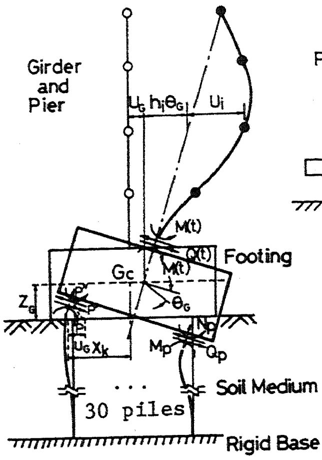



Figure 1. Models for Analysis
(b) Isolated Pier-footing-pile system

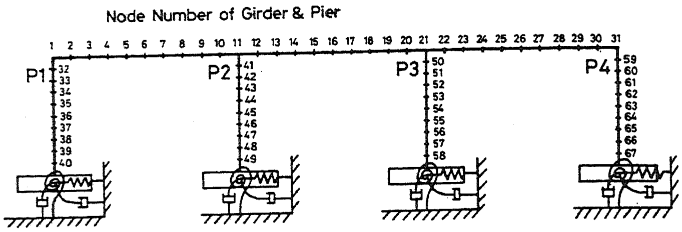
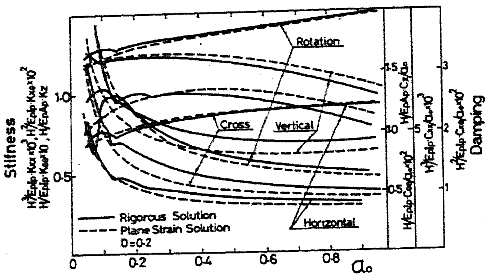
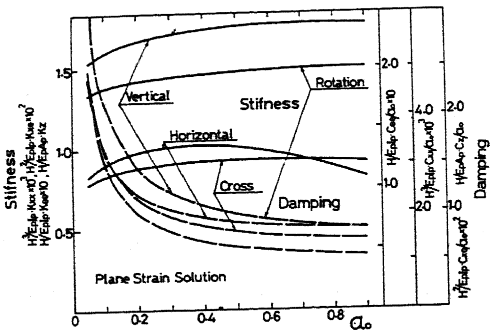



Figure 1. (a) Three-span Continuous Bridge




(a) Single Uniform Layer (Comparison between rigorous and plane strain)




(b) Illustrative multi-layer (Bannosu Area, Kagawa, Japan)

Figure 2. Pile Head Impedance Functions

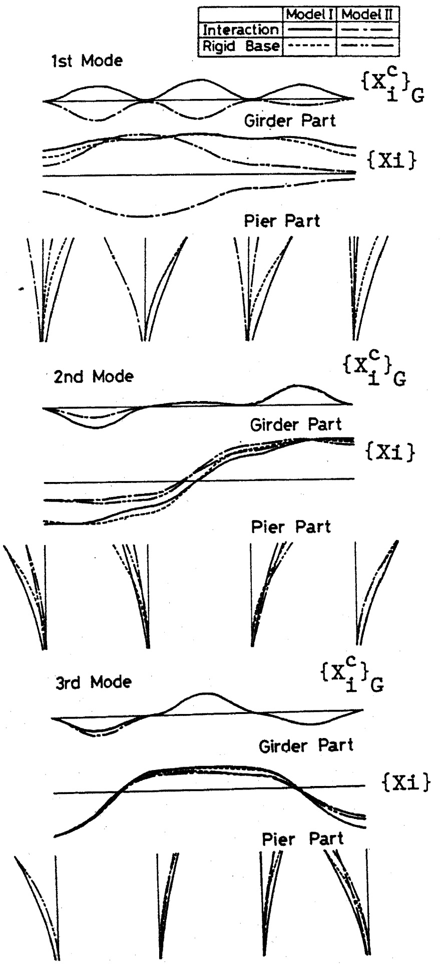



Figure 3. Vibration Modes for Superstructure - Comparison between interaction and rigid base

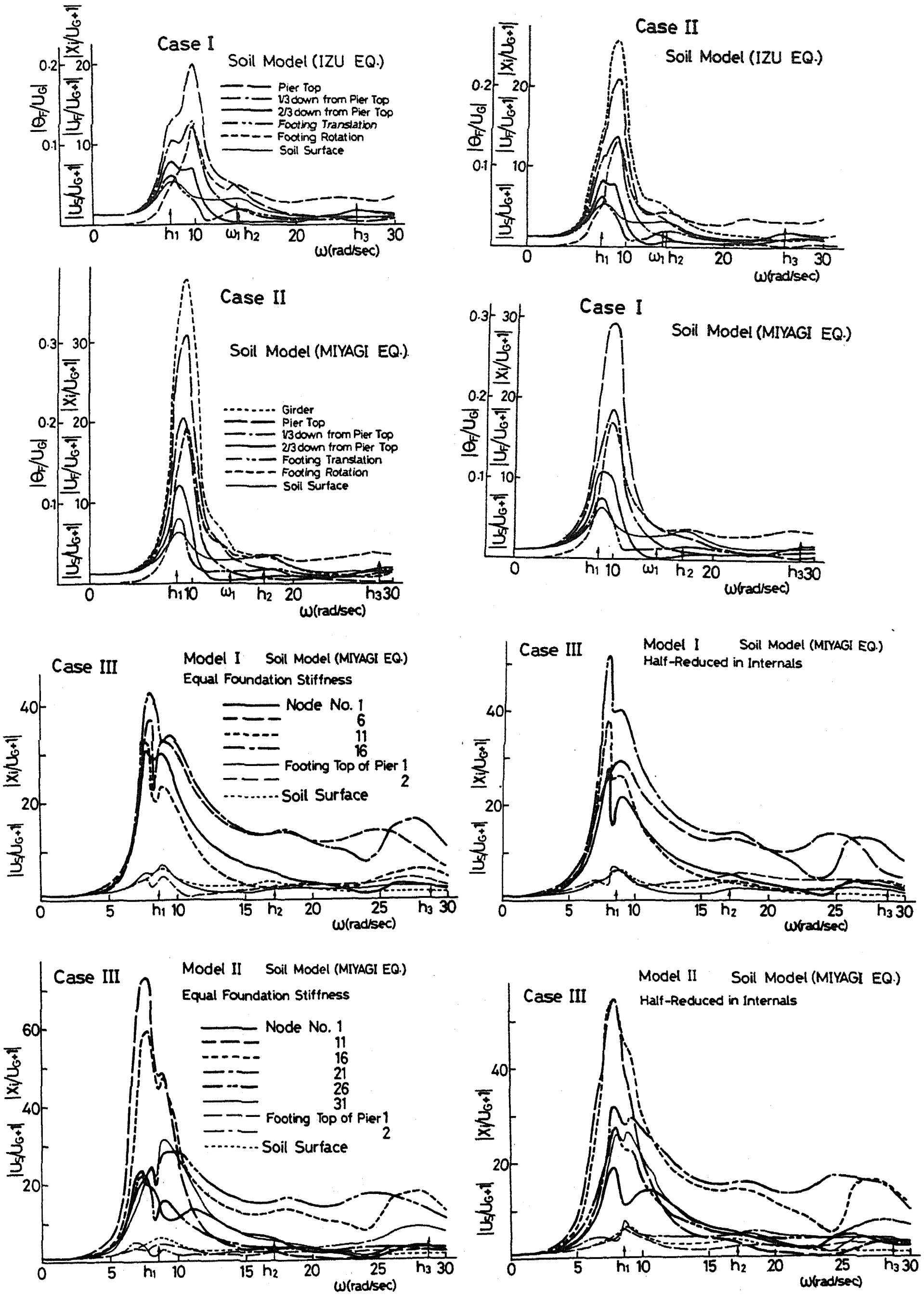
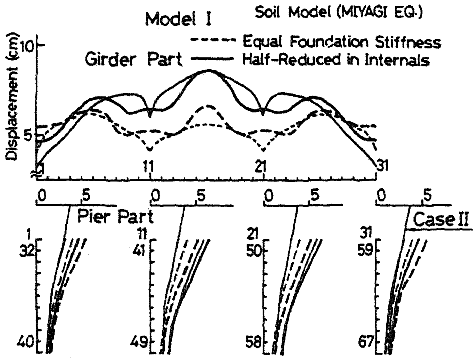
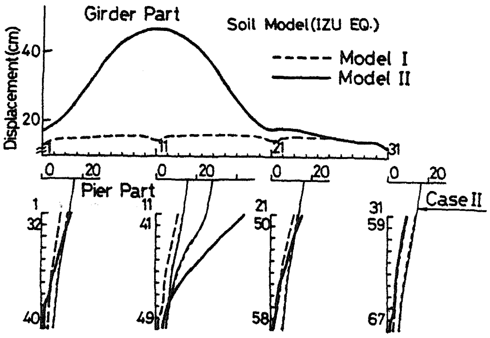



Figure 4. Frequency Response Functions
( 5 % damping for pier and 2 % for girder modes)



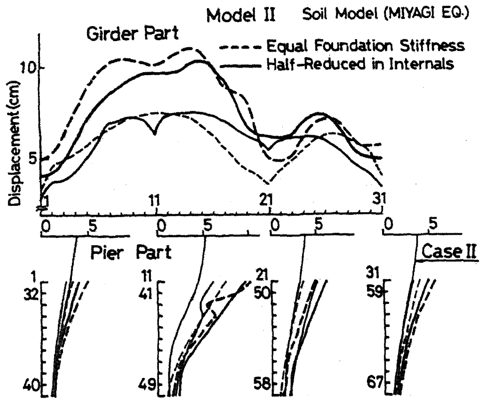




Figure 5. Maximum Relative Displacement

Table 1.a Substructural Dimensions

| Dimensions \ Structural Part | Pile | Footing | Pier |
|---|---|---|---|
| Moment of Inertia ($m^4$) | 3.46 | - | $4.717 \times 10^3$ |
| Mass Moment of Inertia ($t\text{-}m^2$) | - | $1.486 \times 10^6$ | - |
| Young's Modulus ($t/m^2$) | $2.1 \times 10^6$ | - | $2.7 \times 10^6$ |
| Cross Sectional Area ($m^2$) | 4.52 | 36x30 | 52.0 |
| Hight (m) | 52.0 | 5.0 | 58.0 |
| Weight (t/m) | 11.3 | | 116.5 |
| Weight (t) | | 13500 | - |

Table 1.b Three-span Continuous Bridge

| Structures / Dimensions | Model I | | Model II | | |
|---|---|---|---|---|---|
| | Girder | Piers 1 to 4 | Girder | Piers 1,3,4 | Pier 2 |
| Length (m) | 120 | 58 | 120 | 58 | 58 |
| Young's Modulus ($t/m^2$) | $2.1 \times 10^7$ | $2.69 \times 10^6$ | $2.1 \times 10^7$ | $2.69 \times 10^6$ | $2.69 \times 10^6$ |
| Shear Rigidity ($t/m^2$) | $8.1 \times 10^6$ | | $8.1 \times 10^6$ | | |
| Inertia Moment ($m^4$) (bending) | 76.9 | 4720 | 76.9 | 4720 | 1826 |
| Inertia Moment ($m^4$) (torsion) | 115.2 | | 115.2 | | |
| Weight (t/m) | 66.0 | 117 | 66.0 | 117 | 257 |
| Cross Sectional Area ($m^2$) | 0.6 | 52 | 0.6 | 52 | 115 |

Table 2 Natural Frequencies (rad/sec)

| Vibration Modes | Interaction | | | | Rigid Base | |
|---|---|---|---|---|---|---|
| | Miyagi eq. Soil | | Izu eq. Soil | | | |
| | Model I | Model II | Model I | Model II | Model I | Model II |
| First | 7.77 | 7.08 | 7.84 | 7.31 | 7.92 | 7.63 |
| Second | 7.93 | 7.91 | 7.96 | 7.94 | 8.00 | 7.98 |
| Third | 8.05 | 8.05 | 8.05 | 8.05 | 8.06 | 8.06 |
| Fourth | 9.44 | 8.54 | 10.28 | 8.80 | 12.77 | 9.63 |