# A SIMPLIFIED ANALYSIS METHOD ON INTERACTION SYSTEM ABOUT STRUCTURE-FOUNDATION-SOIL SURFACE LAYER

by

Yuzuru Yasui[I]

## SUMMARY

This report describes on analysis method for a structure-foundation-soil surface layer interaction system. This method includes two special features. One is the proposal of a simple treatment of the reflection wave effects of the lower rigid boundary, and the other is approximate solutions of interaction coefficient of a rigid circular disk on an elastic stratum. In this report the author presents the above method and approximate solutions, and next analyzes test data and observed earthquake responses using a dynamic model simulating a structure-foundation-soil surface layer system.

## INTRODUCTION

Numerous studies have been conducted to investigate the effect of the flexibility of the ground foundation on the seismic response of buildings. Most of the studies have assumed the foundation medium to be represented by an elastic half space as in Ref. 1, 2 and 3. In the actual soil grounds under the building foundation, however, it is often experienced that they have several layers. In order to investigate the interaction effect between the structure and the multi-layered ground, two kinds of interaction coefficients about a rigid massless disk on the ground are necessary. As shown by Thau in Ref. 4, one is dynamic stiffness and the other is rigid-fixed forcing function. On the basis of the elastic wave propagation theory, the solutions about the dynamic stiffness were obtained by Kobori, Kashio and Luco in Ref. 5, 6 and 7 respectively. Those solutions are represented by the form of integral or by means of integral equations, so that numerical calculations of the solutions cost us no slight labor.

In this paper, approximate solutions of the interaction coefficients of a rigid circular disk on an elastic stratum are presented. These solutions are the dynamic stiffness functions about swaying, vertical motion and rocking, and the rigid-fixed forcing functions. These are obtained infinite series form involving the second kind modified Bessel functions as used by Tajimi in Ref. 8. Therefore the numerical calculation about the solutions is feasible.

## TYPE OF EQUIVALENT INTERNAL DAMPING

When dealing with the problem of the vibrations of a foundation on an elastic layer resting on an elastic half space, the elastic half space often assumed to be a rigid base. The rigid base assumption causes an infinite amplification of the responses at the natural frequencies of the elastic stratum. To avoid the infinite amplification, internal damping has been

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[I] Researcher, Technical Research Institute, OHBAYASHI-GUMI, LTD.

given in the stratum. As the internal damping, viscous or material damping has been used, for example in Ref. 8, 9 and 10.

The author proposes external viscous damping to the internal damping. This external damping will remove the unfavourable effects due to rigid base assumption completely. Where the external viscous damping means the damping proportional to the relative velocity of the stratum to the rigid base and is estimated by the wave impedance ratio of the stratum to the lower hard layer. For the simplicity one dimensional wave equation is considered. The external damping coefficient $\rho'$ appears in the equation as follows.

$$\rho\frac{\partial^2}{\partial t^2}(u_g + u_x) + \rho'\frac{\partial u_x}{\partial t} = \mu\frac{\partial^2 \ddot{u}_x}{\partial t^2} \tag{1}$$

Where Eq. 1 shows shear wave equation, and $u_x$ is the relative displacement of the stratum to the rigid base, $u_g$ is incident wave at the rigid base, $\rho$ is mass density of the stratum, $\mu$ is shear rigidity of the stratum. The damping factor $h_s$ is related to $\rho'$ as follows.

$$h_s = \frac{\rho'}{2\rho\omega_T} \tag{2}$$

Where $\omega_T$ is the fundamental natural angular frequency of the stratum. If $h_s$ is estimated as to be equal to the radiation damping to the lower hard layer, $h_s$ is given, by the same technique as Tajimi in Ref. 8, as follows.

$$h_s = \frac{2\alpha_s}{\pi}, \quad \alpha_s = \frac{\rho \cdot C_T}{\rho_b \cdot C_{Tb}} \tag{3}$$

Where $\alpha_s$ is the impedance ratio for S-wave, $\rho_b$ is mass density of lower hard layer, $C_T$ and $C_{Tb}$ are S-wave velocity of the stratum and the lower layer respectively. Similarly, when considering vertical motion, the external damping factor $h_v$ is given as follows

$$h_v = \frac{2\alpha_v}{\pi}, \quad \alpha_v = \frac{\rho \cdot C_L}{\rho_b \cdot C_{Lb}} \tag{4}$$

Where $\alpha_v$ is the impedance ratio for P-wave, $C_L$ and $C_{Lb}$ are P-wave velocity of the stratum and the lower layer respectively.

## APPROXIMATE SOLUTIONS ON INTERACTION COEFFICIENTS

In deriving approximate solutions on interaction coefficients about a circular disk on an elastic stratum, a following model is considered as shown in Fig. 2. The massless rigid circular disk of radius a rests on the surface of the stratum of depth H, and the stratum consists of soil column directly under the disk and surrounding stratum. The soil stratum is supported by a rigid half space solid.

For the first time, the approximate solutions on dynamic stiffness functions and base fixed driving force functions for swaying are shown. In deriving the solutions following assumptions are used.

1) The soil column vibrates horizontally as shear beam type.

2) Radiation damping due to wave impedance ratio is considered as the external viscous damping in the stratum.

3) The damping of the stratum itself is material damping.

4) Vertical displacement is neglected.

5) In put motion at lower hard layer (rigid base) is horizontal sinusoidal wave of $u_g = u_g \cdot e^{i\omega t}$

Using the above assumptions and applying three dimensional wave propagation theory to the surrounding soil, the dynamic stiffness function Ks (iω) and the rigid fixed forcing function Es (iω) are obtained as follows.

$$Ks\ (i\omega) = \mu\pi a^2 \left(\frac{\pi}{2H}\right)^2 \frac{H}{2} \cdot \frac{1}{F^*(H,\ \omega)}$$

$$Es\ (i\omega) = \left(\frac{\omega_s}{\omega_T}\right)^2 G^*(H,\ \omega) \cdot \frac{F^*(H,\ 0)}{F^*(H,\ \omega)} \qquad (5)$$

Where

$$F^*\ (H,\ \omega) = \sum_{n=1,3,5}^{\infty} \frac{1}{\xi_n^2 (1+\Omega_n^*)}, \quad G^*\ (H,\ \omega) = \sum_{n=1,3,5}^{\infty} \frac{4}{n\pi} \cdot \frac{(-1)^{\frac{n-1}{2}}}{\xi_n^2}$$

$$\xi_n^2 = n^2 - \left(\frac{\omega}{\omega_T}\right)^2 + i\{2h_T \cdot n^2 + 2h_s\left(\frac{\omega}{\omega T}\right)\},$$

$$\Omega_n^* = \frac{K1\ (\eta^*_{Ln}) + K1\ (\eta^*_{Tn}) \cdot \dfrac{2K1\ (\eta^*_{Ln}) + \eta^*_{Ln}Ko(\eta^*_{Ln})}{2K1\ (\eta^*_{Tn}) + \eta^*_{Tn}Ko(\eta^*_{Tn})}}{K1\ (\eta^*_{Ln}) + \eta^*_{Ln}Ko(\eta^*_{Ln}) - K1(\eta^*_{Tn}) \cdot \dfrac{2K1(\eta^*_{Ln}) + \eta^*_{Ln}Ko(\eta^*_{Ln})}{2K1(\eta^*_{Tn}) + \eta^*_{Tn}Ko(\eta^*_{Tn})}}$$

$$\omega_T = C_T\pi/2H,\ \eta^*_{Tn} = \xi_n\omega_T a/C_T^*,\quad \eta^*_{Ln} = \xi_n\omega_T a/C_L^*$$

$$C_T^* = C_T \cdot \sqrt{1+2h_T},\quad C_L^* = C_L \cdot \sqrt{1+2h_L},\quad C_T = \sqrt{\mu/\rho},$$

$$C_L = \sqrt{(\lambda+2\mu)/\rho},\quad h_T = \mu'/(2\mu),\quad h_L = (\lambda'+2\mu')/\{2(\lambda+2\mu)\}$$

$$\omega s^2 = \text{Real}\ \{Ks(o)\}/m_o,\quad i = \sqrt{-1}$$

Ki(x) = second kind modified Bassel function of i - th order, $m_o$ = mass of the disk which will have, λ, μ = Lamè's constants, λ', μ' = material damping coefficients corresponding to λ and μ respectively

By the same way as of the swaying case, interaction coefficients about vertical vibration can be obtained. In this case above mentioned assumptions are altered as follows. Namely, assumption 1) is altered as "vibrates as longitudinal rod", 4) as "Horizontal displacement is neglected", 5) as "vertical sinusoidal wave". On the basis of those modified assumptions, interaction coefficients can be obtained as follows, where $K_V$ (iω) is the dynamic stiffness function and $E_V$ (iω) is the forcing function.

$$K_V\ (i\omega) = (\lambda+2\mu)\pi a^2 \left(\frac{\pi}{2H}\right)^2 \cdot \frac{H}{2} \cdot \frac{1}{F^*_V(H,\ \omega)} \qquad (6)$$

$$E_v\ (i\omega) = (\frac{\omega_v}{\omega_L})^2 G^*_v\ (H,\ \omega)\ \frac{F^*_v(H,\ 0)}{F^*_v(H,\ \omega)}$$

where

$$F^*_v\ (H,\ \omega) = \sum_{n=1,3,5}^{\infty} \frac{1}{\xi^2_{vn}(1+\Omega^*_{vn})}\ ,\quad G^*_v\ (H,\ \omega) = \sum_{n=1,3,5}^{\infty} \frac{4}{n\pi} \cdot \frac{(-1)^{\frac{n-1}{2}}}{\xi^2_{vn}}$$

$$\xi^2_{vn} = n^2 - (\frac{\omega}{\omega_L})^2 + i\ \{2h_L \cdot n^2 + 2h_v(\frac{\omega}{\omega_L})\}\ ,\quad \omega_L = C_L \cdot \pi/2H$$

$$\omega^2_v = \text{Real}\ \{K_v(o)\}/m_o,$$

$$\Omega^*_{vn} = 2 \cdot \frac{K1(\eta^*_{vn})}{\eta^*_{vn} K_o(\eta^*_{vn})}\ ,\quad \eta^*_{vn} = \xi_{vn} \cdot \omega_L \cdot a/C_T^*$$

For rocking case, only the assumption 1) of the modified ones must be altered as "The soil column deforms vertically so that the displacement shape of the horizontal cross section is triangular". In this case only the dynamic stiffness function $K_R\ (i\omega)$ is obtained as

$$K_R\ (i\omega) = a^3 \mu \frac{\pi^3}{32} (\frac{a}{H}) (\frac{C_L}{C_T})^2 \cdot \frac{1}{F^*_r(H,\ \omega)} \qquad (7)$$

Where

$$F^*_r\ (H,\ \omega) = \sum_{n=1,3,}^{\infty} \frac{1}{\xi^2_{vn}(1+\Omega^*_{rn})}\ ,\quad \Omega^*_{rn} = 4\ \frac{\eta^*_{vn} \cdot K_o(\eta^*_{vn}) + K1(\eta^*_{vn})}{\eta^{*2}_{vn} K1(\eta^*_{vn})}$$

Now, for convenience sake, the stiffness function for swaying is represented as in Ref. 10

$$K_s\ (i\omega) = K_{so}(1 + 2i\beta 1)(k1 + iaoC1) \qquad (8)$$

Where $K_{so}$ is the real part of $K_s\ (i\omega)$ in the static case, $a_o = \omega a/C_T$ is the dimensionless frequency, and k1, C1 are the functions of $a_o$. For rocking and vertical stiffness, the same representations are used, and $K_{Ro}$, $K_{vo}$ are the real part of $K_R\ (i\omega)$, $K_v\ (i\omega)$ in the static case respectively.

## NUMERICAL EXAMPLE

To show the validity of the approximate solutions, a numerical result by this method is compared with an exact solution by Luco in Ref. 7 as shown in Fig. 2. This figure shows the comparison about k1 and C1 for swaying. The original numerical model by Luco has following parameter values, $H/a = 1$, $\alpha_s = 0.34$, $\alpha_v = 0.367$, $\nu = 0.3$ and $\nu_b = 0.367$, where $\nu$ and $\nu_b$ are the Poisson's ratio of the stratum and the lower hard layer. Therefore the calculation about by this method is conducted for the model as $H/a = 1$, $\nu = 0.3$, $h_s = 0.217$, $h_T = h_L = 0$. In Fig. 2 notation (E) represents the model with the external damping, (M) with the material damping in place of the external damping and (I) with the internal viscous damping similarly. It is observed that (E) model best fits the exact solution.

For rocking and vertical mode, strictly speaking, the approximate solutions are valid only for the case $\nu = 0$ ($C_L/C_T = \sqrt{2}$), but for the range $C_L/C_T \leq 3$ those solutions will give rough approximations. It is recommended that the values of static spring constants $K_{so}$, $K_{vo}$, $K_{Ro}$ should be estimaed by the existing formula in Ref. 10 or graph in Ref. 11. Also $h_v = 0$ for rocking case is recommended.

## EQUATION OF MOTION OF INTERACTION SYSTEM

A model of the interaction system about structure-foundation-soil surface layer system and the coordinate system used are shown in Fig. 3. The equation of motion of this system is

$$[m]\{\ddot{x}\} + [K]\{\ddot{x}\} = - [m]\{\ddot{u}_g\} \tag{8}$$

where [m] is mass matrix, [K] is stiffness matrix elements of which are complex $K_s$ (iω) and $K_R$ (iω), {x} is relative displacement vector to the rigid base, and $\{\ddot{u}_g\}$ are input acceleration vector the form of which is given as follows

$$\{\ddot{u}_g\}^T = \{[1 + E_s\ (i\omega)],\ 1,\ 1,\ \ldots,\ 1,\ - E_s\ (i\omega)\cdot s\}\ \ddot{u}_g \cdot e^{i\omega t} \tag{9}$$

The first element of the right hand side of Eq. 9 corresponds to swaying of the foundation and the last to rocking. Eq. 9 contains frequency dependent function $K_s$ (iω), $K_R$ (iω) and $E_s$ (iω), therefore time history response analysis must be conducted by F.F.T. technique.

If the exciting force is not earthquake but a vibrator mounted on the structure, the right hand side of Eq. 8 is replaced by the following vector

$$\{0,\ \ldots,\ 0,\ 1,\ 0,\ \ldots\ 0,\ Hi^e\} m_e r\omega^2 \cdot e^{i\omega t} \tag{10}$$

where unit element corresponds to the mass subjected to exciting force, $Hi^e$ is the height of the mass and $m_e r$ is an eccentric moment of the vibrator.

## COMPARISON WITH EXPERIMENTAL AND OBSERVED RESULTS

As a reference test data the work by Akino et al. in Ref. 12 is used. Fig. 4 shows the test model. The foundation of this model is the R.C. rectangular box of 4 m x 4 m plan and 2.15 m height. The foundation has twin super-structures each of which has three stories. Material of the columns of the twin structures is steel and the slabs are precast concrete. The ground consists of a stratum which is Kwanto loam and the lower hard gravel. The physical properties of this ground are shown in Table 1. When vibration tests were conducted, the vibrator was mounted on the top of the B-structure and used eccentric moment was 64 kg·cm. Observation about earthquake responses was catched by the seismometers shown in Fig. 4.

An analytical model is shown in Fig. 5. The foundation is assumed to be rigid cicular disk the base area of which is equal to that of the rectangular foundation. Table 2 shows the dimensions of the foundation and the static spring constants for swaying and rocking. Table 3 shows the damping factors. Table 4 is the constants of mass and stiffness about the twin super-structures.

Fig. 6 shows the comparison about resonance curves of the top of the twin structure. Good agreement between the experimental data and the calculated results is recognized.

Next, the comparison about the earthquake responses is shown. Fig. 7 is the input wave which was recorded in 6/29/1975. Fig. 8 is the response waves of the base of the foundation, and Fig. 9 shows the responses of the top of the B-structure. Especially about the responses of the foundation good agreement is found.

## CONCLUSIONS

The approximate solutions are obtained in the form of the infinite series, so that parametric studies about interaction system using these solutions are easy. Therefore, the proposed solutions will be a hañdy tool for engineers.

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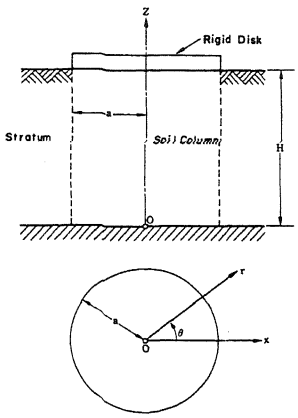


FIG. 1 - SYSTEM CONSIDERED

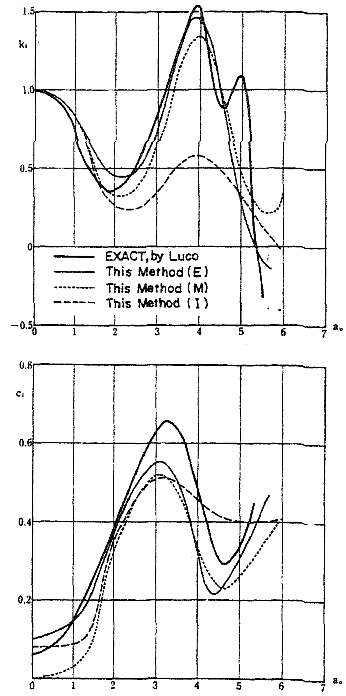


FIG. 2 - COMPARISON WITH EXACT SOLUTION

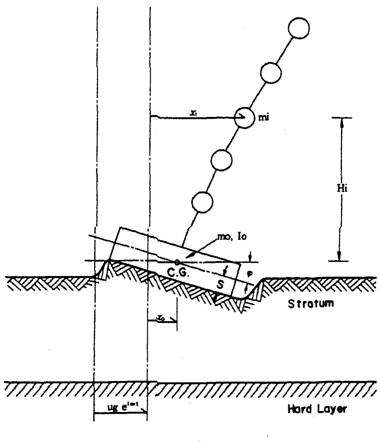


FIG. 3 - INTERACTION SYSTEM

TABLE 1 - SOIL PROPERTY

| | H (m) | $C_T$ (m/s) | $C_L$ (m/s) | $\rho$ (ton.$s^2/m^4$) |
|---|---|---|---|---|
| Stratum | 5.4 | 143 | 380 | 0.122 |
| Hard Layer | — | 466 | 2,100 | 0.214 |

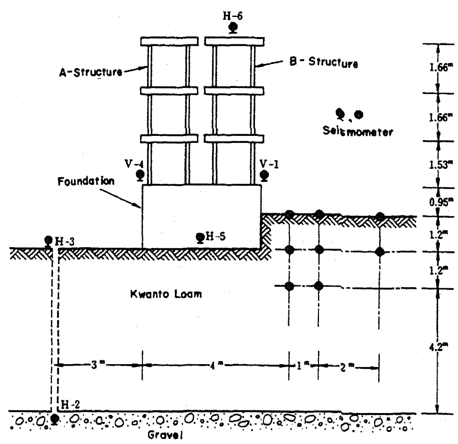


FIG. 4 - EXPERIMENTAL MODEL

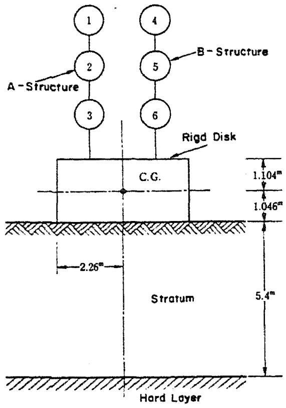


FIG. 5 - ANALYTICAL MODEL

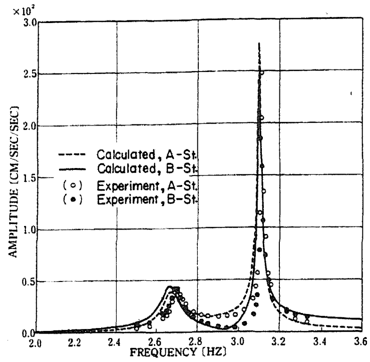


FIG. 6 - RESONANCE CURVE

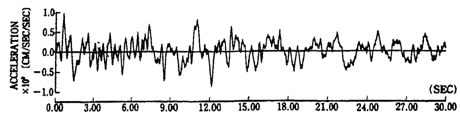


FIG. 7 - INPUT WAVE (GL-5.4m)

TABLE 2 - DIMENSION OF FOUNDATION

| mo (ton·s²/m) | Kso (ton/m) | Io (ton·s²·m) | KRo (ton/rad) |
|---|---|---|---|
| 4.573 | 28,768 | 10.991 | 134,380 |

TABLE 3 - DAMPING FACTOR (%)

| Super-Structure | Stratum | | | |
|---|---|---|---|---|
| | Material Damping | | External Damping | |
| | $h_T$ | $h_L$ | $h_S$ | $h_V$ |
| 0.2 | 4 | 4 | 11.1 | (6.6) |

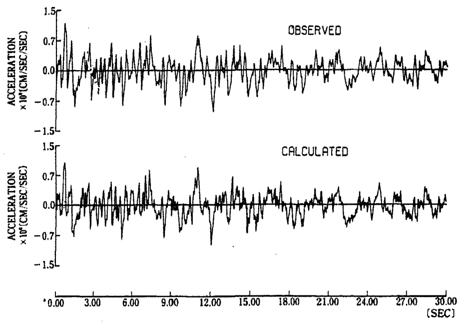


FIG. 8 - RESPONSE AT THE BASE OF FOUNDATION

TABLE 4 - MASS AND STIFFNESS

| (i, j) | mi (ton·s²/m) | kij (ton/m) |
|---|---|---|
| (1,1) | 0.5923 | 1176 |
| (1,2) | —— | -1483 |
| (1,3) | —— | 248.1 |
| (2,2) | 0.5529 | 3284 |
| (2,3) | —— | -1992 |
| (3,3) | 0.5529 | 3736 |
| (4,4) | 0.5923 | 1204 |
| (4,5) | —— | -1554 |
| (4,6) | —— | 256.7 |
| (5,5) | 0.5529 | 3375 |
| (5,6) | —— | -2025 |
| (6,6) | 0.5529 | 393.7 |

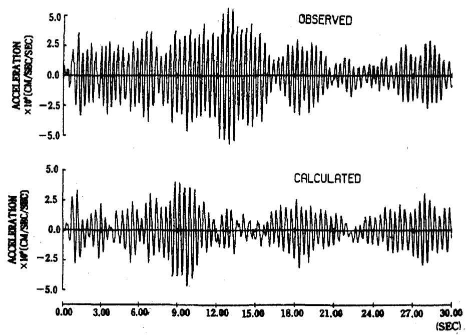


FIG. 9 - RESPONSE AT THE TOP OF B-STRUCTURE