

# EARTHQUAKE ANALYSIS OF COUPLED SHEAR WALL BUILDINGS

by

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## SUMMARY

An efficient technique especially suited for computer analysis of coupled shear wall buildings is outlined. Application of the technique to analysis of earthquake-damaged buildings is demonstrated.

## INTRODUCTION

Coupled shear wall buildings have usually been analyzed by computer programs based on standard methods of building frame analysis. These analysis methods, in determining responses to horizontal ground motions, neglect inertia forces in vertical and rotational degrees of freedom. By static condensation of these degrees of freedom, the dynamic equations are formulated in terms of lateral displacement and the problem size is greatly reduced. However, such a formulation is generally inappropriate for coupled shear wall buildings because vertical inertia effects of walls can be significant in the dynamics of such structures.

Because the stiffness, strength, and stability of coupled shear wall buildings is largely due to the walls, they should be designed to remain essentially undamaged in the event of an earthquake. The benefits of energy dissipation through inelastic action can be provided by yielding of the coupling beams. Therefore, this investigation assumes that the walls are linearly elastic, thus confining yielding to the coupling beams.

The objectives of this paper, which summarizes some results from the complete report<sup>1</sup> on this study, are: (1) to present an efficient technique especially suited for computer analysis of coupled shear walls; and (2) to demonstrate application of the technique to analysis of earthquake-damaged buildings.

## OUTLINE OF ANALYTICAL PROCEDURE

The end shear wall of the building shown in Fig. 1 may be idealized as the structural assemblage shown in Fig. 2, with three wide-column lines (located at the respective neutral axes of the walls), and beams at every floor level coupling adjacent walls. The corner spandrel beams may be neglected; the wall-beam panel zone is idealized by rigid links.

Coupling beams in coupled shear wall buildings are usually deep and subject to high shear, resulting in shear cracking, normally accompanied by yielding of stirrups and flexural reinforcement. Shear cracks and subsequent yielding are not localized at end sections of beams, but are

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spread over a substantial portion of the span. Therefore, instead of using the end sectional moment capacities,  $M_{y1}$  or  $M_{y2}$ , to signify a change in beam stiffness - as is done with flexural beams - the shear  $P_y = (M_{y1} + M_{y2})/S$ , which directly reflects nonlinearities due to diagonal shear cracking or stirrup yielding, is used to decide whether or not a change in stiffness has occurred. A bilinear hysteretic force-deformation relation is assumed for coupling beams, controlled by a bilinear shear-average end rotation relation (Fig. 3a) which can be obtained from laboratory experiments.<sup>2</sup> This model implies simultaneous changes in stiffness in the moment-rotation relation at both ends of the beam, reflecting nonlinear effects distributed throughout the beam span, not just at the ends (Fig. 3b).

By considering bending and axial deformations in walls; bending, axial, and shear deformations in coupling beams; including inertia forces associated with lateral as well as vertical motions; and using variational principles, the equations of motion were first formulated in the nodal point degrees of freedom (DOF): vertical, horizontal, and rotational displacements at each beam-wall (wide column) joint in Fig. 2. The number of these equations increases rapidly as the number of walls and stories increase and their solution requires large computational effort.

In Fig. 4, the first 10 natural vibration mode shapes computed for the McKinley Building<sup>3</sup> are compared to the vibration mode shapes of the individual walls of the building. The general similarity of the two sets of mode shapes suggests that displacements of the structure may be effectively expressed as a linear combination of the natural mode shapes of vibration of individual walls. Thus, the displacements at the nodal points on the  $j$ th wall are expressed as a linear combination of the first few natural mode shapes of the  $j$ th wall, considered as an individual cantilever. Local plastic rotation at the base of a wall may be considered by including the associated rigid body displacement of the structure as an additional shape function. The equations of motion are transformed to the associated generalized coordinates. If a small number of generalized coordinates suffice to predict response accurately, the number of equations and the computational effort would be reduced considerably, as discussed in the next section.

Not only does the numerical step-by-step integration of the reduced system of equations require considerably less computational effort than does the original system, but a larger time step may be used in the integration, because the higher vibration modes, having very short vibration periods and contributing negligibly to structural response, are eliminated by the transformation to generalized coordinates.

#### EVALUATION OF REDUCTION TECHNIQUE

The simple idealization presented in Ref. 3 for the McKinley Building (Fig. 1) was employed to evaluate the effectiveness of the above-described technique for reducing the number of DOF. Coupling beams were assumed to span the two end walls and the middle pier was ignored. A reduced system of equations is designated by  $H_m V_n$ , where  $m$  and  $n$  denote the number of modes of lateral (horizontal) and longitudinal (vertical) vibration, respectively, of each wall included in the analysis. The natural frequencies and mode shapes of the coupled shear wall system, modal stress resultants, and the nonlinear response of the system are computed from the original system

("exact" analysis) of equations in nodal point coordinates, and from the reduced system of equations in generalized coordinates.

The first six natural frequencies and mode shapes of the structure were satisfactorily reproduced by the  $H_4V_4$  system, whereas the first nine modes were more accurately reproduced by the  $H_6V_3$  system (Fig. 5). Because the more significant displacements in the lower modes of vibration of the structure are in the lateral direction, it is effective to include a larger proportion of lateral vibration modes of the walls.

Although the deflected shape of the first antisymmetrical mode was very accurately reproduced by solving the eigenvalue problem for the  $H_{16}V_8$  reduced system, to within 2%, predictions of the associated shear and bending moments in the walls were extremely inaccurate (Fig. 7). Stress resultants were inaccurate because the moments in the walls associated with the deformations in vibration modes of individual walls vary gradually along the height, whereas their actual distribution is discontinuous due to the moments at the ends of coupling beams. However, the predicted wall moment smoothly averaged the discontinuity in moments at the beam level. Shear in coupling beams, however, was predicted accurately (Fig. 7).

The stress resultants for the walls obtained by analyzing the reduced system were corrected by distributing, as shown in Fig. 6, the beam end-moments to the wall above and below each beam-wall joint. A correction was also necessary at the base of the structure. Shear forces in the wall are then correspondingly adjusted to equilibrate corrected bending moments. By applying the above adjustment procedure, the corrected bending moments and shears obtained from analyzing the  $H_6V_3$  reduced system -- a much less refined system than the  $H_{16}V_8$  one -- satisfactorily agreed with the "exact" values (Fig. 7). The  $H_6V_3$  reduced system, with this adjustment, also satisfactorily predicts the stress resultants associated with the third antisymmetrical mode shape (Fig. 8).

Two approaches were used to determine the nonlinear response of the simple idealization for the Mt. McKinley building, wherein yielding of the coupling beams is considered, to a simple ground motion, described by a half-cycle of displacement<sup>4</sup>, with maximum acceleration = 0.5 g in the horizontal direction and one-third of that in the vertical. The  $H_6V_3$  reduced system was analyzed by the procedures outlined earlier and the equations in nodal point coordinates were solved by DRAIN-2D<sup>5</sup>, a computer program based on standard frame analysis procedures. The results (Figs. 9 and 10) indicate that the two analyses lead to essentially the same displacement response, but the forces determined from DRAIN-2D analysis oscillate about those determined from analysis of the reduced system. These oscillations do not disappear even when the integration time step in the DRAIN-2D analysis is reduced to half the value used in analysis of the reduced system. An operation count indicates that the computational effort required for analysis of the reduced system is 20% to 50% of that required for the original system in nodal point coordinates. Obviously, with the use of generalized coordinates, not only is the size of the problem and computational effort greatly reduced but, by eliminating the unimportant higher modes of vibration, the spurious oscillations in the numerical calculations are eliminated.

## ANALYSIS OF EARTHQUAKE-DAMAGED BUILDINGS

Some of the damage to McKinley Building caused by the 1964 Alaska earthquake is apparent in Fig. 1. The response of the mathematical model of an end wall of the building (Fig. 2) to a simulated motion<sup>6</sup>, intended to represent the ground shaking in Anchorage, was determined by the procedure outlined earlier. The results are summarized in Figs. 11-13.

Beams on the second through eighth stories underwent the most extensive yielding, each of which accumulated a total plastic rotation of more than 0.02 radians during more than 20 yielding excursions (Fig. 11). Cyclic rotation ductility demand exceeded 10, an excessive demand for an ordinarily reinforced deep beam. The analysis thus predicted severe inelastic action in these beams which failed due to inadequate ductility. The prediction was generally consistent with the observed damage, beams from the second through the ninth stories having been severely damaged during the earthquake.

Axial force envelopes for walls (Fig. 12) indicate no resulting axial tension, and therefore no possibility of uplifting of the foundation or failure of walls in tension. A significant difference in the magnitude of developed axial compression in two identical walls gave rise to substantially different sectional moment capacities. Therefore, one of the two identical walls with smaller moment capacity was more vulnerable to yielding than the other wall; this is consistent with the observed damage.

Although yielding in walls was not considered in the analysis, it can be examined by studying the force distribution at selected time steps. For example, at  $t = 16.9$  seconds, beams in the third through eighth stories had just undergone three large, consecutive yielding cycles. They were assumed to fail at this point and part of the resistance to story overturning moment, formerly offered by axial forces in the walls, was no longer available at these stories. Wall sections across affected stories had therefore to resist more moment to compensate for the loss of the axial-force couple. This additional moment was assumed to be resisted equally by the two outside walls. The resulting moment distribution is presented in Fig. 13, indicating that wall sections from the fourth story down were stressed beyond yielding capacity. Although actual redistribution of the couple due to axial forces in the walls after some beams have failed is much more complicated, this simple analysis of redistribution of moments indicates a yielding tendency in these lower story wall sections. In fact, yielding did occur in the third story wall section (Fig. 1).

A similar analytical investigation<sup>1</sup> of the performance of the Banco de America building during the Mangua earthquake led to conclusions consistent with the actual damage. Coupling beams underwent significant yielding but the walls were essentially undamaged. The excellent performance of this building suggests that, for coupled shear walls to be most effective as a structural system, walls should be designed to remain elastic, thus justifying the assumption of linearly elastic walls in this analytical procedure.

## CONCLUSION

Under the assumption that inelastic action is confined to the coupling beams, coupled shear wall buildings can be most effectively analyzed by

expressing the deflections as a linear combination of the first few natural mode shapes in lateral (horizontal) and longitudinal (vertical) vibration of individual cantilever walls. In this approach, the vertical inertia, important in the dynamics of coupled shear walls, need not be neglected; and any mechanical model for the coupling beams can be employed. This analysis procedure requires considerably less computational effort than standard computer programs do. Using the technique presented earlier, results were given of earthquake response analyses of two existing coupled shear wall buildings damaged during earthquakes. It was shown that damage predictions based on analytical results are generally consistent with observed damage.

#### ACKNOWLEDGEMENT

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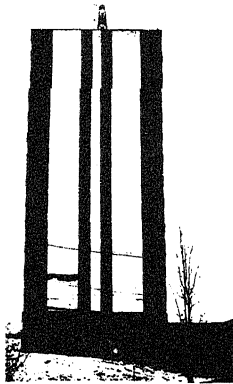


FIG. 1 MCKINLEY BUILDING:  
EARTHQUAKE DAMAGE  
IN NORTH END WALL

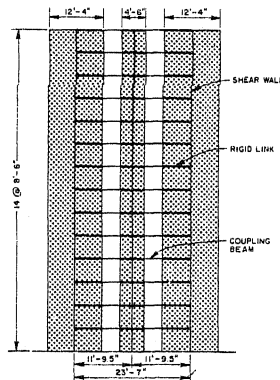


FIG. 2 MCKINLEY BUILDING:  
STRUCTURAL IDEALIZATION  
OF NORTH END WALL

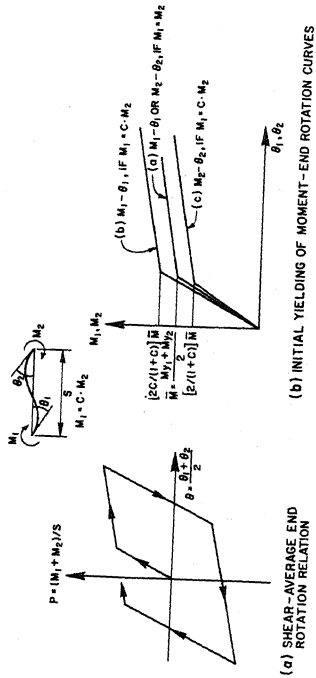


FIG. 3 MECHANICAL MODEL OF COUPLING BEAM

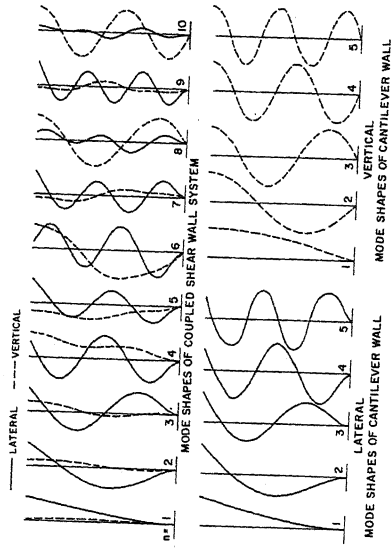


FIG. 4 STRUCTURAL MODE SHAPES AND CANTILEVER WALL MODE SHAPES

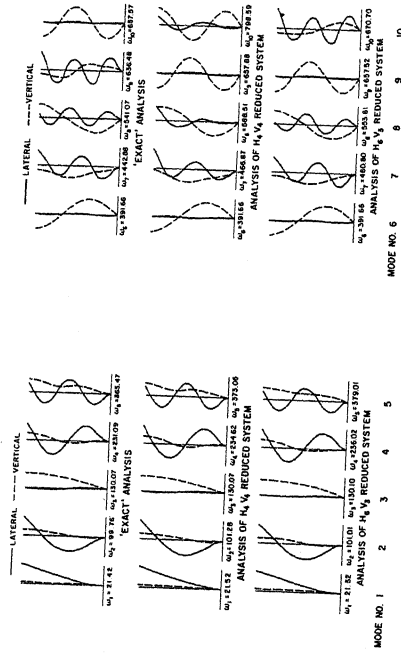


FIG. 5 NATURAL FREQUENCIES AND MODE SHAPES OF VIBRATION OF MCKINLEY BUILDING COMPUTED FROM THREE ANALYSES

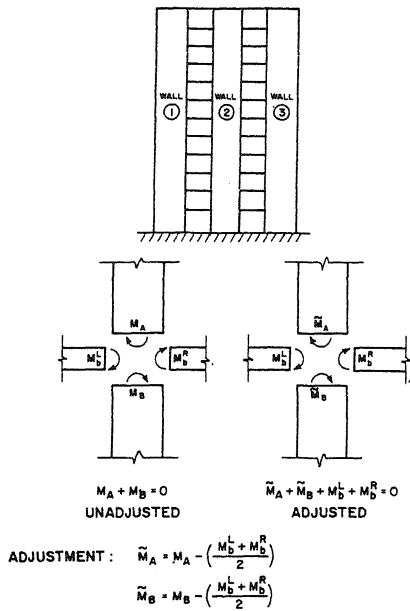


FIG. 6 ADJUSTMENT OF BENDING MOMENT IN WALL

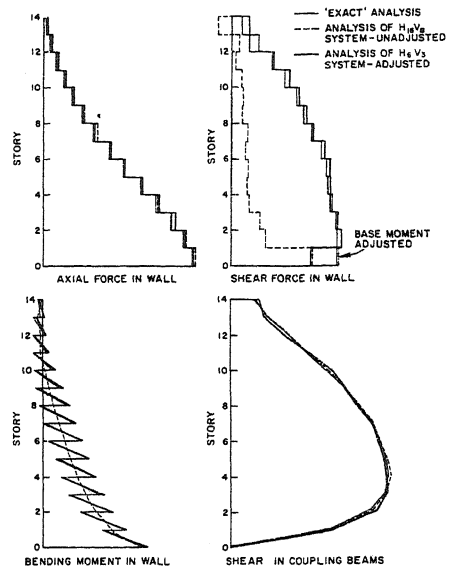


FIG. 7 STRESS RESULTANTS IN FIRST MODE OF VIBRATION OF MCKINLEY BUILDING

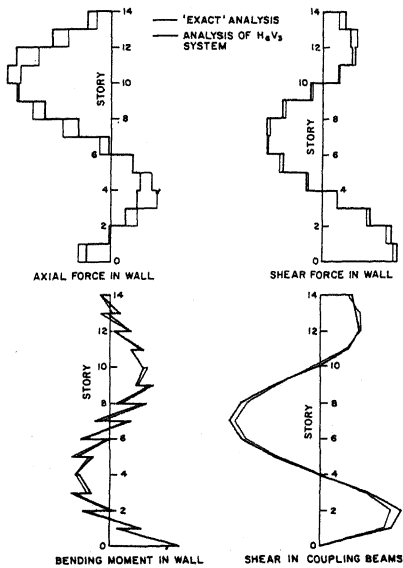


FIG. 8 STRESS RESULTANTS IN THIRD MODE OF VIBRATION OF MCKINLEY BUILDING

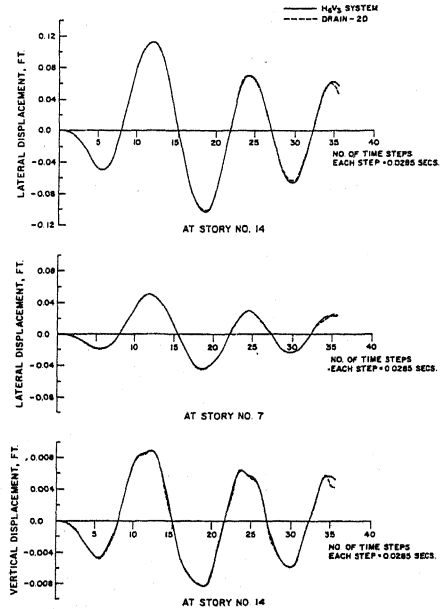


FIG. 9 DISPLACEMENT HISTORY OF WALL 1 OF MCKINLEY BUILDING

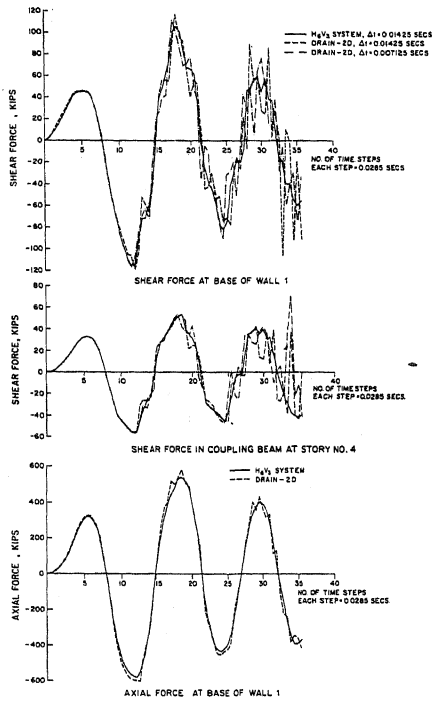


FIG. 10 STRESS RESULTANT HISTORY FOR MCKINLEY BUILDING

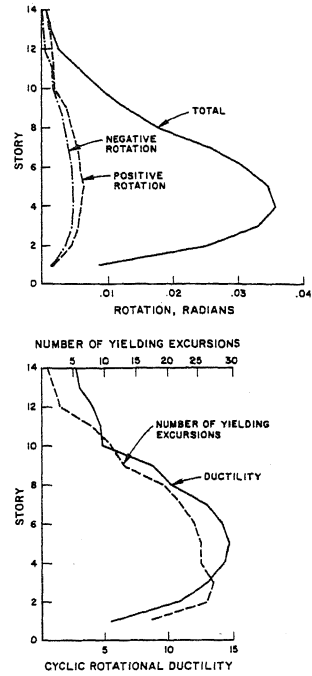


FIG. 11 PLASTIC END ROTATION, CYCLIC ROTATIONAL DUCTILITY DEMAND, AND NUMBER OF YIELDING EXCURSIONS IN COUPLING BEAMS OF MCKINLEY BUILDING

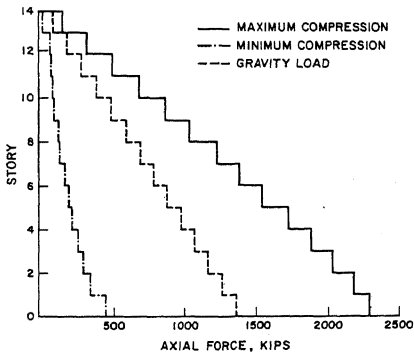


FIG. 12 ENVELOPES OF AXIAL FORCES IN AN END WALL OF MCKINLEY BUILDING

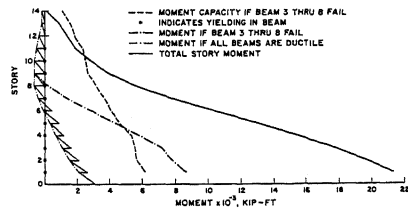


FIG. 13 DISTRIBUTIONS OF MOMENT IN AN END PIER AT TIME = 16.90 SECS.