FLEXURAL RIGIDITY OF R/C COLUMNS IN SIDESWAY FRAMES

Ergin Atımtay[I], Orhan Baydar[II]

The effective flexural rigidity of reinforced concrete columns is studied as affected by level of axial load, longitudinal steel percentage, volumetric percentage of stirrup, width/height ratio, concrete strength, and creep. Use is made of a general expression for σ-ε relationship of concrete which reflects the effect of the above stated parameters. Thrust moment-curvature relationships are developed. It has been observed that level of axial load, longitudinal steel percentage and creep are most effective on column EI. The general form of the ACI expression for column EI has been modified for material non-linearity, cracking and creep. The modified EI expression has been used to predict the moment magnification factors of several test frames. The results are in good agreement.

INTRODUCTION

The columns of a framed structure subject to lateral loading due to an earthquake are designed for the axial force plus the maximum design moment. To reflect the geometric and material non-linearity of the column in sidesway, the common approach is to magnify the first order moments by a magnification factor. In addition, when dynamic analysis is performed, the sway of the structure is to be known. For both cases, a realistic determination of the flexural rigidities of columns becomes essential.

COLUMN FLEXURAL RIGIDITY

To obtain the flexural rigidity of the column as affected by different variables a computer program is developed to give the column N-M-ϕ relationships.

Stress-Strain Relationships

Steel is assumed to be elasto-plastic. No strain hardening is considered.

The general form of the σ-ε relationship of concrete in compression is given by (1) :

$$\sigma = \frac{A(\varepsilon/\varepsilon_o) + (D-1)(\varepsilon/\varepsilon_o)^2}{1 + (A-2)(\varepsilon/\varepsilon_o) + D(\varepsilon/\varepsilon_o)^2} \qquad (1)$$

ε = the compressive strain, ε_o = compressive strain corresponding to maximum stress, and E_c = modulus of elasticity. The parameters A and D are explained below.

$$A = \frac{E_c\ \varepsilon_o}{k_3\ f_{ck}} \qquad (2)$$

I Associate Professor of Civil Engineering, Middle East Technical University, Ankara, Turkey.

II Graduate Student, Middle East Technical University, Ankara, Turkey.

where k_3 is the ratio of the maximum stress to the characteristic cylinder strength. The expression of k_3 can be expressed considering all possible factors.

$$k_3=C(k_{3d})\{k_{3t}(1+k_{3v}\frac{1-C}{C})+10^{-2}\left[0.7\sqrt{\frac{5}{k_d}}+1.5(1-0.25\frac{s}{b_e}\right]\frac{\rho_e''f_{yk}''}{\sqrt{f_{ck}}}\cdot S_f\} \tag{3}$$

$$C=\frac{b_e h_e}{b_w h} \quad \text{and} \quad S_f=\frac{2b_e h_e}{b_e^2+h_e^2} \tag{4}$$

where s = spacing of lateral reinforcement, ρ_e'' = volumetric ratio of lateral reinforcement calculated over enclosed core area, k_{3d} = size effect parameter, k_{3t} = time effect parameter, k_{3v} = cover effectiveness factor, k_d = depth of neutral axis, C = enclosed core ratio, S_f = shape factor, f_{yk}'' = characteristic yield stress of lateral reinforcement, b_e, h_e = width and depth inside to inside of lateral reinforcement, respectively.

Modulus of elasticity E_c is mainly affected by the concrete strength, creep and strain gradient. The expression for E_c is given as follows :

$$E_{ct}=\frac{256262(f_{ck}+1120)^{1/3}}{1+\emptyset_c} \tag{5}$$

Creep effect is introduced into E_c by the coefficient $\emptyset_c$ which is the function of five different coefficients as expressed in CEB Recommendations (2).

The expression of ε_o as a function of time is given below,

$$\varepsilon_o=\varepsilon_u\{0.05(\log^2 t+0.5\log t+18)\} \tag{6}$$

where ε_u is the ultimate strain.

The parameter D is introduced in the σ-ε expression in order to control the slope of the descending branch. Many of the expressions of σ-ε for concrete previously proposed assume either D = 0 or D = 1. In this paper, D is taken as 0.20.

Thrust-Moment-Curvature Relationship (N-M-Ø)

The N-M-Ø relationships are obtained by a computer program which iteratively solves a set of equations to satisfy equilibrium, stress-strain, and compatibility conditions (3). Failure is defined when the compressive strain reaches a value of $\varepsilon_u = 2\varepsilon_o$.

Equivalent Flexural Rigidity of Columns

Using the developed computer program, N-M-Ø relationships were obtained to bring out the effect of the variables under study. Attention was paid to keep all the other variables constant except the variable whose effect was searched. The secant flexural rigidity drawn at a point which corresponds to $0.85M_u$ (M_u is the peak moment of N-M-Ø curves) was taken as the basis of comparison.

A study of the results have shown that the most influencing parameters are steel percentage,, level of axial load (N/N_o), and creep.

The general expression of effective EI is taken as given in the ACI Code (4).

$$EI = \frac{\frac{EI_g}{\alpha} + E_e I_e}{1 + \beta} \qquad (7)$$

where EI_g = gross flexural rigidity, $E_e I_e$ = rigidity contributed by steel, α = factor to represent the effect of material non-linearity and cracking, β = factor to represent the effect of creep.

The factors α and β are studied as a function of N/N_0, Fig.1 and Fig. 2, for all combinations of N/N_o and time intervals of 1 week to 1 year. The factors α and β are presented in Table 1 and Table 2.

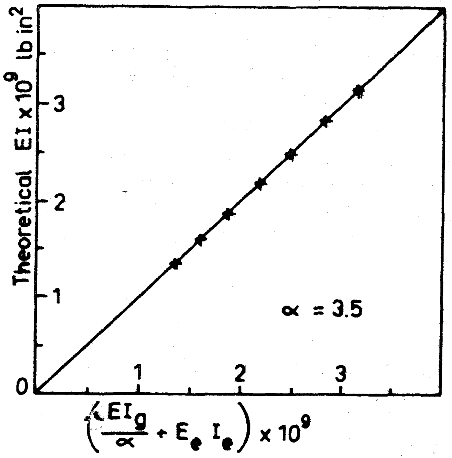

FIG. 1 $N/N_0 = 0.50$

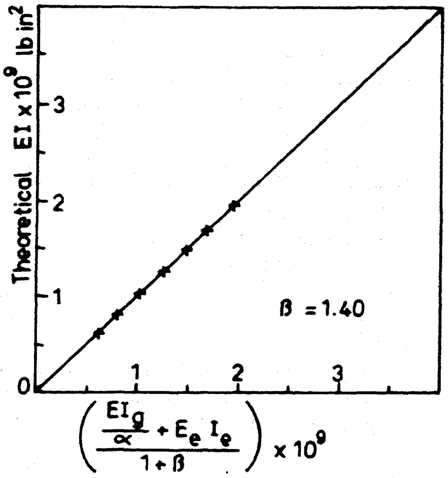

FIG. 2 $N/N_0 = 0.50$, $\alpha = 3.5$ T = 1 Year

TABLE 1

N/N_o	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80
α	3.5	3.5	3.5	3.5	3.5	3.5	4.0	4.6	5.5

TABLE 2

T_1 = 1 week T_2 = 2 months T_3 = 1 year

N/N_o	0.40			0.45			0.50		
Time	T_1	T_2	T_3	T_1	T_2	T_3	T_1	T_2	T_3
β	0.50	0.90	1.10	0.50	0.90	1.20	0.50	1.0	1.30

N/N_o	0.55			0.60			0.65		
Time	T_1	T_2	T_3	T_1	T_2	T_3	T_1	T_2	T_3
β	0.60	1.10	1.40	0.70	1.20	1.50	1.0	1.30	1.80

Table 2 (Contn'd)

N/N_o	0.70			0.75			0.80		
Time	T_1	T_2	T_3	T_1	T_2	T_3	T_1	T_2	T_3
	1.20	1.50	2.0	1.60	1.90	2.5	2.3	2.5	2.9

COMPARISON OF PROPOSED EI WITH EXPERIMENTAL DATA

To test the validity of the developed expression, two sets of frame tests done by Ferguson and Breen (5), and Rad (6) were considered. The experimental F-values (moment magnification factors) are compared with those obtained by using Eq. 7 (with Table 2). Results obtained by ACI expression of EI (α=5.0) are also included. The comparisons are given in Table 3 and Table 4.

TABLE 3

F-Values of Ferguson and Breen's Frames.

Frame	Test	From Eq. 7	From ACI	N/N_o
L1	4.7	5.5	4.0	0.35
L2	2.35	2.18	3.04	0.25
L5	1.4	1.36	1.42	0.40
L6	2.3	1.60	1.62	0.55
L7	3.4	3.0	4.64	0.45

e = eccentricity,
t = cross-sectional depth,
ℓ = height of column.

TABLE 4

F-values of Rad's Frames

Frame	Test	From Eq. 7	From ACI	N/N_o
A1	1.33	1.56	2.22	0.60
A2	1.86	1.99	2.50	0.50
A3*	1.84	1.76	2.38	0.55
A4*	1.50	2.02	3.09	0.50
A5	1.87	1.86	2.83	0.60

* Frames A3 and A4 have columns of unequal area and moment of inertia.

CONCLUSIONS

It can be concluded that α= 3.5 can be used for normal ranges of N/N_o 0.65 in evaluating the flexural stiffness of a column. More experimental data are needed to test the validity of the developed expression under high N/N_o, e/t, and ℓ/t ratios.

Experimental results are lacking to test the validity of the proposed values of β. Such experiments are strongly needed, even though costly and time taking. However, the proposed values of β seem to be a refinement in reflecting creep effect than merely using M_g and M_p (g=dead load; p=live load) as ACI proposes.

REFERENCES

1. Sargın, M., Stress-Strain Relationship for Concrete and the Analysis of Structural Concrete Sections, Ph.D. Dissertation, University of Waterloo, Ontario, 1971.
2. CEB, Bulletin D'information, No. 111, October, 1975.
3. Atımtay, E., and Cengizkan, M., "A Numerical Approach to Creep of Reinforced Concrete", METU Journal of Pure and Applied Sciences, Vol.8, No.3, pp. 343-365, Dec., 1975.
4. ACI (318-77), American Concrete Institute, Detroit, Michigan, 1977.
5. Ferguson, P.M., and Breen, J.E., "Investigation of the Long Concrete Column in a Frame Subject to Lateral Loads", Symposium on Reinforced Concrete Columns, SP-13, ACI, Detroit, Michigan, 1966.
6. Rad, F.N., "Behavior of Single Story Two-Column Reinforced Concrete Frames Under Combined Loading", Ph.D. Dissertation, Univ. of Texas, Austin, Dec. 1972.