ENERGY CONCENTRATION OF MULTI-STORY BUILDINGS

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SUMMARY

Plasticized multi-story buildings are very susceptible to the energy concentration which causes the collapse of a single story as exhibited by past earthquake hazard. The most important problem in the limit state design approach of multi-story buildings is how to overcome the concentration of inelastic strain energy.

In this paper a basic law governing the energy distribution in shear-type buildings is presented and applied to the analysis of previous structural damages, and based on the balance of the input energy by an earthquake and the energy absorbed by a structure, a proposal is made on the general design method allowing for the energy concentration.

1. IMPORTANCE OF ENERGY CONCENTRATION IN MULTI-STORY BUILDINGS

Since the earthquake is a probabilistic phenomenon occurence of which during the life time of a building is quite a matter of uncertainty and the energy input to a building exerted by an earthquake is not boundless, the energy absorption due to inelastic deformation of structures may be considered to be preferable performance which permits the more economical structural designing. The total energy input made by an earthquake depends mainly on the total mass and the fundamental natural period of the structure and is scarcely affected by the strength and the type of restoring force characteritics and the energy concept initiated by Housner may be most sound basis for developing the aseismic design method for buildings(1). However, to utilize inelastic energy absorption is not so easy as is made by simple extension or modification of the elastic design method.

Fig.1 shows a typical contrast of structural behavior of multi-story framed structures. When weak beams and strong columns simply supported at their bases are applied, beams undergo inelastic deformation while elastic columns allow the structure to keep a deflectional mode almost same as the fundamental translational mode of elastic vibration of the structure. Inelastic strain energy is evenly dispersed in the weak beam structures shown in Fig.1(a) and such a structure, being free of concentration of damages, can be analytically reduced to a single mass system. On the other hand, when strong beams and weak columns are applied, columns suffer inelastic deformation while elastic beams are unable to prevent excessive deformation of columns. Thus, the weak column structures are inevitably exposed to the concentration of damages.

It is important to note that most of existing buildings must be classified into the weak column structures and most of catastrophic damages of buildings are ascribed to the concentration of energy input into the relatively weak story. We can easily find several reasons of dominant presence of the weak column structures. These are,

1. Beams are primarily proportioned for bending stresses due to vertical loads. To reduce strength and increase ductility can be attained only by the sacrifice of efficient use of materials.
2. Collaboration of slabs on beams is considerable and hinders to

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realize weak beams.

3. Beams are usually crossly displaced and are originally stiffened against seismic loads applied in the diagonal direction.
4. To exclude any possibilities of column yielding requires a highly sophisticated design technique which has not been developed yet.

Therefore, we must recognize the fact that the weak column structures are most common and prevailing and the importance of energy concentration in multi-story buildings should not be left behind. There have been several design criteria for inelastically stressed structures(2), but most of them are rooted in response analyses of single mass systems which correspond to weak beam structures and do not take into account the energy concentration inherent to multi-story buildings.

In the following chapters, principal features of the energy concentration in the weak-column structures are described based on the energy concept. Adopted relationship between story displacement and story shear force is assumed to be one of the fundamental restoring force characteristics as shown in Fig.2.

2. ESTIMATION OF ENERGY CONCENTRATION

2.1. Invariability of Total Energy Input

Total energy input, E into a single mass system is written as

$$E = - M \int_0^{t_g} \ddot{z}_0 \dot{y} dt \quad , \qquad (1)$$

where M: total mass, $\ddot{z}_0$: acceleration of ground,
$\dot{y}$: relative velocity of mass, t_g: duration of ground motion.

With respect to the total energy input, it has been already clarified that the total energy input is scarcely influenced by the types of restoring force characteristics, and is mainly governed by the total mass and the fundamental natural period(4). According to response analyses of single mass systems, more detailed feature of the energy input is described as follows.

For elastic systems with damping,

$$E = \frac{1}{\Delta T} \int_{T_0-\Delta T/2}^{T_0+\Delta T/2} E_0(T) dT \quad , \qquad (2)$$

and for inelastic systems without damping,

$$E = \frac{1}{\Delta T} \int_{T_0}^{T_0+\Delta T} E_0(T) dT \quad , \qquad (3)$$

where E_0: total energy input of undamped elastic systems,
T_0: natural period of the system considered in elastic range,
ΔT: band of integration, T: natural period in elastic range.

Eq.3 implies that the total energy input of general cases can be obtained by averaging the total energy input of undamped elastic systems with respect to T. The difference of these two systems is found only in the difference of bounds of integration in Eq.2 and Eq.3. In the case of elastic systems with damping, the period of vibration is fixed to T_0 and averaging band spreads around T_0 with increasing ΔT as the damping capacity increases. On the other hand, since period of vibration becomes longer as the inelastic deformation increases, averaging band for inelastic systems extends beyond T_0. The extent of inelastic deformation can be measured by the cumulative ductility factor, η defined by summation of increments of ductility, $\Delta\eta$ shown in Fig.2 as follows.

$$\eta = \Sigma\Delta\eta \qquad (4)$$

Fig.3 shows the result of response analyses for single mass systems subjected to E-W component of the Tokachi-oki Earthquake (recorded at Hachinohe Harbor in 1968). The total energy input is converted to an equivalent velocity, V_E defined by the following equation.

$$V_E = \sqrt{\frac{2E}{M}} \qquad (5)$$

As the amount of damping capacity or the cumulative ductility increases, the fluctuation of V_E diminishes due to the progress of averaging rate. In this sense the total energy input can be assumed invariable regardless of specific parameters involved in a system. For natural periods below one second, the energy input of inelastic system tends to swell beyond that of elastic systems. This tendency may be understood by considering that E_o is an increasing function of T in this region and the averaging band of this case extends beyond T_o. After some manipulation a simplified bi-linear curve is obtained for the design purpose as shown by broken lines in Fig.3.

2.2. Expression of Damages

The energy absorbed by the inelastic deformation in each story, W_{pi} can be written as

$$W_{pi} = Q_{Yi}\,\delta_{Yi}\ , \qquad (6)$$

where Q_{Yi}: yield shear force in i-th story from the ground level,
δ_{Yi}: yield story displacement.

Introducing the equivalent spring constant, k_{eq}, and the yield shear force coefficient, α_i, W_{pi} is expressed as follows (3).

$$W_{pi} = c_i\,\alpha_i^2\,\eta_i \times \frac{M\,g^2\,T^2}{4\pi^2}\ , \qquad (7)$$

where $c_i = \left(\sum_{j=i}^{N} m_j/M\right)^2/\bar{k}_i$, m_i = mass of i-th story,

$\bar{k}_i = k_i/k_{eq}$, $k_{eq} = 4\pi^2 M/T^2$

k_i: spring constant of i-th story = Q_{Yi}/δ_{Yi} ,

g: acceleration of gravity, N = number of story.

Therefore, total amount of inelastic strain energy, W_p is written as

$$W_p = \sum_{i=1}^{N} c_i\,\alpha_i^2\,\eta_i \times \frac{M\,g^2\,T^2}{4\pi^2}\ . \qquad (8)$$

2.3. Optimum Distribution of α_i

The distribution of damages, W_{pi} is primarily affected by the distribution of α_i. The optimum distribution of α_i is defined by one which produces uniform distribution of $\eta_i(=\eta_o)$. Under such a distribution of α_i, the distribution of damages can be written as

$$D_i = \frac{W_{pi}}{W_p} = \frac{s_i}{\sum_{j=1}^{N} s_j} \tag{9}$$

where $s_j = c_j\, \bar{\alpha}_j^2\, \overline{k_1}$, $\bar{\alpha}_i = \alpha_i/\alpha_1$ in the optimum distribution. Although $\bar{\alpha}_i$ depends slightly on T, the amount of η, and k_i/k_1, $\bar{\alpha}_i$ is represented by a following unified curve with some permissible errors (4).

$$\bar{\alpha}_i = f\left(\frac{i-1}{N}\right) , \tag{10}$$

$$f(x) = 1 + 1.5927x - 11.8519x^2 + 42.5833x^3 - 59.4826x^4 + 30.1586x^5 .$$

2.4. Law of Energy Concentration

For the general distribution of α_i/α_1, it was found through vast amount of numerical analyses that D_i can be expressed by the following equation, which applies to multi-story buildings with such a restoring force characteristics as shown in Fig.2(4).

$$D_i = \frac{s_i\, p_i^{-12}}{\sum_{j=1}^{N} s_j\, p_j^{-12}} , \tag{11}$$

where $p_j = \alpha_j/\bar{\alpha}_j$

From Eq.11 it can be seen that the distribution of damages are very sensitive to the deviation of α_i/α_1 from $\bar{\alpha}_i$. If the condition of $\alpha_i/\alpha_1 = \bar{\alpha}_i$ could be satisfied, even weak column systems might be compatible to weak beam systems as for the distribution of damages. However, such a condition is hardly hoped for following reasons.

1. Fluctuation of strength of materials is uncontrolable.
2. Structural analyses adopted in the practical design is not reliable enough to shoot narrow targets.
3. Dimensions of structural members must be regulated by requests from planners and fabricators for other reasons than structural rationality.

Considering these factors, the concentration of damages in the individual story must be checked. In the design of buildings which are aimed to equip with the optimum distribution of strength, the estimation of energy concentration in the observed k-th story may be done by specifying p_j as follows.

$$p_k = 1.0, \text{ and } p_{j \neq k} = a > 1.0 \quad . \tag{12}$$

where a: a constant value influenced by aforementioned factors. The value of "a" should be at least larger than 1.1.

3. PAST EARTHQUAKE DAMAGES

There are many examples which demonstrate the seriousness of energy concentration in multi-storied reinforced concrete buildings. Simplest case is the main building of Olive View Hospital nearly collapsed on the San Fernando Earthquake in 1971 which had five stories above the ground level

upper four stories of which were sufficiently stiffened by shear walls. The weakest first story was exposed to the energy concentration(5).
Similar case is the south wing of Hachinohe Technical College heavily damaged in the longitudinal direction on the Tokachi-oki Earthquake in 1968 which was a three-story rigid frame structure free of shear walls(6). Relative weakness of the first story resulted in the energy concentration in that story.

On the occasion of recent Miyagiken-oki Earthquake in 1977, many steel structures were heavily hitted(7). The energy concentration peculiar to steel buildings was exhibited by M-building which was a four stories high rigid frame structure composed of rolled H-section members and reinforced concrete slabs. Abrupt change of member size caused inclination of 0.056 radian in the third story due to the energy concentration.

The energy concentration in these buildings can be readily predicted by using Eq.11. Table 1 indicates necessary parameters quoted from references (6),(7), and the predicted distribution of damages, D_i in the abovementioned two buildings. $\overline{D}_i$ in these tables is that calculated by Eq.9 which corresponds to the case where the distribution of strength is optimum. Discrepancy of D_i from $\overline{D}_i$ in the story which suffered damages tells us the important ill-effect of the energy concentration.

4. ASEISMIC DESIGN BASED ON ENERGY CONCEPT

Although the energy concentration is very serious in multi-story buildings, it can be overcome by a proper application of the energy concept.

Balance of the input energy and the absorbed energy in a structure is expressed as

$$W_e + W_p + W_h = E \quad , \tag{13}$$

where W_e: elastic strain energy,
W_p: inelastically absorbed energy,
W_h: energy absorbed by miscellaneous damping effects.

W_h can be eliminated by using the following empirical formula(3).

$$W_e + W_p = E\, w_h^2 \tag{14}$$

$$w_h = 1.0/(1+3h+1.2\sqrt{h}\,) \quad ,$$

where h: fraction of critical damping.

W_e can be also approximately expressed as follows(4).

$$W_e = \frac{\alpha_1^2}{2} \times \frac{Mg^2T^2}{4\pi^2} \tag{15}$$

W_p can be expressed in terms of the damage of the first story, W_{p1} as

$$W_p = \frac{W_{p1}}{D_1} = \frac{c_1\ \alpha_1{}^2\ \eta_1}{D_1} \tag{16}$$

Using V_E given by Eq.5, Eq.13 yields the following relation.

$$\frac{\alpha_1^2}{2} + \frac{c_1 \alpha_1^2 \eta_1}{D_1} = \frac{2\pi^2 (V_E w_h)^2}{g^2 T^2} \tag{17}$$

The yield story displacement δ_Y may be considered to be almost constant and the aimed distribution of strength should be α_i. Hence, the distribution of stiffness is determined as

$$\frac{k_i}{k_1} = \frac{\alpha_i \sum_{j=i}^{N} m_j}{\alpha_1 M} \tag{18}$$

The fundamental natural period is expressed as

$$T = 2\pi \sqrt{\frac{\delta_Y \bar{k}_1}{g \alpha_1}} \tag{19}$$

where $\bar{k}_1 = k_1/k_{eq}$ being a function of k_i/k_1 and m_i/m_1 .

For a realistic example of rigid frame structures, following parameters are selected which may be consistent with both usual steel structures and reinforced concrete structures.

$$\frac{m_i}{m_1} = 1.0 , \quad \delta_Y = 2.33 \text{ cm}, \quad a = 1.185 \tag{20}$$

The energy input of the strongest earthquakes is assumed as shown in Fig.4. which may produce the maximum acceleration of 1g in elastic single mass systems. In Fig.4 the yield shear force coefficient, α_1 calculated by Eq.17 is depicted with respect to the cumulative ductility in the base story, η_1. η_1 of 4.0 is deemed attainable for carefully designed reinforced concrete structures and η_1 of 10.0 is also a realistic value for steel structures. For comparison's sake the case of a = 1.0 where no energy concentration occurs was analysed and is shown in the same figure. The figure shows that optimism neglecting the energy concentration leads to serious underestimation of the required strength of multi-story buildings under earthquakes.

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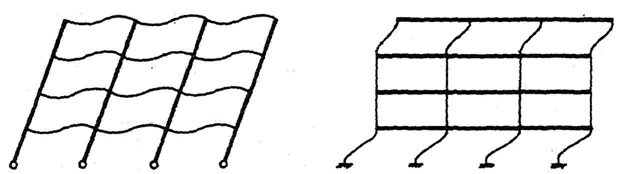
i	$\frac{m_i}{m_1}$	$\frac{k_i}{k_1}$	α_i	$\overline{\alpha}_i$	p_i	c_i	s_i	$\overline{D}_1$	D_1
3	1.055	0.799	1.366	1.63	0.837	0.0771	0.4030	0.192	0.001
2	0.982	0.902	0.816	1.18	0.691	0.2545	0.6958	0.332	0.016
* 1	1.000	1.000	0.505	1.00	0.505	0.5102	1.0000	0.476	0.983

(a) Hachinohe Technical College, $\overline{k}_1 = 1.96$

i	$\frac{m_i}{m_1}$	$\frac{k_i}{k_1}$	α_i	$\overline{\alpha}_i$	p_i	c_i	s_i	$\overline{D}_1$	D_1
4	1.034	0.408	0.749	1.83	0.409	0.0409	0.5397	0.138	0.003
* 3	1.000	0.395	0.373	1.38	0.270	0.1633	1.2249	0.314	0.906
2	1.000	0.625	0.373	1.12	0.330	0.2296	1.1348	0.291	0.075
1	1.000	1.000	0.372	1.00	0.372	0.2538	1.0000	0.256	0.016

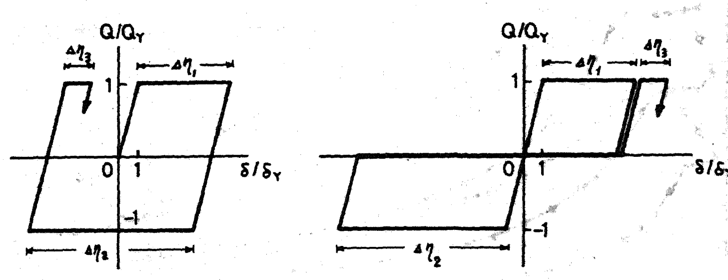
(b) M-Building, $\overline{k}_1 = 3.94$

Table 1. DAMAGE DISTRIBUTION IN NEARLY COLLAPSED BUILDINGS



(a) Weak Beam System (b) Weak Column System

Fig.1. CONTRAST IN MODES OF DEFLECTION



(a) Elastic-Plastic Model (b) Slip Model

Fig.2.TYPICAL RESTORING FORCE CHARACTERISTICS

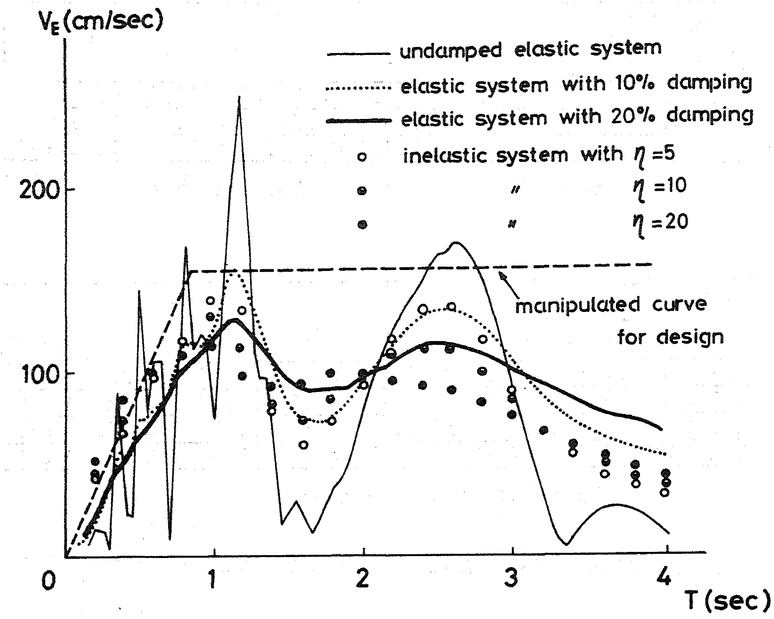


Fig.3. BASIC FEATURES OF TOTAL ENERGY INPUT

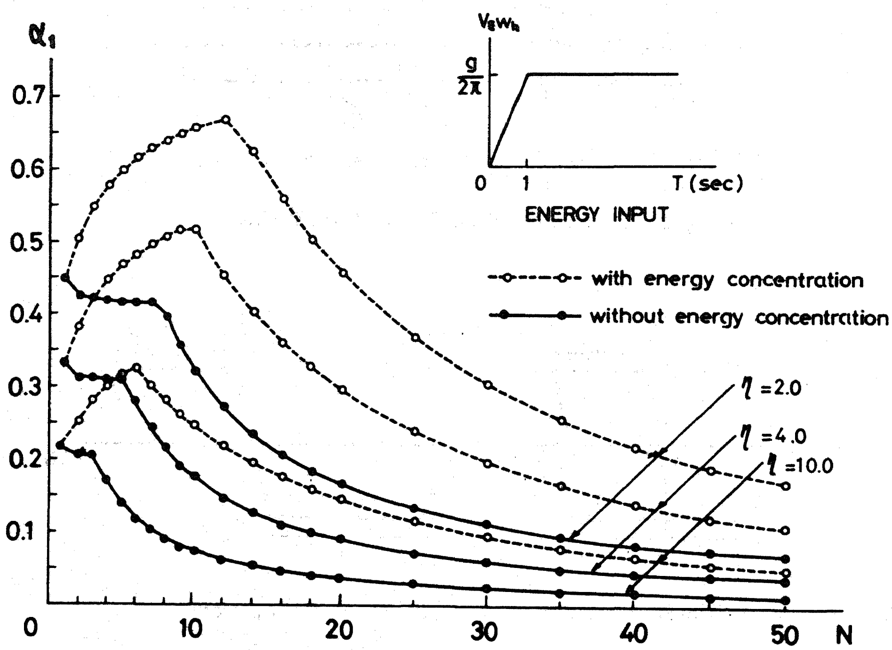


Fig.4. INFLUENCE OF ENERGY CONCENTRATION ON REQUIRED STRENGTH OF BUILDINGS