

## OPTIMUM DESIGN OF ASEISMIC COUPLED SHEAR WALLS

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### SUMMARY

An optimization procedure providing the most economical design of aseismic coupled shear walls is presented. When the seismic load, width, height and relative distance of the walls are fixed, the problem is reduced to three design variables only: thickness, maximum ductility of the beams and maximum drift. On them do in fact depend all the other variables through conditions which guarantee a good behavior of the structure when subjected to a strong earthquake. A few numerical examples are also given.

### INTRODUCTION

Coupled shear walls effectively solve the problem of structural design within seismic areas. In fact they combine a high stiffness, which reduces damages to non-structural components in consequence of the decreased deformability of the structure, with a considerable capability of dissipating energy during the hysteresis cycles of the beam elements connecting the walls, if suitably designed.

The authors, in two previous works [3,4], showed how to attain the limit design of coupled shear walls for a fixed value of the static load equivalent to the seismic action. The limit design is based on appropriate limitations on the maximum ductility of the connecting beams, so as to avoid failures due to excessive deformation, and on the maximum drift of the walls, so as to limit damages to non-structural components.

In this paper, we briefly sketch the fundamentals of the above mentioned analysis and present the relevant design formulae; afterwards an optimization method which provides the design of minimum cost will be described.

### FORMULATION OF THE PROBLEM

The shear wall under consideration, has only a vertical row of openings (see fig. 1) and it is subjected to a seismic equivalent static load of triangular shape with zero value at the bottom. The actual connecting beams are idealized as a continuous connection between the coupled walls [1] with a total stiffness and strength equal to the sum of the corresponding values of the single beams. In reality the structure collapses when both coupled walls as well the connecting medium attain yielding; in the present so as in the previous works, in order to reduce structural and non-structural damages, the structure is considered to be collapsed when the connecting medium and one wall only attain yielding. Moreover, the yielding of nearly the whole ( $\geq 90\%$ ) connecting

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medium is requested before the first yielding of any of the coupled walls, to warrant a ductile behaviour of the structure.

The thickness of the walls "s", the maximum ductility of the connecting beams " $\hat{\eta}$ " and the maximum drift of the walls " $\hat{\lambda}$ " are chosen as design variables of the optimization procedure, whereas the geometric quantities  $b_1$ ,  $b_2$ ,  $b$ ,  $h$ ,  $H$  have fixed and known values. The aforementioned positions lead (2) to the following formulae (the suffix "L" stands for quantities at limit state):

$$J_c = \frac{b^3 h}{6H^2 \left( \frac{1}{A_1} + \frac{1}{A_2} + \frac{l^2}{J_0} \right)} \left( \frac{11}{40} \frac{W_L H^2 \hat{\eta}}{E J_0 \hat{\lambda}} - 1.5 \nu \hat{\eta} \right) ; \quad q_L = \frac{12 E l J_c}{b^3} \frac{\hat{\lambda}}{\hat{\eta}}$$

$$N_{1,2L} = P_{1,2}(z) + q_L \frac{H-z}{h} ; \quad M_{1,2L} = \frac{J_{1,2}}{J_1 + J_2} \left( M_0(z) - \frac{l}{h} q_L (H-z) \right)$$

where:

$W_L$  is the value of the seismic equivalent static load at collapse;

$M_0(z)$  is the bending moment due to  $W_L$ ;

$J_c$  is the moment of area of each connecting beam,  $h_c$  its depth and  $q_L$  its limit shear force;

$P_{1,2}(z)$  is the normal force due to vertical loads,

$M_{1,2L}$  and  $N_{1,2L}$  are the limit bending moment and normal force in any cross section of the walls 1 and 2 respectively.

The aforementioned quantities and therefore the reinforcements of beams and walls, can be calculated in terms of the design variables through a rapidly convergent iterative procedure, controlled by the " $\nu$ " ratio of the average value of ductility to its maximum value all over the height.

The abovesaid collapse conditions are fulfilled by the structure if only  $J_c$  (and hence  $q_L$ ) is not less than zero and  $W_L$  is not less than the value  $W_y$  of the seismic equivalent static load at the yielding of the 90% of the connecting medium. The existence dominion of the limit design solution [4], in the space of the variables  $s$ ,  $\hat{\eta}$ ,  $\hat{\lambda}$ , is so defined; such dominion may be represented in a  $\lambda_0/\hat{\lambda}$ ,  $\nu \hat{\eta}$  plane, with  $\lambda_0 = 11 W_L H^2 / [60 E (J_1 + J_2) \nu]$ , for each minimum fixed value  $\omega_0 \geq 1$  of the  $\omega = W_L / W_y$  ratio (see fig. 2). The straight line  $\lambda_0/\hat{\lambda} = 1$  represents the  $J_c = 0$  condition while the curves  $\omega_0 = \text{const.}$  represent the  $\omega = \omega_0$  conditions; the straight line and the curve relevant to the  $\omega_0$  chosen value, are the boundary of the existence dominion of the solution. The above exposition clearly shows that, for a given design problem (i.e. fixed  $W_L$  and geometric quantities), there are infinite solutions  $s$ ,  $\hat{\eta}$ ,  $\hat{\lambda}$  within the existence dominion. The optimum design corresponds to choosing the minimum cost solution among such infinity.

#### OPTIMIZATION PROCEDURE

The optimization problem treated is a nonlinear programming problem and needs numerical solution. Firstly, the maximum and minimum allowable values of the design variables are evaluated. In the so defined field in the space of the variables  $s$ ,  $\hat{\eta}$ ,  $\hat{\lambda}$ , the object function to be minimized is given by the sum of a cost function and a penalty function. The penalty function is given by the sum of two terms; the former warrants that the found minimum belongs to the existence dominion defined in the previous paragraph, the second warrants that the condition  $h_{\min} \leq h_c \leq h_{\max}$  is verified, with  $h_{\min}$  and  $h_{\max}$

minimum and maximum allowable values of  $h_c$ . The cost function is given by the sum of the volumes of concrete and reinforcements multiplied by their unitary cost. The volume of vertical reinforcements required by the walls is evaluated interpolating the values corresponding to the cross sections at the bottom and at the midheight of each wall; such reinforcements are obtained via an independent optimization method.

The volume of reinforcements required by the beams is evaluated choosing its best arrangement according to  $h_c/b$  ratio. Minimum reinforcements are imposed both horizontally and vertically, all over the structure.

The conjugate direction method [2] provides the direction along which the search of the minimum has to be performed, by the golden section method. The starting directions are assumed to be parallel to the coordinate axes.

The whole procedure is performed for a fixed minimum value  $\omega_0$  of  $\omega$  and obviously the attained optimized solution change according to the chosen  $\omega_0$  value.

### NUMERICAL EXAMPLES AND CONCLUSIONS

Four different walls have been optimized via the above described procedure; tab. 1 shows the fixed geometric quantities and load of each type of wall. The minimum and maximum allowable values of the design variables are:  $S_{shear} \leq S \leq 1.2$  m,  $1.0 \leq \eta \leq 13.0$ ,  $1 \times 10^{-3} \leq \lambda \leq 8 \times 10^{-3}$ . Furthermore it has been assumed  $\omega_0 = 1.2$  for the S301, S303, S601 walls,  $\omega_0 = 1.0$  for the S603 walls and  $h_{min} = 0.0$  m,  $h_{max} = 1.00$  m. The obtained results are presented

Type	H (m)	b (m)	b <sub>1</sub> (m)	b <sub>2</sub> (m)	h (m)	Floor Weight (N)
S 301	30.	1.	5.5	5.5	3.	4.23x10 <sup>5</sup>
S 303	30.	3.	4.5	4.5	3.	4.23x10 <sup>5</sup>
S 601	60.	1.	5.5	5.5	3.	4.23x10 <sup>5</sup>
S 603	60.	3.	4.5	4.5	3.	4.23x10 <sup>5</sup>

Tab. 1

in Tab.2; three different seismic coefficients (horizontal to vertical load ratio), indicated by the first digit in the type specification, has been considered. In particular such coefficients are: 0.2, 0.4, 0.6 for the S301-S303 walls

and 0.2, 0.3, 0.4 for the S601-S603 walls.

The results in Tab.2 provide some suggestions for a choice of  $s$ ,  $\eta$ ,  $\lambda$  leading to the optimum design:

- 1) The thickness should assume always the minimum shear value;
- 2) The ductility should not assume the maximum allowable value for walls with wide holes;
- 3) The drift should not assume the maximum allowable value for short walls.

By such optimal choice of the design variables, the  $\omega$  value attains  $\omega_0$  for tall walls only while  $h_c$  attains its maximum value for walls with wide holes. At last it must be underlined that all the previous observations seem to be independent from the value of the seismic coefficient.

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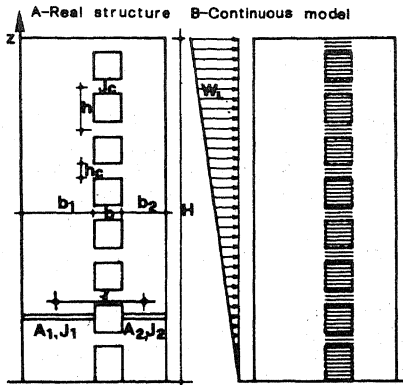


Fig. 1

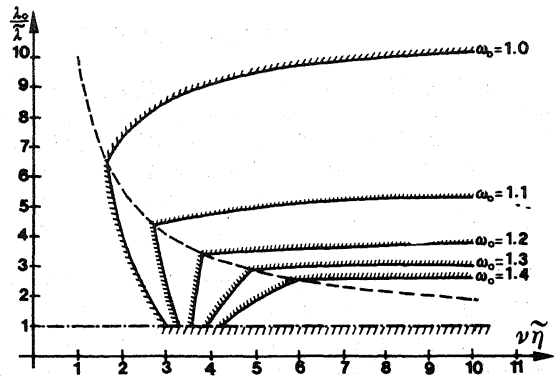


Fig. 2

Type	s (m)	$\tilde{\eta}$	$\tilde{\lambda} \times 10^3$	$q_L$ (KN x m)	$h_c$ (m)	Steel (m <sup>3</sup> )	Concrete (m <sup>3</sup> )	Cost	$\omega$
2S301	0.063	12.86	2.56	137.12	0.658	0.159	21.24	37.14	1.32
4S301	0.143	12.95	2.44	319.33	0.676	0.420	48.32	90.31	1.30
6S301	0.249	12.64	2.49	550.05	0.665	0.791	84.65	162.61	1.30
2S303	0.077	8.02	5.99	92.90	0.995	0.175	21.59	39.15	1.53
4S303	0.175	8.00	6.01	210.52	0.993	0.475	49.09	96.60	1.53
6S303	0.305	8.00	6.01	366.01	0.993	0.897	85.33	175.07	1.53
2S601	0.177	12.85	7.91	343.35	0.435	1.272	118.60	249.52	1.26
3S601	0.298	12.85	7.91	615.87	0.445	2.476	198.77	452.61	1.24
4S601	0.445	12.97	7.91	989.44	0.457	4.179	237.76	729.12	1.22
2S603	0.175	5.23	7.91	370.57	0.950	1.068	98.03	204.87	1.04
3S603	0.317	5.52	7.90	649.03	0.970	2.286	176.73	405.32	1.04
4S603	0.483	6.11	7.91	1020.93	1.000	4.510	270.55	657.26	1.04

Tab. 2