

EFFECT OF DAMPING AND TYPE OF MATERIAL  
NONLINEARITY ON EARTHQUAKE RESPONSE

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SUMMARY

Dynamic responses of single-degree-of-freedom nonlinear systems were computed for 10 different earthquake records for three types of resistance functions: elasto-plastic, bilinear, and stiffness degrading; and for relative damping values of 2, 5, and 10 percent of critical. Statistical studies of the results indicate that: (a) damping becomes less important as ductility increases; (b) ordinates of the mean spectra do not differ significantly for the 3 types of resistance function used; and (c) in general mean spectra for elasto-plastic systems give in almost all cases conservative estimates for the other types of systems considered.

INTRODUCTION

The dynamic response of single-degree-of-freedom nonlinear systems subjected to earthquake excitations has been recently considered (Ref. 1) with the purpose of (a) evaluating the effect of damping combined with inelastic behavior and the influence of the type of material nonlinearity on inelastic response, and (b) to propose new recommendations for deriving inelastic design spectra that take into account, explicitly, the effect of the previously mentioned factors. The principal findings and conclusions related to part (a) are discussed herein; the application to the derivation of design spectra, part (b), is presented in a companion paper (Ref. 2).

SYSTEM AND LOAD-DEFORMATION MODELS CONSIDERED

A simple single-degree-of-freedom system was considered, as shown in Fig. 1.a. The force in the spring, or resistance function  $R$ , depends on the relative displacement of the mass with respect to the ground. Three types of resistance functions were used, as shown in Fig. 1. The elasto-plastic model was selected both for its simplicity and because it has been widely used in nonlinear analyses, thus, comparison with other models is of great interest. The bilinear model, also frequently used, is meant to represent an upper bound to the hysteretic behavior of intrinsically ductile, nondeteriorating systems, like steel members, eccentrically braced steel frames, and unbraced steel frames with moderate axial loads. The stiffness degrading model represents the characteristic behavior of reinforced concrete members, frames, and walls under predominantly flexural stresses.

A few additional comments regarding the selection of the models are necessary. Notwithstanding the experimental results already available from

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cyclic tests of members, subassemblages and reduced scale models, it must be borne in mind that the hysteretic behavior of a real structure under strong ground shaking is still an unresolved question. A review of the available evidence, seems to indicate that the various hysteretic shapes can be grouped into a few patterns of some generality (Ref. 1). Two of these types, which cover a wide range of structural behavior, can be represented, in a simplified manner, by the mentioned bilinear and stiffness degrading models. It must be also noted that the models shown in Fig. 1 do not present strength deterioration, i.e., the maximum capacity can be developed and maintained regardless of the amplitude and number of yield excursions; the behavior may not be so in some cases, as occurs with reinforced concrete systems presenting reinforcement deficiencies or subjected to high shear stresses. Decay in strength is undesirable and efforts must be made to avoid it.

In addition to the energy dissipated by the system by inelastic behavior, in the structural mechanics sense, there are other sources of energy loss which are customarily taken into account by means of a damping factor. Damping values of 2, 5, and 10 percent of critical were used in combination with the elasto-plastic model. Damping of 5 percent was used for bilinear and degrading systems to permit comparison with the elasto-plastic case.

#### GROUND MOTIONS CONSIDERED AND RESULTS OF RESPONSE COMPUTATIONS

Because of the large number of systems considered, a limited number of earthquake records could be used. For this reason, no attempt of grouping the records according to similar characteristics was made; on the contrary, the ground motions selected cover a variety of situations regarding site conditions, distance to source, intensity, duration of motion, etc. It is thus believed that the sample of records indicated in Table 1 is representative of the randomness of earthquake excitations.

Step by step integration of the equation of motion was performed by means of standard procedures (Ref. 3). Special precautions were taken to handle records with specified initial conditions for ground velocity and displacement (Ref. 4). An iterative procedure was used to obtain responses associated to six preselected ductility factors, the results were considered satisfactory if within 1 percent of the desired ductility (Ref. 1).

About 40 frequencies were considered for each record and condition, thus, responses for about 12,000 different systems were computed, and the corresponding results summarized in the form of Inelastic Yield Spectra. These are tripartite logarithmic plots featuring  $\omega u_y$  as ordinates, and  $u_y$  and  $\omega^2 u_y$  in the  $45^\circ$  axes, where  $u_y$  is the yield displacement corresponding to the specified ductility  $\mu$ , and  $\omega^2 u_y$  is the acceleration associated with the yield point. The latter quantity multiplied by the mass  $m$  gives the yield force or maximum base shear if the system is elasto-plastic; in order to obtain the maximum force in systems with strain-hardening slope, one must multiply by  $m[1+s(\mu-1)]$ , where  $s$  is defined as shown in Fig. 1. The ductility factor is defined as  $\mu = u_m/u_y$ , where  $u_m$  is the absolute value of the maximum displacement of the system with respect to the ground.

## ANALYSIS OF THE DATA

As a first step in the statistical analysis presented in Ref. 1, the spectra was normalized to peak ground acceleration, velocity and displacement, and statistics were computed for each frequency, so that mean and mean plus one standard deviation spectra could be plotted. These sort of plots are relevant for a number of reasons. First, the appropriateness of the normalization to three parameters, as used in studies of elastic spectra, is verified for the inelastic case. Second, observation of mean spectra is paramount to devise a procedure to define spectral regions for which frequency band statistics can be computed; this process leads to factors for constructing inelastic design spectra (Ref. 2). Lastly, by means of comparisons of mean spectra for the various conditions considered in the study, observations can be made regarding the effect of damping and the type of resistance function on the response of inelastic systems, as discussed in the next sections.

### EFFECT OF DAMPING COMBINED WITH INELASTIC BEHAVIOR

Mean spectra for elasto-plastic systems with 2, 5, and 10 percent damping are compared in Fig. 2. The spectra shown are normalized to 1 in/sec. ground velocity; it must be noted, however, that whatever scaling parameter is used is immaterial for the purpose of this discussion. It is apparent that:

(a) At the low frequency end of the spectrum the effect of damping is negligible regardless of the ductility level. The same is in general true at the high frequency end; however, there is still some reduction for a ductility factor of 10.

(b) The effect of damping becomes less important as inelastic deformations increase. In the velocity and acceleration regions of the spectrum, say between 0.4 and 8 cps, the response of elastic systems is reduced, on the average, by about 40 percent when damping increases from 2 to 10 percent of critical, while for elasto-plastic systems with displacement ductility of 10 the mean response decreases by only 20 percent for the same damping range.

(c) In the displacement region, say between 0.1 and 0.4 cps, the effect of damping is more uniform for the various ductility values. Increasing the damping factor from 2 to 10 percent causes, on the average, a response reduction of 27 percent for elastic systems, 22 percent for systems with ductility of 1.5 and approximately 20 percent for ductilities between 2 and 10.

### EFFECT OF THE TYPE OF NONLINEARITY

Before considering mean spectra to discuss the effect of the type of nonlinearity on earthquake response, it is instructive to compare spectra for elasto-plastic, bilinear, and stiffness degrading systems, for 5 percent damping, computed for a particular ground motion. The spectra for El Centro, shown in Fig. 3, feature characteristics which are representative of the spectra for all the records considered in the study. It can be

seen that: (a) In general, differences are not significant in the low and high frequency ends of the spectrum; (b) for intermediate frequencies, the responses of bilinear and elasto-plastic systems with ductilities less than or equal to 2 are almost identical, while for larger ductilities, the maximum responses of bilinear systems are generally smaller than those of the associated elasto-plastic systems; (c) for intermediate frequencies, the responses of degrading systems are generally between about 0.5 to 1.5 times the response of the corresponding elasto-plastic systems; and (d) notably, spectra for degrading systems have a tendency to go below the peaks and above the troughs of spectra for elasto-plastic systems.

The differences or similarities in the response of particular systems with different types of resistance, for all other parameters the same, can be explained by means of the corresponding response time histories. The latter reveal that the energy dissipation mechanism and the specific characteristics of the ground motion itself interrelate in an extremely complex manner, thus making it practically impossible to predict accurately the response of a particular system to a particular earthquake motion. Furthermore, even systems with the same type of nonlinearity and amount of damping, and excited by the same ground motion, may present entirely different hysteretic behavior depending on their yield point resistances (Ref. 1).

It is apparent that no conclusive statements can be made from the observation of particular systems. On the contrary, comparisons of average spectra for a number of records clearly reveal some general trends. From the spectra shown in Fig. 4 one concludes that: (a) The ordinates of the mean spectra do not vary significantly when various nonlinear models are used; differences occur mainly for intermediate frequencies and large ductilities, and are practically negligible at the low and high frequency ends of the spectrum; (b) use of the elasto-plastic idealization provides, in almost every case, a conservative estimate of the average response to a number of earthquake motions; and (c) it is particularly significant that, on the average, the stiffness degradation phenomenon is not as critical as one might expect a priori.

#### ACKNOWLEDGMENT

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REFERENCES

1. Riddell, R., and Newmark, N. M., 1979, "Statistical Analysis of the Response of Nonlinear Systems Subjected to Earthquakes," Civil Engineering Studies, Structural Research Series No. 468, University of Illinois, Urbana, 291 pp.
2. Newmark, N. M., and Riddell, R., 1980, "Inelastic Spectra for Seismic Design," 7th World Conference on Earthquake Engineering, Istanbul, Turkey.
3. Newmark, N. M., 1962, "A Method of Computation for Structural Dynamics," Transactions, ASCE, Vol. 127, pp. 1406-1435.
4. Pecknold, D. A., and Riddell, R., 1978, "Effect of Initial Base Motion on Response Spectra," Journal of the Engineering Mechanics Division, ASCE, Vol. 104, No. EM2, pp. 485-491.

TABLE 1 EARTHQUAKE ACCELEROGRAMS CONSIDERED

Earthquake	Year	Recording Station	Comp.	Peak Accel.(g)
Imperial Valley, Ca.	1940	El Centro	EW	.214
Western Washington	1949	Olympia	N86E	.280
San Francisco, Ca.	1957	Golden Gate Park	S80E	.105
Parkfield, Ca.	1966	Cholame, St. 5	N85E	.434
San Fernando, Ca.	1971	Pacoima Dam	S16E	1.171
		Castaic	N21E	.316
Off Peru Coast	1970	Lima, Inst. Geo.	N82W	.107
Off Chile Coast	1971	Santiago, U of Ch.	N10W	.159
Managua, Nicaragua	1972	Managua, ESSO	EW	.383
San Juan, Argentina	1977	San Juan, Inpres	EW	.193

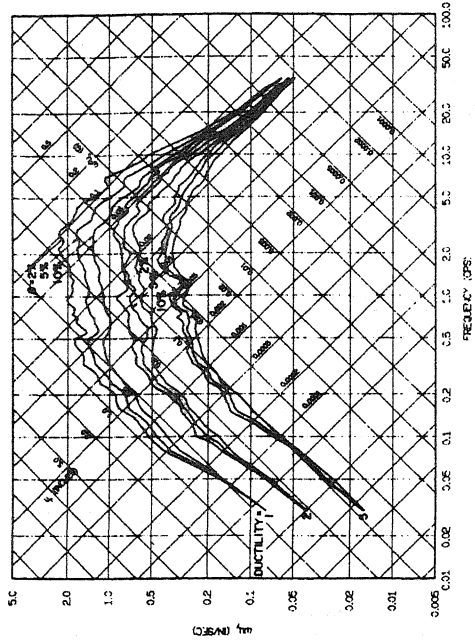
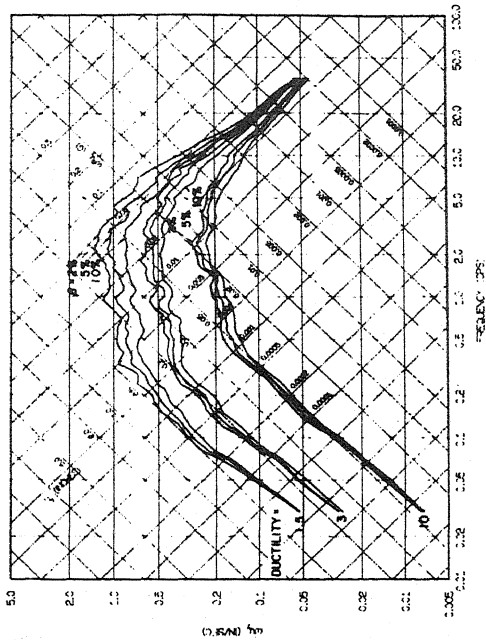


FIG. 2. COMPARISON OF MEAN SPECTRA FOR ELASTOPLASTIC SYSTEMS WITH 1, 5 AND 10% DAMPING.

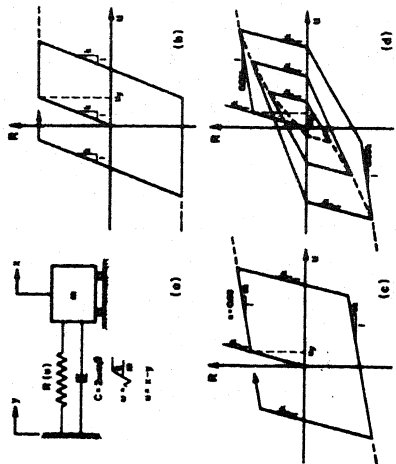


FIG. 1. CHARACTERISTICS OF THE SYSTEM AND RESISTANCE FUNCTIONS CONSIDERED

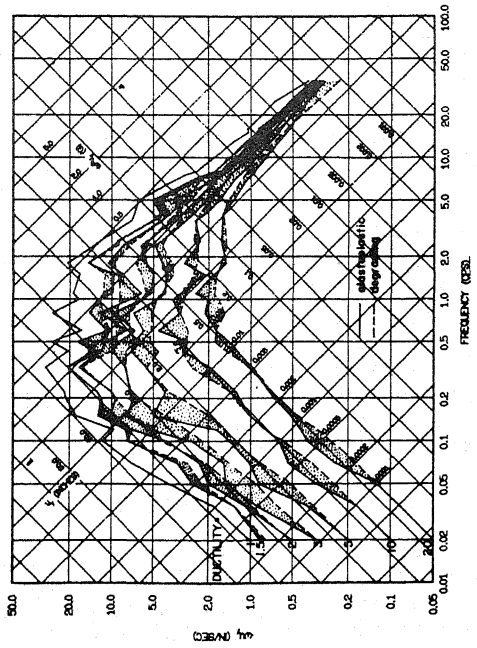
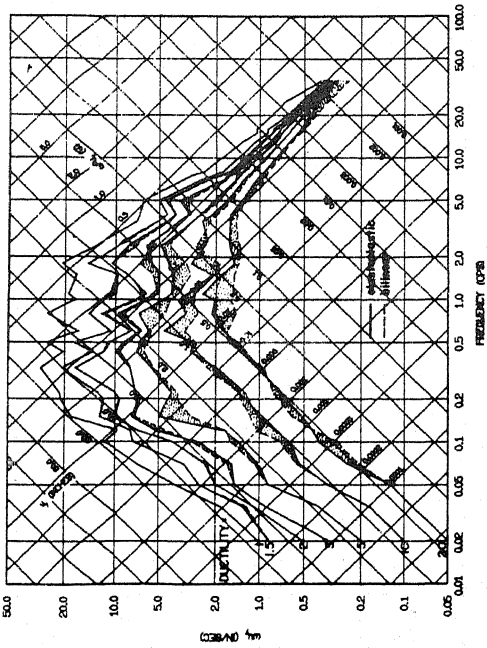


FIG. 3 COMPARISON OF SPECTRA FOR ELASTOPLASTIC, BILINEAR AND DEGRADING SYSTEMS WITH 1% DAMP. EL CENTRO EQ.

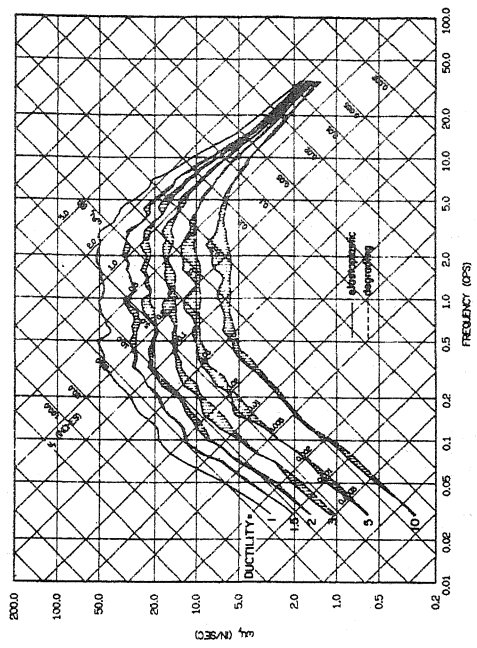
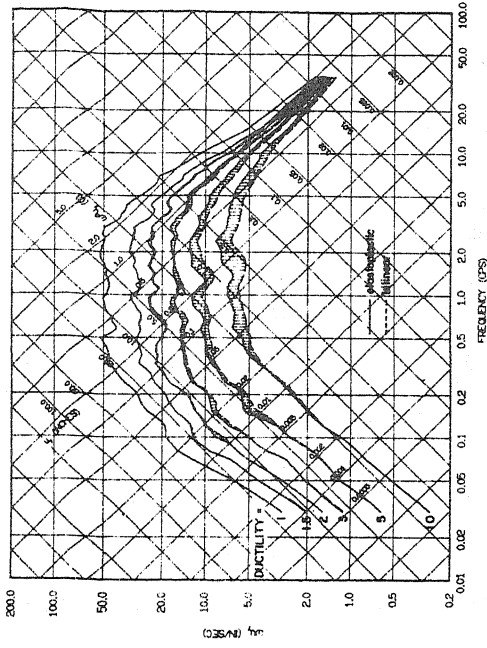


FIG. 4 COMPARISON OF MEAN SPECTRA FOR ELASTOPLASTIC, BILINEAR AND DEGRADING SYSTEMS WITH 5% DAMPING.