ON DEFORMATIONS DUE TO HORIZONTAL FORCES

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SUMMARY

One bay multistorey frames loaded by horizontal forces are considered. Three angle equation is developed and general solution is given for uniform frames. From the solution for a frame subjected to a horizontal force at the top floor numerical results are obtained for a 10 storeys frame for various values of beam-to-column stifness ratio and some conclusions are drawn about the influence of beam stifness on deformations.

1. INTRODUCTION

Forces due to earthquakes and winds cause drift of structures. Excessive drift has several structural, non structural and human effects so the calculation of drift holds an important place in the analysis of tall structures and has been treated by many authors. See for instance well known work of K.Muto [1] . It is necessary to fix some permissible limits for lateral deflections. This is usually set at 0 , 1 to 0.35 per cent of height.

For approximate analysis lateral displacements can be considered as consisting of two components: Racking component due to bending of columns and girders without change in their lengths and cantilever component due to bending of the frame as a cantilever.

The racking component is dominant unless the structure is a tall or slender one in which case the cantilever component may be as large as the racking component. In the following the racking component in uniform frames is considered and the influence of the column and girder stifnesses on deformations is determined.

2. FUNDAMENTAL RELATIONS

One bay multistorey frames subjected to horizontal forces are considered and notations are as shown in Fig.1.

Slope deflection formulae and the j^{th} storey column equilibrium equations are

$$M_{j\ell}=S_j(2\theta_{j-1}+\theta_j-3\psi_j)\,,\ M_{ju}=S_j(2\theta_j+\theta_{j-1}-3\psi_j)\,,\ M_{ig}=3C_j\theta_j \tag{2.1}$$

$$h_jQ_j+M_{ju}+M_{j\ell}=0 \tag{2.2}$$

and substituting from Eq.2.1 into Eq.2.2 it is obtained that

$$\psi_j=\psi_j^*+\frac{1}{2}(\theta_{j-1}+\theta_j) \tag{2.3}$$

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in which

$$\psi_j^* = \frac{h_j Q_j}{6 S_j} = \frac{h_j \bar{Q}_j}{12 S_j} = \frac{A_j}{12 S_j} \tag{2.4}$$

then

$$M_{j\ell} = \frac{S_j}{2}(\theta_{j-1} - \theta_j - 6\psi_j^*),\ M_{ju} = \frac{S_j}{2}(\theta_j - \theta_{j-1} - 6\psi_j^*),\ M_{jg} = 3C_j\theta_j \tag{2.5}$$

ψ_j^* is the sway angle if the columns were clamped at both ends against rotation.

The j^{th} floor joint equilibrium with Eq.2.5 gives three angles equation for $1 \leqslant j \leqslant n-1$

$$-S_j\theta_{j-1} + (S_j + S_{j+1} + 6C_j)\theta_j - S_{j+1}\theta_{j+1} = 6(S_j\psi_j^* + S_{j+1}\psi_{j+1}^*) \quad 1 \leq j \leq n-1 \tag{2.6}$$

For top floor joint

$$-S_n \cdot \theta_{n-1} + (S_n + 6C_n) \cdot \theta_n = 6\,S_n\psi_n^* \tag{2.7}$$

3. UNIFORM FRAMES

Frames considered here are assumed to have the same column stifness, beam stifnesses and storey heights throughout and Eq.2.6 and Eq.2.7 become

$$-\theta_{j-1} + 2(1+3\lambda)\theta_j - \theta_{j+1} = 6(\psi_j^* + \psi_{j+1}^*) \tag{3.1}$$

$$-\theta_{n-1} + (1+6\lambda)\theta_n = 6\psi_n^* \tag{3.2}$$

in which.

$$\lambda = C/S \tag{3.3}$$

with

$$r = (1+3\lambda) - \sqrt{(1+3\lambda)^2 - 1} \tag{3.4}$$

and θ_{jp} being a particular solution of Eq.3.1 its general solution is

$$\theta_j = B_1 \cdot r^{j-i} + B_2 r^{k-j} + \theta_{jP} \tag{3.5}$$

in which B_1 and B_2 are constants as yet unknown but to be determined from boundary conditions and i and k numbers of two for some reasons appropriate floors (for instance for boundary conditions).

In the following in order to get quantitative idea about the influence of the girder stifnesses on deformations the loading shown in Fig.2 will be considered. For this case

$$\psi_j^* = \psi^* = \frac{h \cdot P}{12S}, \quad \theta_{jP} = \theta_\infty = \frac{2\psi^*}{\lambda} \tag{3.6}$$

$$\theta_j = B_1 r^{j-i} + B_2 r^{k-j} + \theta_\infty \tag{3.7}$$

and with boundary conditions θ_o and θ_n and introducing

$$H(r,p) = r^{-p} - r^{p}, \quad D(r,p) = r^{-p} + r^{p} \tag{3.8}$$

so

$$6\lambda = D(r,1) - 2 = \left[H(r,\tfrac{1}{2})\right]^2 \tag{3.8'}$$

it is obtained that

$$\theta_j = \left[1 - \frac{D(r,\frac{j-j''}{2})}{D(r,\frac{n}{2})}\right] \cdot \theta_\infty + \frac{H(r,j'') \cdot \theta_o + H(r,j) \cdot \theta_n}{H(r,n)} \tag{3.9}$$

$$\quad ① \qquad ② \qquad\qquad ③ \qquad\qquad ④$$

In Eq.3.9 ① gives rotations at joints infinitely many storeys away from both ends in an infinitely many storeys high frame. ①+② is solution for a frame which is fixed against rotation at both ends, Fig.2b. ③, ④ and ③+④ correspond to the end rotation loadings shown in Figs2c,d and e.

Considering tha boundary condition Eq.3.2, later Eq.2.3 and the horizontal displacement $\Delta_j = \sum_{j=1}^{j} h\psi_j$ of the j^{th} floor one gets

$$\theta_j = \left[1 - \frac{D(r,j''+\frac{1}{2}) + \frac{1}{2}\left[H(r,\frac{1}{2})H(r,j)\right]}{D(r,n+\frac{1}{2})}\right] \cdot \theta_\infty + \frac{D(r,j''+\frac{1}{2})}{D(r,n+\frac{1}{2})}\theta_o \tag{3.10}$$

$$② \qquad\qquad ③ \qquad\qquad ④$$

$$\underset{\textcircled{1}}{\psi_j = \psi^*} + \underset{\textcircled{2}}{\theta_\infty} - \underset{\textcircled{3}}{\frac{D(r,\frac{1}{2})D(r,j''+1)}{2D(r,n+\frac{1}{2})}\left[1+\frac{H(r,\frac{1}{2})H(r,j-\frac{1}{2})}{2D(r,j''+1)}\right]\cdot\theta_\infty} + \underset{\textcircled{4}}{\frac{D(r,\frac{1}{2})D(r,j''+1)}{2D(r,n+\frac{1}{2})}\cdot\theta_0} \tag{3.11}$$

$$\frac{\Delta_j}{h} = j(\underset{\textcircled{1}}{\psi^*} + \underset{\textcircled{2}}{\theta_\infty}) - \underset{\textcircled{3}}{\frac{D(r,\frac{1}{2})H(r,\frac{j}{2})D(r,n+\frac{1}{2}-\frac{j}{2})}{2H(r,\frac{1}{2})D(r,n+\frac{1}{2})}\left[1+\frac{H(r,\frac{1}{2})H(r,\frac{j}{2})}{2D(r,n+\frac{1}{2}-\frac{j}{2})}\right]\theta_\infty}$$

$$+ \underset{\textcircled{4}}{\frac{D(r,\frac{1}{2})H(r,\frac{j}{2})D(r,n+\frac{1}{2}-\frac{j}{2})}{2H(r,\frac{1}{2})D(r,n+\frac{1}{2})}\theta_0} \tag{3.12}$$

in which (1) corresponds to the case of girders of infinite stifness, (2) to flexibility of girders in an infinitely many storeys frame, (3) is correction due to finite number of storeys and (4) shows the influence of the rotation at the bottom. (1)+(2)+(3) is the solution for frame fixed at the bottom.

4. QUANTITATIVE RESULTS AND CONCLUSIONS

In order to draw quantitative conclusions about the influence of beam stifness on deformations due to lateral forces a frame of ten storeys height loaded at the top floor and of various beam-to-column stifness ratios $\lambda = C/S$ has been considered. Results obtained from Eqs.3.10 , 3.11 and 3.12 for a frame fixed at the bottom are given plotted in Figs.3 to 6.

It is seen that for a given column stifness beam-to-column stifness ratio λ has a great influence on the magnitude of deformations due lateral forces. For the extreme cases of $\lambda = 0$ and $\lambda \to \infty$ the ratio of the top floor displacements is $4n^3 h\psi^*/nh\psi^* = 4n^2$ and of the top storey sway angles $2(3n^2-1)\psi^*/\psi^* = 2(3n^2-1)$, for instance for a ten storeys high building 400 and 598, respectively. This ratio increases rapidly as the number of storeys of frame increases. However there will be no substantial increase in the rigidity of the frame for values of λ greater than say 5 as seen in Figs. 3 to 6.

ACKNOWLEDGEMENT

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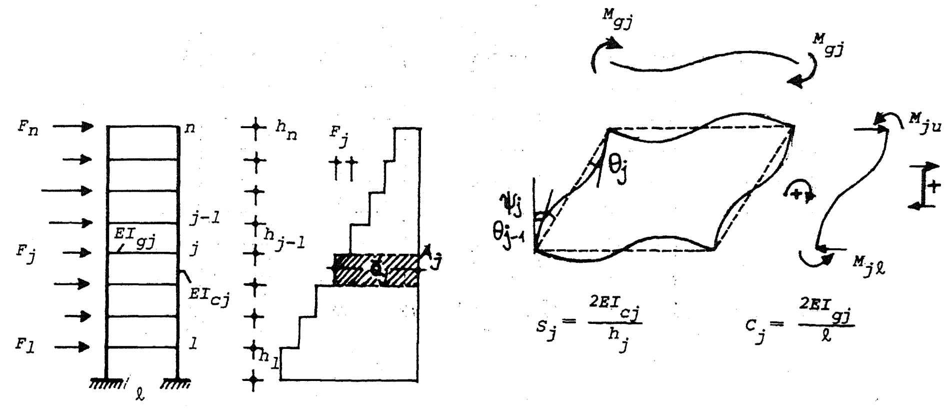
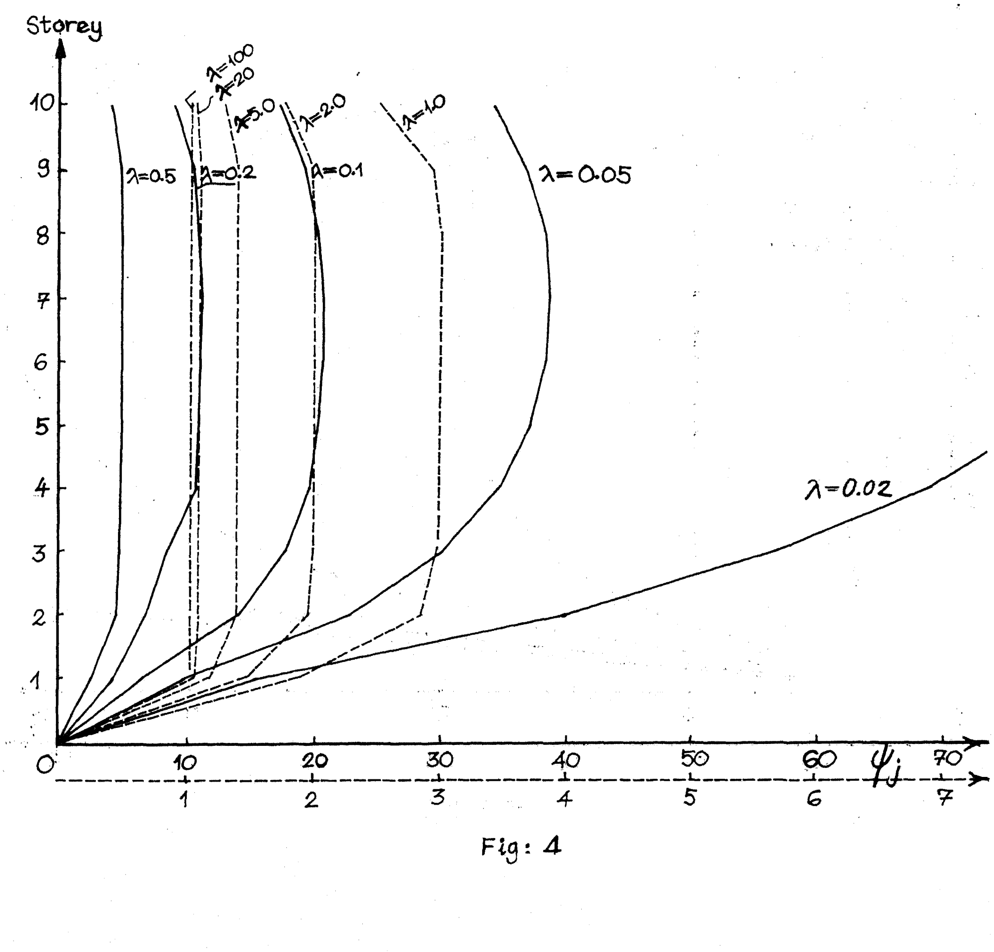
REFERENCE

[1] Muto,K., 1974, Asesismic Design Analysis of Buildings,Maruzen Co.Ltd., Tokyo, Japan.



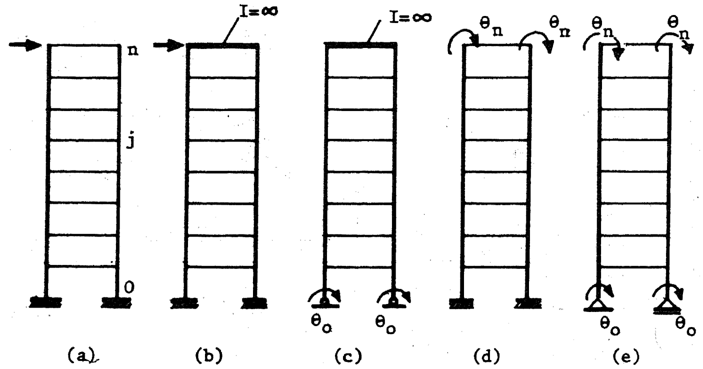
F_n
F_j
F_1
n
j-1
j
1
EI_{gj}
EI_{cj}
ℓ
(a)
h_n
h_{j-1}
h_1
F_j
(b)
M_{gj}
M_{gj}
M_{ju}
$M_{j\ell}$
θ_j
ψ_j
θ_{j-1}
$S_j = \frac{2EI_{cj}}{h_j}$
$C_j = \frac{2EI_{gj}}{\ell}$

Fig : (1)



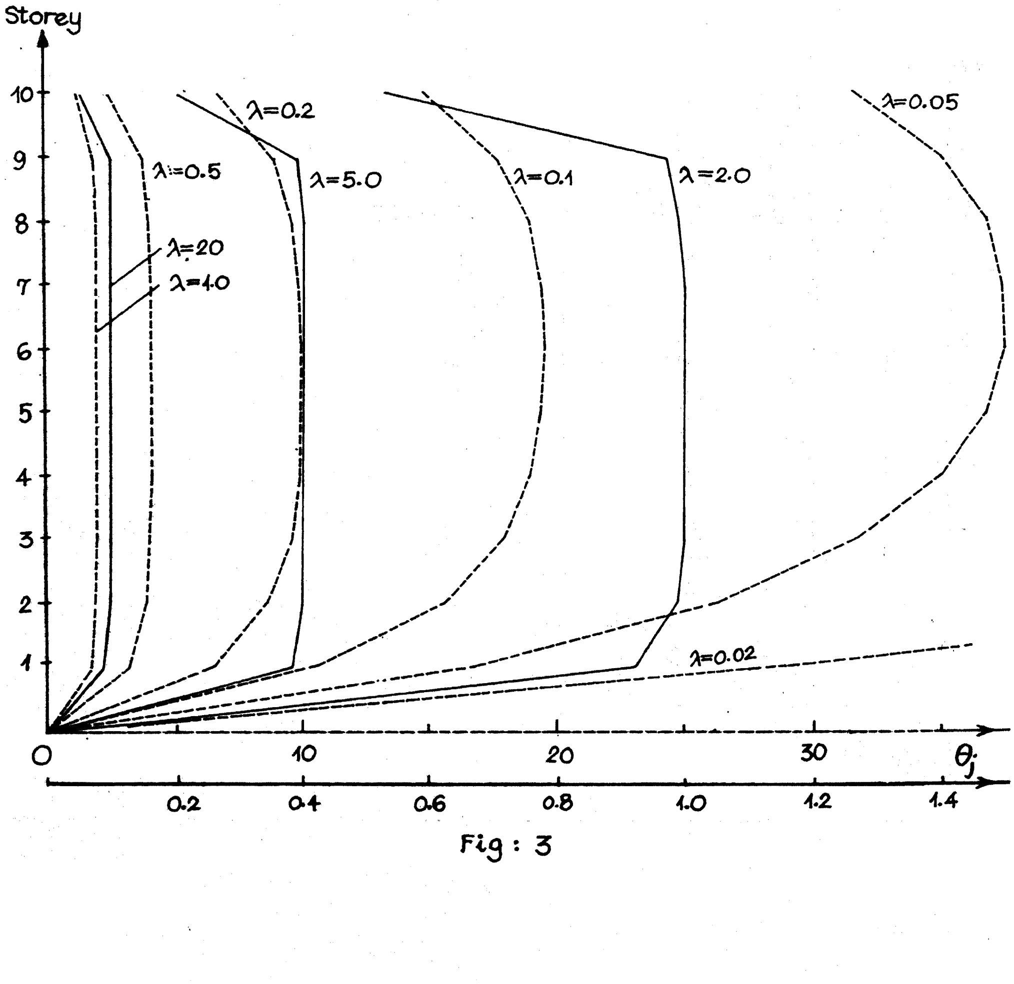
Storey
λ=100
λ=20
λ=5.0
λ=2.0
λ=1.0
λ=0.5
λ=0.2
λ=0.1
λ=0.05
λ=0.02
ψ_j
0
10
20
30
40
50
60
70
1
2
3
4
5
6
7
8
9
10

Fig: 4



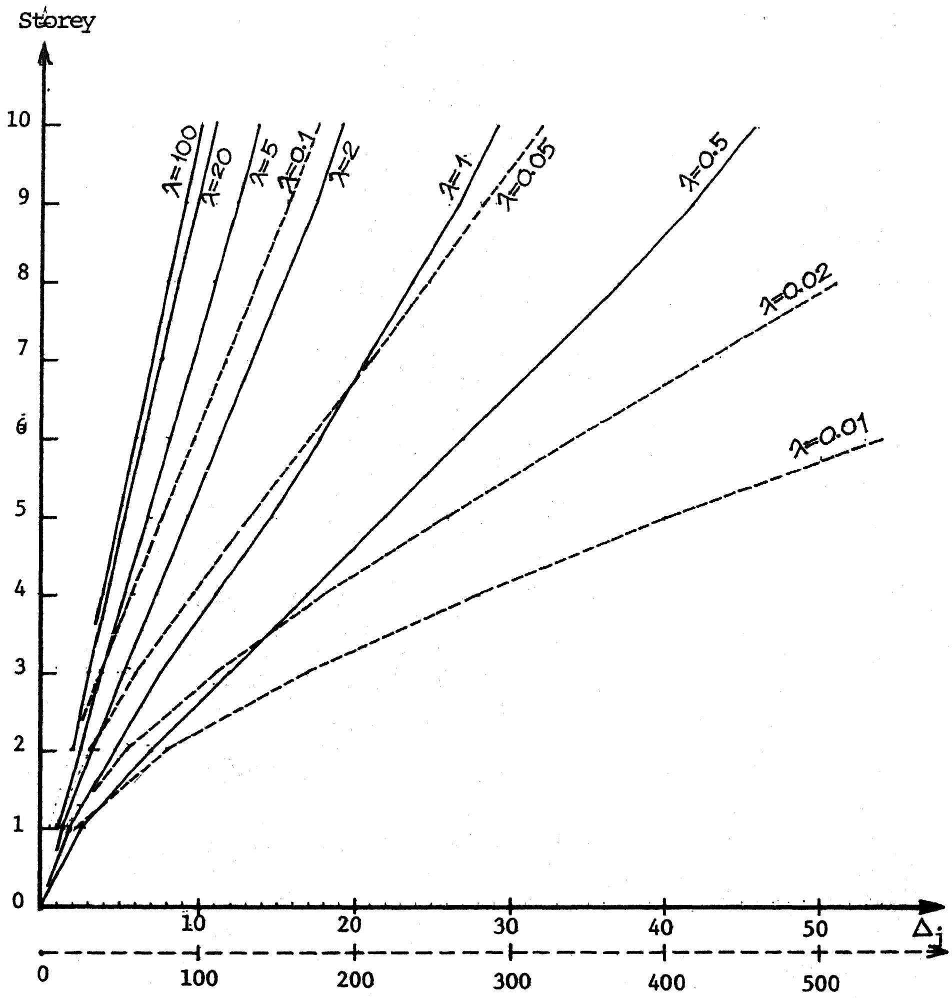
I=∞
I=∞
n
j
0
θn
θn
θn
θn
θo
θo
θo
θo
(a)
(b)
(c)
(d)
(e)

Fig. 2.



Storey
10
9
8
7
6
5
4
3
2
1
λ=0.2
λ=0.05
λ=0.5
λ=5.0
λ=0.1
λ=2.0
λ=20
λ=4.0
λ=0.02
0
10
20
30
θj
0.2
0.4
0.6
0.8
1.0
1.2
1.4

Fig : 3



Storey
10
9
8
7
6
5
4
3
2
1
0
λ=100
λ=20
λ=5
λ=0.1
λ=2
λ=1
λ=0.05
λ=0.5
λ=0.02
λ=0.01
10
20
30
40
50
Δj
0
100
200
300
400
500

Figure : 5

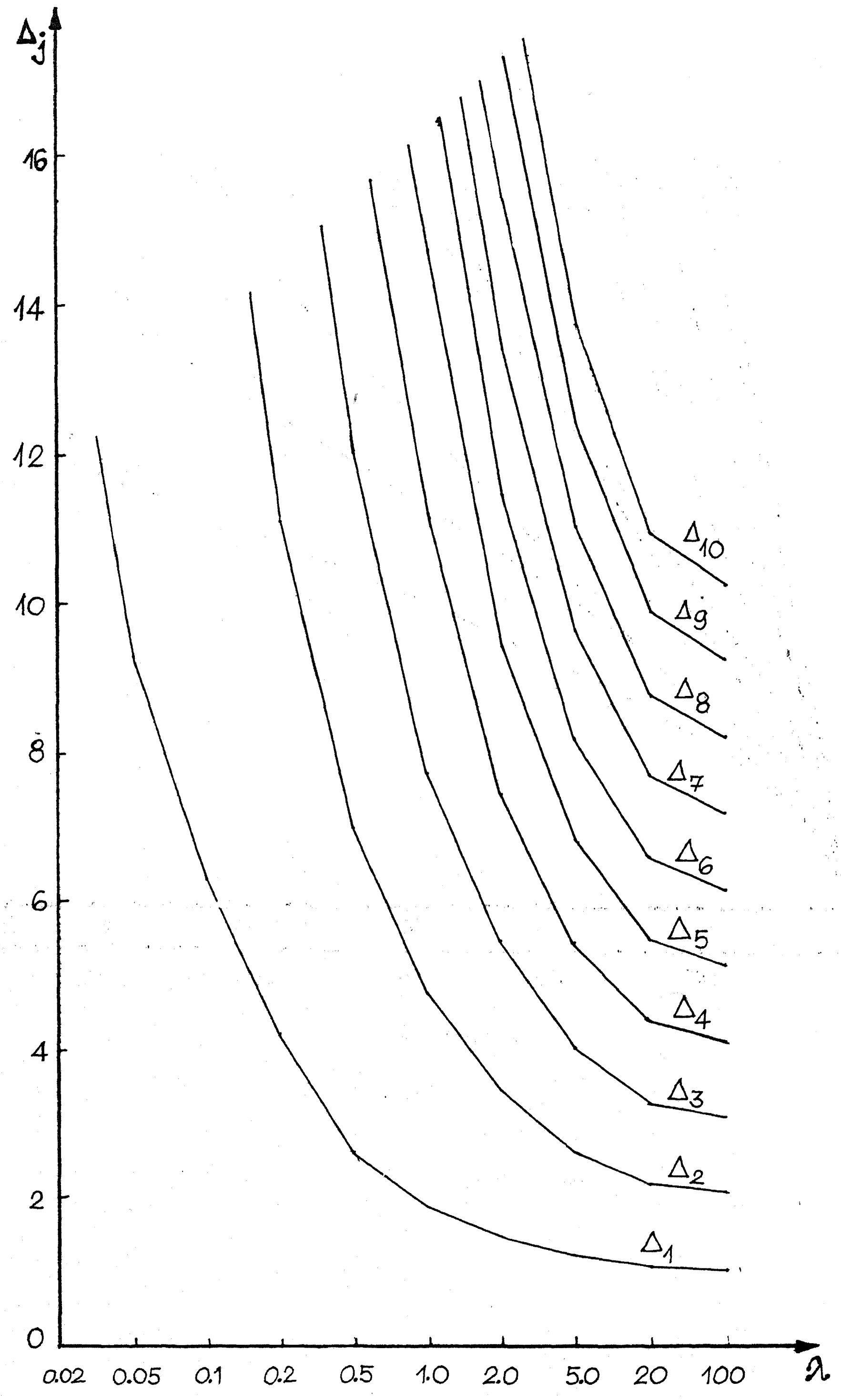

Δ_j
16
14
12
10
8
6
4
2
0
0.02
0.05
0.1
0.2
0.5
1.0
2.0
5.0
20
100
λ
Δ_{10}
Δ_9
Δ_8
Δ_7
Δ_6
Δ_5
Δ_4
Δ_3
Δ_2
Δ_1

Fig: 6