## SPATIAL RESPONSE SPECTRUM

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SUMMARY

A new conceptual tool, the Spatial Response Spectrum, is proposed for the direct evaluation of the modal maximum effects on a structure due to two earthquake components acting contemporaneously. The SRS is, for every value of the damping factor, a surface in the space S,T,β, where β is the parameter of a linear combination of the two acceleration components, $\cos\beta\,\ddot{S}_1 + \sin\beta\,\ddot{S}_2$, and it is related to the structure as a function of the geographical position and, for every mode, of the partecipation factors.

INTRODUCTION

It is known that, to superpose the maximum modal effects produced by a single earthquake component computed via the spectral technique, several approximate formulas are used, the most common of which is the so called SRSS formula:

$$S = \sqrt{\sum_{i=1}^{n} S_i^2} \qquad (1)$$

Furthermore, when, always in the linear case, more simultaneous earthquake components are considered, the commonly used superposition criterium is based on the natural extension of the SRSS formula [1,2,3,4]

$$S = \sqrt{\sum_{i=1}^{n} (S_{i1}^2 + S_{i2}^2 + S_{i3}^2)}, \qquad (2)$$

where $S_{ik}$ is the (maximum) $i^{th}$ modal effect produced by the $K^{th}$ component (K=1,2,3), evaluated from the response spectrum associated with the component itself.

It is clear, however, that the expression (2) introduces further possibilities of approximations and dispersions in the computed response. If two components are considered only, namely the horizontal ones, that are the most important for many structures, it is possible to introduce, compute and use a new conceptual tool, the Spatial Response Spectrum, SRS, which allows the practically "exact" evaluation of the global effect of the two components for every mode, so as to perform the final superposition through the original SRSS formula (1).

DEFINITION OF THE SRS

Let x and y be the orthogonal reference axis in plan for a generic spatial structure reduced to an appropriate discrete n - d.o.f. system. In normal coordinates the motion equations, uncoupled, are, with obvious notation:

$$\ddot{\varphi}_i + 2\nu_i\,\omega_i\,\dot{\varphi}_i + \omega_i^2\varphi_i = -g_{ix}\,\ddot{x} - g_{iy}\,\ddot{y}\ ;\ i = 1,2,\ldots,\ n; \qquad (3)$$

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$g_{ix}$ and $g_{iy}$,partecipation factors, are expressed by

$$g_{ix} = \sum_{j=1}^{n} \sum_{k=1}^{n_x} u_{ij}\, m_{jk} \quad , \quad g_{iy} = \sum_{j=1}^{n} \sum_{k=n_x+1}^{n_y} u_{ij}\, m_{jk} \; , \qquad (4)$$

with $u_{ij}$ eigenvectors components, $m_{jk}$ coefficients of the inertia matrix, and with the following d.o.f. pattern: from 1 to $n_x$ the parameters related to x, from $n_x+1$ to $n_y$ those related to y, from $n_y+1$ to n the remaining ones.

Generally the input will not be defined directly in terms of x and y, but along the geographical directions N-S and E-W; let $S_1$and $S_2$ be such directions and $\alpha$ the angle $x\hat{S}_1$ (Fig.1). From the figure we obtain

$$\begin{cases} \ddot{x} = \ddot{S}_1\cos\alpha - \ddot{S}_2\sin\alpha \; , \\ \ddot{y} = \ddot{S}_1\sin\alpha + \ddot{S}_2\cos\alpha \; ; \end{cases} \qquad (5)$$

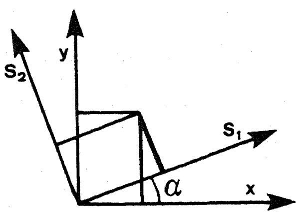

Fig. 1

with such expressions the r.h.s. of (3) becomes:

$$-(g_{ix}\cos\alpha + g_{iy}\sin\alpha)\,\ddot{S}_1 - (-g_{ix}\sin\alpha + g_{iy}\cos\alpha)\,\ddot{S}_2 \qquad (6)$$

In order to solve immediately the generic equation (3÷6) one should have at his disposal tne response spectrum associated with the linear combination (6), while commonly the two separated spectra of $S_1$ (N-S) and $S_2$ (E-W) are provided only, each of them obtained numerically with $\ddot{S}_1(t)$ and $\ddot{S}_2(t)$ input respectively.

From the coefficients of $\ddot{S}_1$ and $\ddot{S}_2$ in (6) let us define a new quantity $g_i$ :

$$(.........)^2 + (.........)^2 = g_{ix}^2 + g_{iy}^2 = g_i^2 \; ,$$

$$g_i = \sqrt{g_{ix}^2 + g_{iy}^2} \; ; \qquad (7)$$

then let us define an angle $\beta$ such that:

$$\begin{cases} \cos\beta = \dfrac{1}{g_i}\,(g_{ix}\cos\alpha + g_{iy}\sin\alpha) \; , \\ \sin\beta = \dfrac{1}{g_i}\,(-g_{ix}\sin\alpha + g_{iy}\cos\alpha) \; ; \end{cases} \qquad (8)$$

with these definitions (3÷6) becomes:

$$\ddot{\varphi}_i + 2\nu_i\,\omega_i\,\dot{\varphi}_i + \omega_i^2\varphi_i = -g_i\,(\cos\beta\;\ddot{S}_1 + \sin\beta\;\ddot{S}_2) \qquad (9)$$

Now, but for the intensity factor $g_i$ , the linear combination of the two components $S_1$ and $S_2$ depends only on the $\beta$ factor, related to the angle $\alpha$ and to the partecipation factors by the (8); such observation suggests the definition of the Spatial Response Spectrum of the couple of components $S_1$ $S_2$, as the family of the response spectra of the linear combination $\cos\beta\;\ddot{S}_1 + \sin\beta\;\ddot{S}_2$ for $0 \leqslant \beta \leqslant 180°$.

The computation of such a spatial spectrum may be easily performed, with a numerical effort not much heavier than that required for the computation of the two usual N-S and E-W spectra. In fact, if we consider the two motion equations

$$\ddot{\varphi} + 2\nu\omega\dot{\varphi} + \omega^2\varphi = -\ddot{S}_1 \quad (10a)$$

$$\text{"} \quad \text{"} \quad \text{"} = -\ddot{S}_2 \quad (10b)$$

it is sufficient, for every $\omega$ and $\nu$, to perform contemporaneously the step by step integration of (10a) and (10b) and, at every time station for which the two actual responses are usually compared with the stored maxima, to consider the linear combination

$$\cos\beta \cdot (\text{response to } S_1) + \sin\beta \cdot (\text{response to } S_2) \quad (11)$$

for an appropriately chosen series of $\beta$ values, and to operate as previously indicated for every component of such series. At the end of the computation one obtains, instead of two ordinates of the two N-S and E-W spectra, a series of ordinates of the spectra family, i.e. of the spatial spectrum. Of course one still obtains, for $\beta = 0°$ and $90°$, the N-S and E-W spectra.

## USE OF THE SRS

The way of using the SRS is quite evident. Known $\alpha$, the partecipation factors $g_{ix}$ and $g_{iy}$ are calculated for every mode; $g_i$ and $\beta$ are then computed by means of (7) and (8) and the SRS, (e.g. the ecceleration spectrum), with the present values of $\beta$, $\omega_i$ and $\nu_i$, is entered, thus finding the modal acceleration maximum which, multiplied by $g_i$ allows the computation of the researched modal maximum effect. The analysis is then concluded by the application of (1), as in the case of a single component input.

## A PHYSICAL INTERPRETATION OF THE DISPLACEMENTS SRS

Let us consider a system with 2 d.o.f. along x and y, with equal natural periods $T_1 = T_2$, under the N-S + E-W input. Being $T_1 = T_2$ the distinction between $S_1$ and x, $S_2$ and y is unessential and the two motion eqs. can be written ($\omega_1 = \omega_2 = \omega$)

$$\begin{cases} \ddot{x} + 2\nu\omega\dot{x} + \omega^2 x = -\ddot{S}_1 \\ \ddot{y} + 2\nu\omega\dot{y} + \omega^2 y = -\ddot{S}_2 \end{cases} \quad (12)$$

The solution of the equations defines the trajectory in the plane xy ($\equiv S_1S_2$) of the oscillator's mass M.

If we consider the displacements SRS the expression (11) becomes

$$x\cos\beta + y\sin\beta \, , \quad (13)$$

and the maximum of that quantity is given by the SRS. Now it is easy to recognize that the expression (13) gives the length of the segment OP in the plane xy (Fig.2), where OP is the projection of M on the straight line from the origin with $\beta$ inclination on the x-axis.

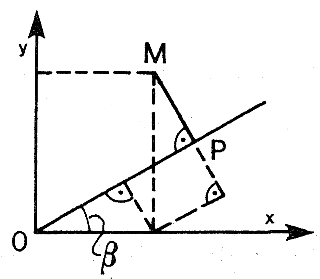

Fig. 2

We thus realize that: to look, for a given time history, for the maximum of the expression (13), i.e. for an ordinate of displacements SRS, is equivalent to look, in the plane xy where the motion of the 2 d.o.f. system takes place, for the maximum of the segment OP associated with the trajectory defined by the T.H. itself (Fig.3). Of course every trajectory allows to individuate the segments $OP_j$ related to a whole series of $\beta$-values (Fig.4) and so gives a direct representation of the SRS: indeed it is sufficient to connect the points $P_j$ to obtain the curves T = const. in the polar plane: max displ., β.

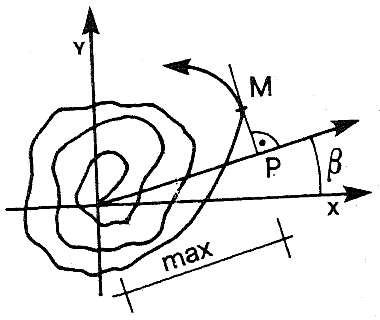


Fig. 3

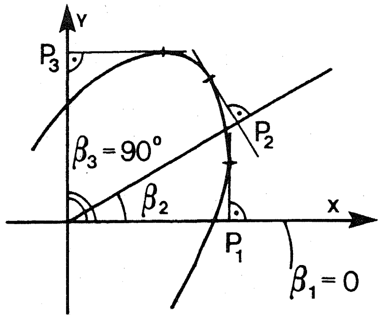


Fig. 4

Finally we can observe that the interpretation now illustrated allows also, at least in principle, to recognize a possible instrumental deduction of the displ. SRS.

REPRESENTATION OF THE SRS

The above indicated representation has a theoretical rather than a practical interest; moreover, it concerns only the displacements SRS. Considering now more practical representations, the most suitable seems to be that defined by sections β = const., e.g. as shown in Fig.5 which is related to the registration of the main shock of the Friuli earthquake (Tolmezzo, may 6th 1976). Is is easy to see the gradual passage from the N-S to the E-W spectrum.

An effective overall representation is then obtained by assembling all the sections so as to get a 3D picture (Fig.6).

One can observe that every section β = const. has a proper meaning as the response spectrum associated with the accelerogram defined by the linear combination $\cos\beta\, \ddot{S}_1 + \sin\beta\, \ddot{S}_2$, i.e. related to the plane system set in the β-oriented vertical plane.

COMPARISONS

Without the computation which leads to the definition of the SRS, the possible necessity to compute a spectrum for β-oriented plane (vertical) systems on the basis of the N-S and E-W spectra only, should be logically satisfied via the application of a criterium like formula (2), i.e.:

$$S_\beta = \sqrt{(\cos\beta \cdot S_1)^2 + (\sin\beta \cdot S_2)^2} \qquad (14)$$

Such a spectrum, which is obviously approximate, may be compared with the "exact" spectrum obtained as a section of the SRS. Such a comparison is performed in Fig.7, as an example, for β = 45°. One can see that the difference is, in the present case, rather large.

CONCLUSIONS

The SRS appears a tool rather simple to build up and to use; its use may allow the deduction of better results than those obtained via the formula (2) from the two separate N-S and E-W spectra and without doubt this is an interesting theoretical result.

In the field of applications the interest of the SRS for the possible definition of a (spatial) design tool is related to a characterization of the usual design response spectra in respect of the geographical directions. Such a detailed definition seems not yet to be praticable at the present time; only further studies and computations could possibly, in the future, give practical interest to the proposal.

Finally it can be observed that the same concept of SRS may be applied to the case of a vertical plane system for which the contemporaneous action of one horizontal component and the vertical component is considered.

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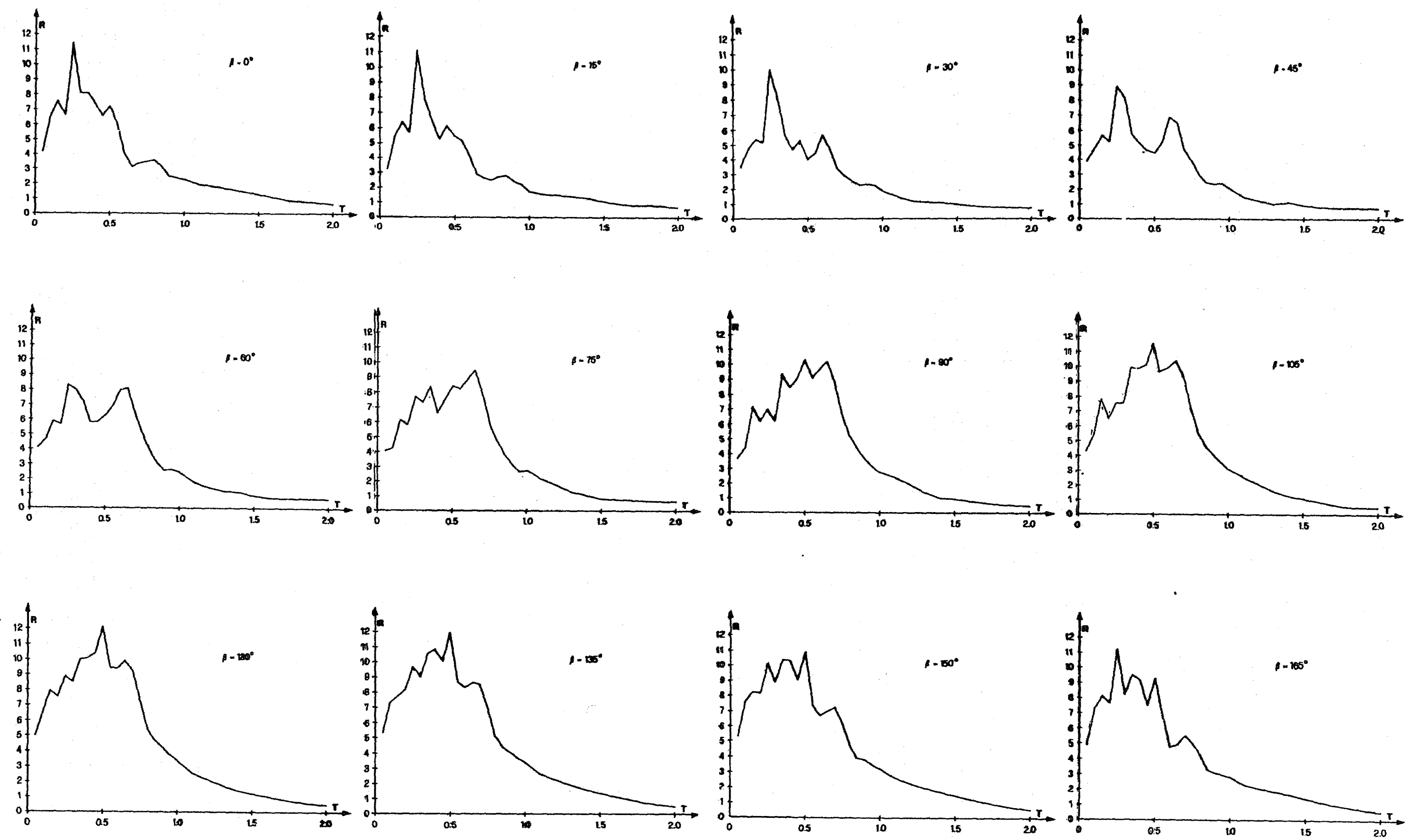


Fig. 5 – υ = 5%

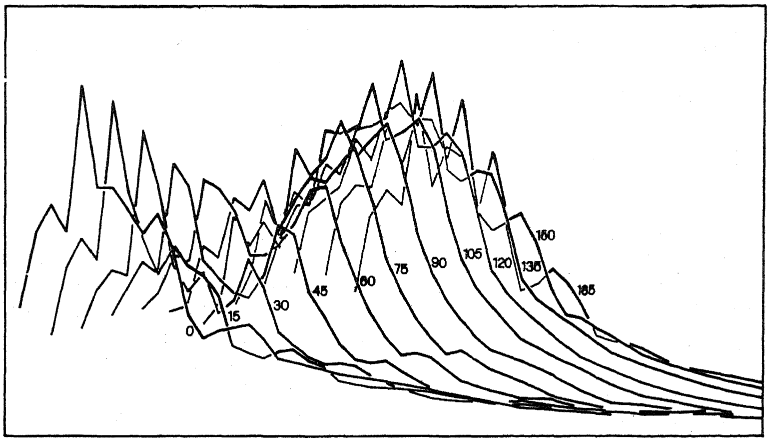
0
15
30
45
60
75
90
105
120
135
150
165

Fig. 6

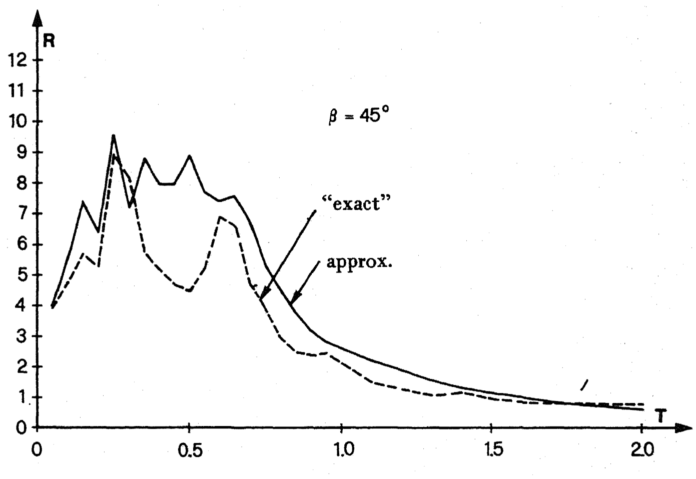
R
12
11
10
9
8
7
6
5
4
3
2
1
0
β = 45°
"exact"
approx.
T
0
0.5
1.0
1.5
2.0

Fig. 7