A DESIGNER'S DEFINITION OF AN OPEN-SECTION SHEAR WALL

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SUMMARY

The general differential equation of an open-section shear wall and its solution are presented which considers the action of flexural twists and bimoments. Different combinations of applied twists and bimoments and the resulting load effects which cause warping of the cross-section are developed. Such general solutions are evaluated for a wide range of k-values, which is found to be a good measure of warping. It has been shown that if certain k-values are obtained in preliminary design, the open-section shear wall can be treated as a thick-walled beam by applying classical formulas. The solutions are applied to a practical problem.

INTRODUCTION

Open-section shear walls are frequently used as lateral load resisting elements in tall structures subject to earthquake forces. Even though the analysis of planar shear walls is quite well-known, there is much ambiguity in the analysis of open-section shear walls, as that open section shear walls develop cross-sectional deformations and stresses which cannot be foreseen nor treated by classical methods of analysis.

The general approach to the analysis of open-section shear walls is to treat them as thin-walled beams. The term "thin-walled beam" then acquires an important meaning, since it does not only define the appearance of a beam, but also its behavior under load.

The designer wants to know an exact definition of a thin-walled shear wall, since the actual thickness of the walls does not decide wheather a shear wall will behave as a thin or thick-walled one. Theoretically, a thin-walled shear wall can be defined as any beam of which the principal sectorial moment of inertia is not equal to zero, but from the practical point of view this definition is too general. From an engineering point of view, when the bimoment and flexural twist diminish to an insignificant value in a short distance from the loaded point, then these forces can be treated as local, and the shear wall can be analyzed with classical formulas as a thick-walled beam. Therefore, it is of paramount importance to analyze open-section shear walls under most commonly occuring loading cases and study the "speed" with which the function of bimoment and flexural twist disappear. It is only then possible to say when an open-section shear wall can be treated as a thick-walled beam or when to go to a more refined analysis applying Vlassov's Theorems.

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GENERAL EQUATION OF OPEN-SECTION SHEAR WALLS

Consider an open section shear wall loaded by a twisting moment at its tip as shown in Fig. 1.a., the final distortion of which (Fig. 1.b) can be represented as the summation of distortions shown in Fig. 1.c. and Fig. 1.d. A twisting moment causing bending of flanges around their respective minor axes of symmetry is called a "Flexural Twist" T_w. A flexural twist causes a pair or pairs of bending moment which are called "Bimoments, B".

The relationship between external loading and internal forces, i.e. bimoment and flexural twist, and their distribution along the length of a beam are governed by a differential equation as derived below (1).

The relationship between the angle of twist and internal bimoment is as follows,

$$\frac{d^2\phi}{dz^2} = -\frac{B(z)}{E_1 I_w} \tag{1}$$

where ϕ = angle of twist of a beam over a distance z; z = longitudinal axis; B(z) = bimoment at z; I_w = principal sectorial moment of inertia; $E_1 = E/(1-\nu^2)$; ν = Poisson's ratio. The flexural twist caused by B(z) is obtained by the first derivative of B(z), as

$$T_w = B'(z) = -E_1 I_w \frac{d^3\phi}{dz^3} \tag{2}$$

The relationship between the angle of twist $\phi(z)$ and the external or internal St. Venant's twisting moment T_v is given by

$$\phi(z) = T_v(z)\frac{z}{GC} \tag{3}$$

where G = shear modulus of elasticity; and C = torsional constant.

The length dz of an open-section wall loaded by internal and external forces are shown in Fig. 2. From the condition of equilibrium, the following can be written.

$$d\,T_v + d\,T_w = dT = m\,dz \tag{4}$$

Using Eq. (2) and Eq. (3) in Eq. (4), the differential equation of bimoment and flexural twist as functions of twisting moment, and their distribution along the length of a shear wall is obtained.

$$\frac{d^4\phi}{dz^4} - k^2\frac{d^2\phi}{dz^2} = -\frac{1}{E_1 I_w}\frac{dT(z)}{dz} = -\frac{m}{E_1 I_w} \tag{5}$$

$$k^2 = \frac{GC}{E_1 I_w} \tag{6}$$

The solution of Eq. (5) for a straight shear wall having a constant cross-section is as follows (1).

$$\phi_{(z)} = \phi_o + \frac{d\phi_o}{dz}\,\frac{\sinh(kz)}{k} + \frac{B_o}{GC}\{1 - \cosh(kz)\}$$

$$+ \frac{T_o}{kGC}\{k_z - \sinh(kz)\} + \frac{1}{kGC}\int_o^z \{k(z-t)-\sinh(z-t)\}\, m_t dt \qquad (7)$$

where $m_t = dT_t/dt$; ϕ_o = angle of twist at $z = 0$; B_o = external bimoment at $z = 0$; T_o = external twisting moment at $z = 0$. The function m_t represents all external twisting moments acting on a beam over a distance, z. Differentiating Eq. (7) by dz twice and making use of Eq. (1), the expression for bimoment is obtained.

$$B_{(z)} = - \frac{d\phi_o}{dz}\,\frac{GC}{k}\sinh(kz) + B_o\cosh(kz) + \frac{T_o}{k}\sinh(kz)$$

$$+ \frac{1}{k}\int_o^z \sinh k(z-t) m_t\, dt \qquad (8)$$

The proper signs of all load effects mentioned in Eq. (8) are shown in Fig. 3.

<u>External Uniform Twist</u>

Boundary conditions : $\frac{d\phi_o}{dz} = 0$; $T_o = -mL$; $B(L) = 0$

$$B(z) = \frac{m}{k^2 \cosh kL}\{kL \sinh k(L-z) + \cosh kz - \cosh kL\} \qquad (9)$$

$$T_w(z) = \frac{d\,B(z)}{dz} = \frac{m}{k \cosh kL}\{-kL \cosh k(L-z) + \sinh kz\} \qquad (10)$$

The graphs of Eq. (9) and Eq. (10) are shown in Fig. 4 for different values of k.

<u>External Point Twist</u>

Boundary conditions : $T_o = -T$; $\frac{d\phi_o}{dz} = 0$; $B(L) = 0$

Point twist is defined as a uniformly distributed twisting moment over a distance Δc, or

$$T = m\,\Delta c \rightarrow m = T/\Delta c \qquad (11)$$

$$B(z) = \frac{T}{k}\left\{\frac{\sinh kL - \sinh k(L-z_t)}{\cos kL}\cosh kz - \sinh kz + \underbrace{\sinh k(z-z_t)}_{\text{for } z \geq z_t}\right\} \qquad (12)$$

$$T_w(z) = T\,\{\frac{\sinh kL - \sinh k(L-z_t)}{\cosh kL}\sinh kz - \cosh kz$$
$$+ \underbrace{\cosh k(z-z_t)\}}_{\text{for } z \geq z_t} \qquad (13)$$

When $z_t = L$

$$B(z) = \frac{T}{k}\,\{\frac{\sinh kL}{\cosh kL}\cosh kz - \sinh kz\} \qquad (14)$$

$$T_w(z) = T\,\{\frac{\sinh kL}{\cosh kL}\sinh kz - \cosh kz\} \qquad (15)$$

The graphs of Eq. (14) and Eq. (15) are shown in Fig. 5 for different values of k.

<u>External Point Bimoment</u>

Boundary conditions, $\frac{d\phi_o}{dz} = 0$; $T_o = 0$; $B(L) = 0$

In this case m_t in the integral part of Eq. (13) must represent an external bimoment acting at z_B. From definition, we can represent a bimoment as a bending or twisting moments.

$$B = T\Delta \rightarrow T = B/\Delta \qquad (16)$$

$$B(z) = B\{\frac{\cosh k(z_B-L)}{\cosh kL}\cosh kz - \cosh k(z_B-z)\} \qquad (17)$$

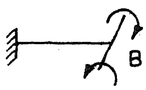
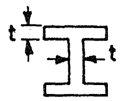
$$T_w(z) = B\,k\{\frac{\cosh(z_B-L)}{\cosh kL}\sinh kz + \underbrace{\sinh k(z_B-z)\}}_{\text{for } z \geq z_B} \qquad (18)$$

If Eq. (17) and Eq. (18) are evaluated and the effects superposed at increments from z = 0 to z = L, the case for a uniformly distributed bimoment is developed, Fig.6.

ILLUSTRATIVE PROBLEM

An I-shaped shear wall is subjected to a bimoment ($1t\text{-}m^2$) and twist (1t-m) applied at the tip. Similar internal load effects are obtained for both loading cases. For the case of external bimoment at the tip, the geometric properties and resulting flexural twist and internal bimoment at z/H = 0.5 are presented as a percentage of maximum values occuring at the tip. One of the cases is presented in graphical form in Fig.7.

TABLE 1

Loading	H(m)	h(m)	b(m)	t(m)	$k(m^{-1})$	% $T_w(0.5)$	% B(0.5)
	50	3.0	3.0	0.20	0.072	28	34
	50	4.0	3.0	0.20	0.056	23	24
	50	5.0	3.0	0.20	0.047	16	17
	50	5.0	3.0	0.20	0.047	28	34
	50	5.0	4.0	0.20	0.033	34	50
	50	5.0	5.0	0.20	0.026	37	63
t = uniform	30	5.0	3.0	0.20	0.047	28	34
wall	40	5.0	3.0	0.20	0.047	32	45
thickness	45	5.0	3.0	0.20	0.047	34	50

CONCLUSIONS

It can be observed from Figs. 4-6 that the value of k (see Eq. 6) is a good measure of warping effects of flexural twist and bimoment. When the value of k is greater or equal to 0.1, at midheight of the shear wall, the flexural twist and bimoment are about 10% of the value at the point of application.

For externally applied twists or bimoments, the internal bimoments and flexural twists created die away approximately at equal rates.

If a much faster speed of decay is desired from a design point of view, a much greater value of k must be obtained, such as 0.5. But, from the illustrative example, it is seen that such a high value of k is quite difficult to achieve in practice.

The height of the shear wall does not seem to be significant in the occurrence of the warping effects.

For preliminary design, constructing graphs, as shown in Fig. 4-6, will be helpful for the choice of cross-sectional shape and dimensions.

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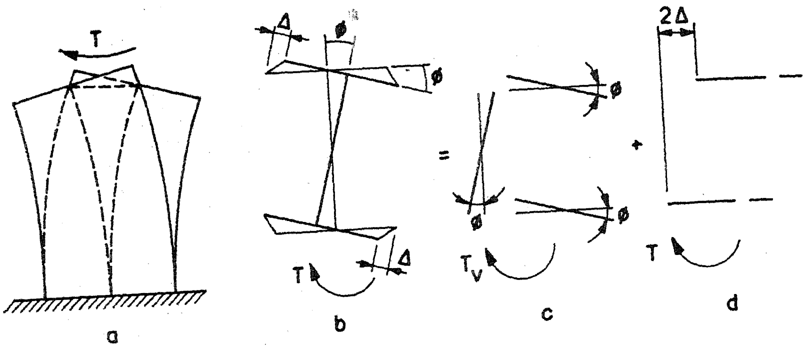

FIGURE 1

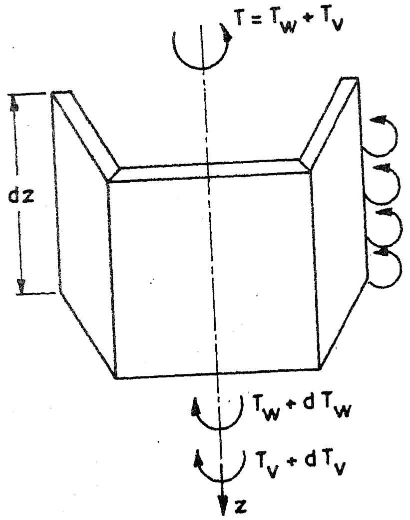

FIGURE 2

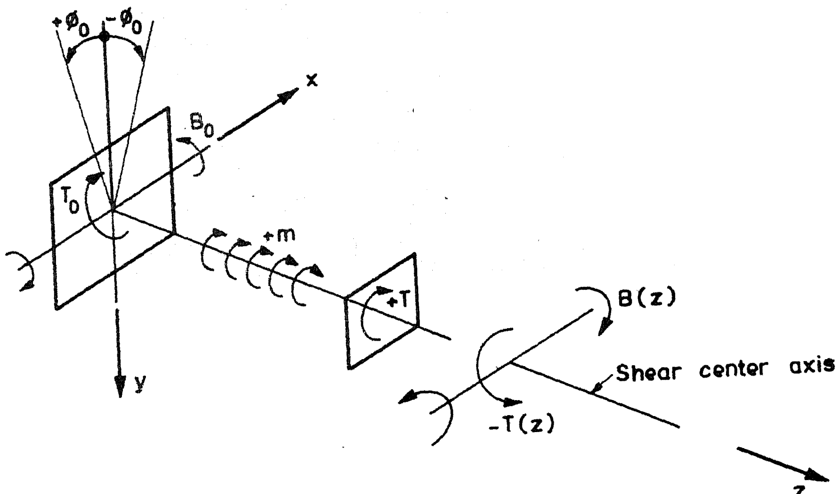

FIGURE 3

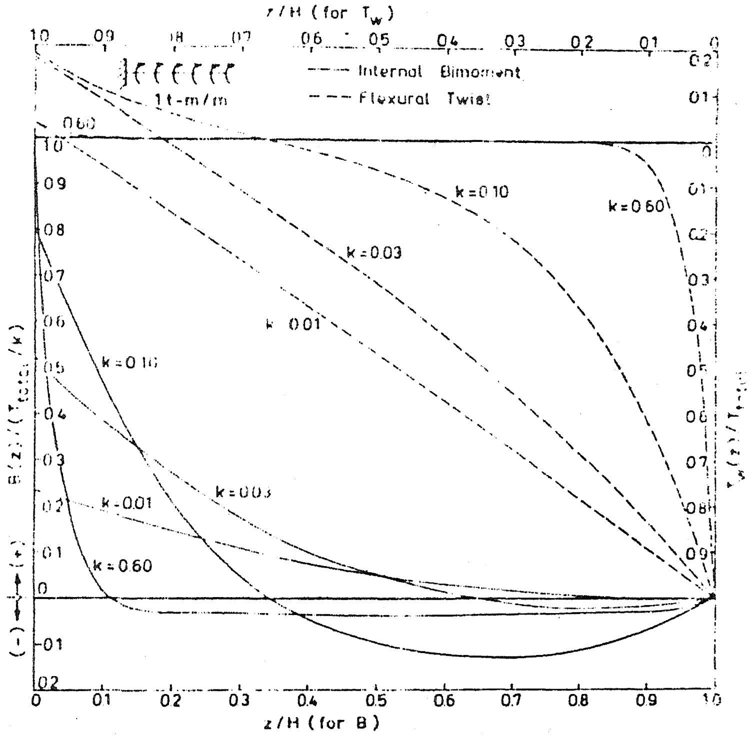

FIGURE 4

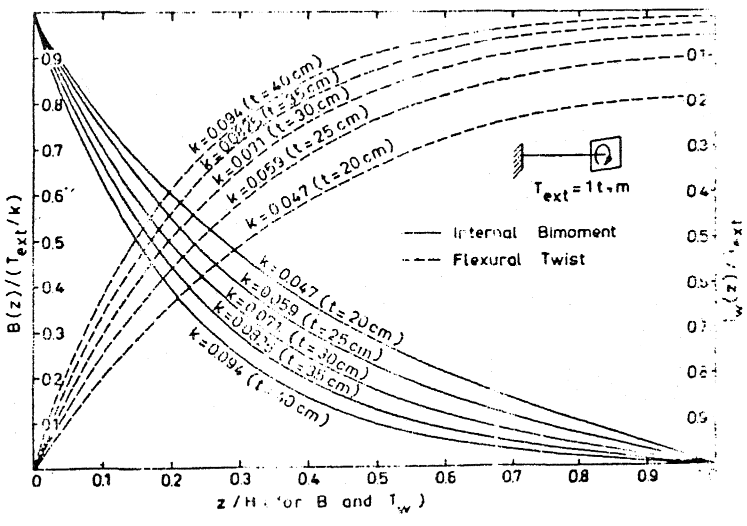

FIGURE 5

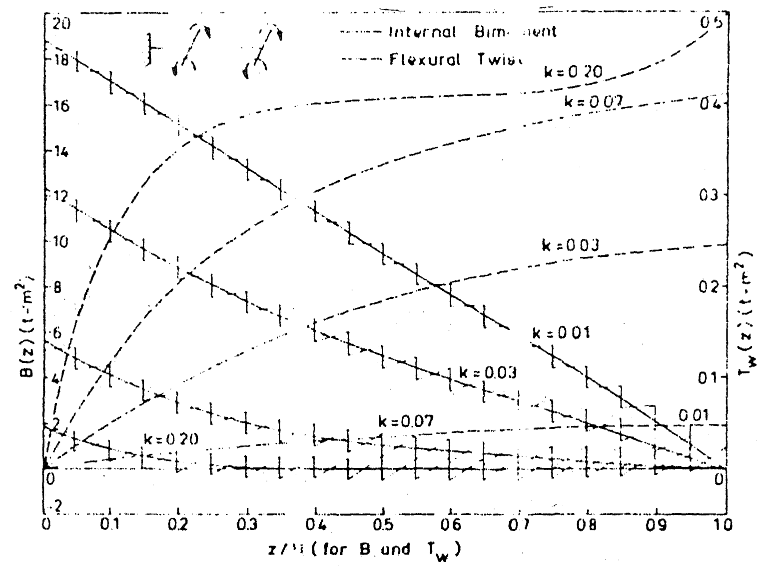

FIGURE 6

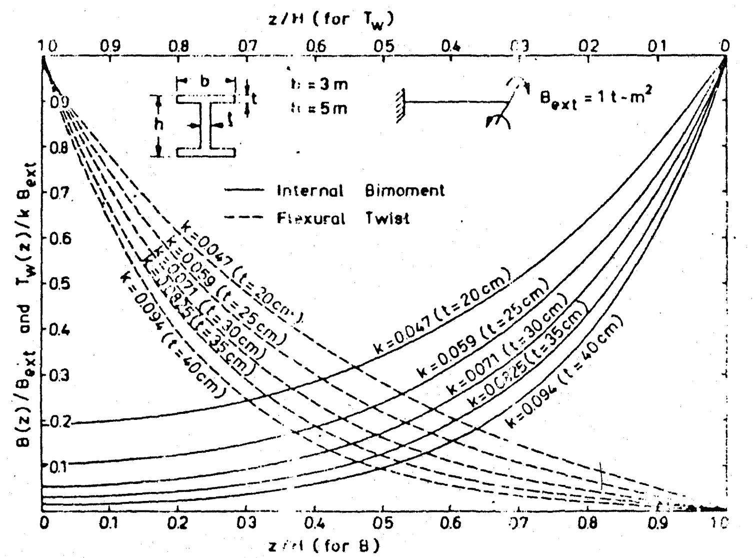

FIGURE 7