UNFAVORABLE SEISMIC DIRECTIONS IN EARTHQUAKE-RESISTANT DESIGN

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SUMMARY

The equation of motion for linear structural systems is derived for the general case of seismic vibrations and using this equation the maximum value of a specified effect and the corresponding seismic direction is determined. A numerical procedure is given for the calculation of the largest maximum value together with the corresponding instant and seismic direction. An approximate formula for the largest maximum value is also given and the conditions under which this formula is exact are indicated.

1- INTRODUCTION

In most codes for earthquake-resistant design it is recommended that earthquake effects be computed independently for two orthogonal directions and structural members be designed according to the maximum value of the internal forces or stresses thus obtained. This design value, however, is not the true maximum except in the case of structures symmetrical in two orthogonal directions, with lateral load-bearing members parallel to the axes of symmetry. In all other cases the design values are less than the true maxima and the overall safety factor for the structure is therefore reduced.

Hence, the determination of both the seismic direction corresponding to the true maximum of any specified internal force, stress or displacement and the magnitude of this maximum value appears as an important problem in seismic analysis of structures.

In this paper first the equation of motion for linear structural systems is derived for the general case and later, using these equations, the determination of both the maximum value of any effect such as an internal force, stress or displacement and the corresponding seismic direction at each instant is shown. The determination of the largest of the maximum values of any specified effect together with the corresponding instant and seismic direction is also explained.

An approximate formula for the calculation of the largest of the maximum values of any specified effect during the whole of the vibration period is derived in terms of the maximum components obtained for each seismic vibration component of the foundation. It is shown that the results obtained from this formula are always on the safe side. The conditions under which this formula becomes exact are also indicated.

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2- NOTATION AND DEFINITIONS

The symbols used in the equation of motion for the first-order seismic analysis of linear elastic structures are defined as follows :

$[d] = [D(t)]$: column matrix composed of the nodal displacement component functions $D_j(t)$ which define the displaced configuration of the system at any instant t.

$u_x(t)$, $u_y(t)$, $u_z(t)$: displacement functions of the foundation in the directions of orthogonal axes x,y,z during the ground motion.

$[U_x]$, $[U_y]$, $[U_z]$: column matrices composed of the rigid body displacement components of the system produced by $u_x = 1$, $u_y = 1$, $u_z = 1$ respectively, i.e. unit foundation displacements in the directions of orthogonal axes x,y,z.

$[\delta] = [\delta(t)]$: column matrix composed of the relative nodal displacement components $\delta_j(t)$ of the system with respect to the foundation. Through the above definitions,

$$[d] = [\delta] - \sum_i [U_i] . u_i \qquad (1)$$

$$(i = x,y,z)$$

$-[S]$: a square matrix where each k th column is composed of the elastic forces at the nodes produced by $\delta_k = 1$ and acting in the direction of nodal displacement components D_j, i.e. system stiffness matrix.

$-[M]$: a square matrix where each k th column is composed of the inertial forces at the nodes produced by $\ddot{D}_k = 1$ and acting in the direction of D_j , i.e. system mass matrix.

$-[C]$: a square matrix where each k th column is composed of the damping forces at the nodes produces by the unit relative velocity component $\dot{\delta}_k = 1$ and acting in the direction of D_j, i.e. system damping matrix.

In the above definitions the total number of dots on symbols denotes the corresponding derivate with respect to time.

In systems where the variations in the magnitude of axial forces are negligibly small during vibration, the second-order theory counterpart of the stiffness matrix $[S]$ may be used.

3- EQUATION OF MOTION

Through the above definitions the dynamic equilibrium of the system may be expressed as,

$$[M][\ddot{d}] = -[S][\delta]-[C][\dot{\delta}]$$

$$[S][\delta]+[C][\dot{\delta}]+[M][\ddot{d}] = 0 \qquad (2)$$

The terms in Eq. 2 denote the elastic forces, the damping forces and the inertial forces respectively.

Differentiating Eq. 1 twice with respect to time and substituting $[\ddot{d}]$ in Eq. 2 yields the equation of motion,

$$[S][\delta]+[C][\dot{\delta}]+[M][\ddot{\delta}] = -\sum_i [M][U_i].\ddot{u}_i \qquad (3)$$

$$(i = x,y,z)$$

4- DETERMINATION OF THE MAXIMUM OF A SPECIFIED EFFECT AND THE CORRESPONDING SEISMIC DIRECTION

4.1- The special case of identical displacement functions

In this section the special case of identical displacement functions i.e.,

$$u_x(t) \equiv u_y(t) \equiv u_z(t) \equiv u(t) \qquad (4)$$

shall be investigated.

The value of any internal force, stress or displacement is denoted by $F = F(t)$ and the values of $F(t)$ produced by seismic vibrations u_x, u_y, u_z in the directions of orthogonal axes x,y,z are denoted by $F_x(t)$, $F_y(t)$, $F_z(t)$ respectively. The acceleration components of the foundation produced by an earthquake of acceleration u and direction cosines λ_x, λ_y, λ_z are $\lambda_x.u$, $\lambda_y.u$, $\lambda_z.u$ respectively. Consequently, through Eq. 3 , the value of F is, by superposition,

$$F = F_x.\lambda_x - F_y.\lambda_y-F_z.\lambda_z \qquad (5)$$

Eq. 5 may be expressed as the scalar product of the two vectors $\vec{F}_1$ and $\vec{a}$,

$$F = \vec{F}_1.\vec{a} = |\vec{F}_1| \ . \ |\vec{a}| \ . \ \cos\gamma = |\vec{F}_1|.1.\cos\gamma \qquad (6)$$

where,

$\vec{F}_1 = \vec{F}_1(t)$: a vector with components F_x, F_y, F_z.

$\vec{a}$: unit vector in the earthquake direction defined by $\lambda_x,\lambda_y,\lambda_z$.

γ : angle between vectors $\vec{F}_1$ and $\vec{a}$.

From Eq. 6 it is clear that for F to be maximum at any instant t,

$$\cos\gamma = 1 \quad , \quad \gamma = 0 \tag{7}$$

i.e. the earthquake direction should coincide with that of F . From Eqs. 6 , 7 the maximum value of F at any instant t is,

$$\max.F(t) = |\vec{F}_1(t)| = \sqrt{\sum_i F_i^2(t)} \tag{8}$$

$$(i = x,y,z)$$

The largest of the max.F values which has to be considered in the design of the structure shall occur at a certain instant t_m . This value, denoted by max.max.F, is the value of max.F at the instant t_m, i.e.,

$$\max.\max.F = \max.F(t_m) = \sqrt{\sum_i F_i^2(t_m)} \tag{9}$$

$$(i = x,y,z)$$

Each of the F_x,F_y,F_z components of F(t) may be determined by integrating Eq. 3 seperately for seismic vibrations in each of the x,y,z directions. The component functions thus obtained are plotted schematically in Fig. 1 a,b,c . The maximum value of F(t) at any instant t may then be determined numerically through Eq. 8 . The max.F(t) corresponding to these component functions is shown in Fig.1 d . The max.max.F value produced during the whole of the vibration period is the maximum ordinate in Fig. 1 d.

The same procedure may similarly be applied to the case where rotational vibrations defined by

$$u_{xx}(t) \equiv u_{yy}(t) \equiv u_{zz}(t) \equiv u(t) \tag{10}$$

also exist about the orthogonal axes x,y,z. In this case each of the vectors $\vec{F}$ and $\vec{a}$ has 6 components instead of 3 . Column matrices $[U_{xx}]$, $[U_{yy}]$, $[U_{zz}]$ composed of the rigid body displacement components of the system produced by $u_{xx} = 1$, $u_{yy} = 1$, $u_{zz} = 1$,i.e. unit foundation rotations about the orthogonal

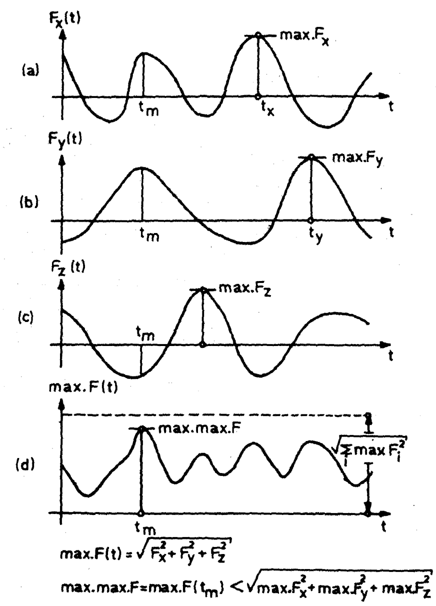

Fig.1 Vibration diagrams

axes x,y,z , are first calculated. Then the additional functions $F_{xx}(t)$, $F_{yy}(t)$, $F_{zz}(t)$ are obtained by integrating Eq. 3 seperately for the additional terms on the right hand side. These terms are $-[M].[U_{xx}].\ddot{u}$, $-[M].[U_{yy}].\ddot{u}$ and $-[M].[U_{zz}].\ddot{u}$ respectively. The max.F values and max.max.F are found from Eqs. 8 , 9 bearing in mind that (i=x,y,z,xx,yy,zz).

4.2- The general case

In this section the case of foundation vibrations which can be represented by the non-identical displacement functions,

$$u_x = a_x.\bar{u}_x \quad , \quad u_y = a_y.\bar{u}_y \quad , \quad u_z = a_z.\bar{u}_z$$
$$u_{xx} = a_{xx}.\bar{u}_{xx} \quad , \quad u_{yy} = a_{yy}.\bar{u}_{yy} \quad , \quad u_{zz} = a_{zz}.\bar{u}_{zz} \qquad (11)$$

shall be investigated. Here $\bar{u}_x$, $\bar{u}_y$, $\bar{u}_z$, $\bar{u}_{xx}$, $\bar{u}_{yy}$, $\bar{u}_{zz}$ represent the known seismic vibrations acting on the foundation. The first three functions denote vibrational displacements in the directions of the orthogonal axes x,y,z and the last three denote vibrations about the same axes. $a_x, a_y, a_z, a_{xx}, a_{yy}, a_{zz}$ are coefficients which define the magnitude of foundation displacements produced by $\bar{u}_i (i=x,y,\ldots,zz)$. It is seen that the earthquake effect is defined such that the magnitude $a=|\vec{a}|$ of the vector $\vec{a}$ whose components are $a_i (i=x,\ldots,zz)$, remains constant during the period of ground motion.

Each of the acceleration components u_i of the foundation produced by the earthquake thus defined will be,

$$\ddot{u}_i = a_i.\ddot{\bar{u}}_i \quad (i=x,y,z,xx,yy,zz) \qquad (12)$$

and for each acceleration component the right hand side of Eq. 3 will take the form,

$$-[M]\,[U_i]\,a_i.\ddot{\bar{u}}_i \qquad (13)$$

The value of any internal force, stress or displacement is denoted by F(t) and the values of F(t) produced by each of the seismic vibration functions $\bar{u}_i$ are denoted by $F_i = F_i(t)$. It is clear that each F_i may be determined by integrating Eq. 3 for each vibration function $\bar{u}_i$, i.e.,

$$[S]\,[\delta_i]+[C]\,[\dot{\delta}_i]+[M]\,[\ddot{\delta}_i] = -[M]\,[U_i]\ddot{\bar{u}}_i \rightarrow \delta_i = \delta_i(t) \qquad (3a)$$

$$\delta_i \rightarrow F_i = F_i(t) \qquad (i = x,y,z,xx,yy,zz)$$

The value of F produced by the earthquake defined in (11) is, by superposition,

$$F = \sum_i F_i.a_i \qquad (i = x,y,z,xx,yy,zz) \qquad (14)$$

Eq. 14 may be expressed as the scalar product of the vectors $\vec{F}_1$ and

$\vec{a}$, i.e.,

$$F = \vec{F}_1 \cdot \vec{a} \tag{15}$$

where $\vec{F}_1$ *and* $\vec{a}$ *are the vectors with components* F_i *and* a_i *respectively.*

From Eq. 15 it is seen that for F to be maximum at any instant t, the ratios between the components F_i and a_i have to be constant, i.e., the direction of $\vec{F}_1$ should coincide with that of $\vec{a}$. The maximum value of F at any instant t is, therefore,

$$\max.F(t) = |\vec{F}_1|.|\vec{a}| = a\sqrt{\sum_i F_i^2(t)} \tag{16}$$

where $a = |\vec{a}|$ is the coefficient which defines the magnitude of seismic vibrations. It is seen that the results obtained in Sections 4.1 and 4.2 are similar.

The largest of the max.F values which has to be considered in the design of the structure shall occur at a certain instant t_m . From Eq. 16 this value is,

$$\max.\max.F = \max.F(t_m) = a\sqrt{\sum_i F_i^2(t_m)} \tag{17}$$

$$(i = x,y,z,xx,yy,zz)$$

The numerical procedure described in Section 4.1 may similarly be applied for the calculation of max.max.F given by Eq. 17 .

4.3. Approximate formula for the largest of the maximum values

In design calculations it is mostly the $\max.F_i = F_i(t_i)$ values that are computed instead of the functions $F_i(t)$. Since the instant t_i at which $\max.F_i$ occurs is different for each vibrational displacement component $u_i = a_i.\bar{u}_i$, then, for all t_i,

$$\max.\max.F < a\sqrt{\sum_i \max.F_i^2(t_i)} \tag{18}$$

$$a = |\vec{a}| \quad , \quad (i = x,y,z,xx,yy,zz)$$

For example, the value of the expression in the right hand side of inequality (18) corresponding to the component functions in Fig.1 a,b,c is indicated by the dashed horizontal line in Fig.1 d.

According to the above inequality the largest of the maximum values to be considered in the design of the structure may safely be approximated as,

$$\max.\max.F = a\sqrt{\sum_i \max.F_i^2} \tag{19}$$

The approximate formula (19) becomes exact only when all of the calculated $\max.F_i(t_i)$ values occur at the same instant t_r ,i.e.,

$$t_i = t_r = \text{constant} \quad (i = x,y,z,xx,yy,zz) \tag{20}$$

Condition (20) is truly satisfied and formula (19) becomes exact if the functions $F_i(t)$ are similar with respect to t since in this case the ratios among the ordinates of the functions $F_i(t)$ are independent of t and all the $\max.F_i$ values occur at the same instant.

In most practical applications only the horizontal vibration components u_x, u_y of the foundation are taken into account. In these cases the largest of the maximum values may safely be approximated as,

$$\max.\max.F = a\sqrt{\max.F_x^2 + \max.F_y^2} \qquad (19a)$$

$$a = \sqrt{a_x^2 + a_y^2}$$

For structures in which the dynamic characteristics in each of the x and y directions are equal, the functions $F_x(t)$ and $F_y(t)$ will be proportional, leading to the equality,

$$\max.\max.F = a\sqrt{\max.F_x^2 + \max.F_y^2} \qquad (19b)$$

It is evident that for structures in which the dynamic characteristics in the x,y directions are close to each other, the formula (19a) will give a good and safe approximation for max.max.F.

5- EXAMPLES

5.1- Example 1

The structure shown in Fig.2 is symmetrical in the x,y directions and has shear walls which are non-parallel to x and y.

If this structure is subjected to identical foundation vibrations $u_x(t) \equiv u_y(t) \equiv u(t)$ and is designed such that the normal modes, the natural frequencies and the damping ratios in both x and y directions are equal to each other then the functions $F_x(t)$ and $F_y(t)$ which correspond to any internal force F in one of the shear walls are proportional. In this case the equality 19b is valid and max.max.F may be obtained by substituting the $\max.F_x$ and $\max.F_y$ values computed through spectral analysis into (19b), bearing in mind that $|\vec{a}| = 1$. If the first mode alone is considered in the design it is sufficient that only the natural frequencies and the damping ratios corresponding to the first mode in the x and y directions be equal for the formula (19b) to give the exact

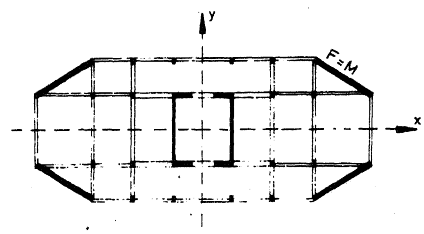

Fig.2 Floor plan of Example 1

value of max.max.F. For the special case of $max.F_x = max.F_y$ the value of max.max.F will be,

$$max.max.F = \sqrt{2}.max.F_x$$

5.2- Example 2

The structure shown in Fig. 3 has identical frames in both x and y directions and the cross sections of all columns are square. It is again assumed that $u_x(t) \equiv u_y(t) \equiv u(t)$.

Since the $F_x(t)$, $F_y(t)$ functions for the corner stress at "a" in column 4-4 are identical because of symmetry, the formula (19b) is valid. The maximum values $max.F_x$, $max.F_y$ computed through spectral analysis are equal and therefore,

$$max.max.F = \sqrt{2}.max.F_x$$

This example shows that in earthquake-resistant design the sole consideration of the larger of the $max.F_x$ and $max.F_y$ values for the design of sections may, in certain cases, lead to quite unsafe results.

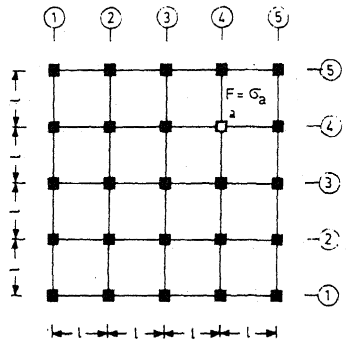

Fig.3 Floor plan of Example 2