PEAK DUCTILITY DEMANDS IN SIMPLE TORSIONALLY UNBALANCED BUILDING MODELS SUBJECTED TO EARTHQUAKE EXCITATION

by H. M. Irvine[I] and G. E. Kountouris[II]

SUMMARY

An investigation of the inelastic seismic response of a simple torsionally unbalanced building is reported. The study concerns a two-degree-of-freedom model in which two identical frames support a diaphragm, the center of mass of which may be offset from the center of stiffness. A comprehensive parameter study was undertaken to identify trends in the peak ductility demand of the worst situated frame, and a regression analysis of the data performed to provide simple confidence levels for these peak ductility demands. Surprisingly, eccentricity does not appear to be a particularly significant parameter.

INTRODUCTION

There have been numerous investigations reported of the elastic seismic response of torsionally unbalanced structures, but little work has been done on their inelastic response, at least insofar as systematic studies are concerned. Torsional provisions in current codes of practice are based almost exclusively on the findings of elastic studies and while these are undoubtedly helpful, there must always remain the question of the influence of inelastic behavior. Although the present study is not completely comprehensive, it does allow some conclusions that seem important to be tentatively drawn.

Necessarily, the present paper is but an abstract of the actual investigation. A full account may be found in the report from which it has been drawn (1). Other recent reports that are germane to this investigation include those by Kan and Chopra (2) and by Batts, Berg & Hanson (3).

THE MODEL

The model consists of two identical single-bay frames parallel to each other supporting a rigid diaphragm of given mass and radius of gyration. The frames are assumed to be massless and to exhibit elastic-plastic behavior. The diaphragm's center of mass is offset from the center of stiffness, so that there is an eccentricity and the model is torsionally unbalanced. For earthquake excitation which coincides with the plane of the frames, both translation and rotation will be present in the response—there are thus two degrees of freedom. The symmetric model, in which there is no eccentricity, is a useful reference case.

If the mass of the diaphragm is M (with radius of gyration r) and if $K_v/2$ is the elastic stiffness of one frame (which lies at a distance, ℓ,

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[I] Edgerton Associate Professor of Civil Engineering, Massachusetts Institute of Technology, Cambridge, Massachusetts, 02139, U.S.A.

[II] Research Assistant, Massachusetts Institute of Technology.

from the other), then the "translational frequency" is $(K_v/M)^{1/2}$ and the ratio of the "torsional frequency" to this translational frequency is $[\{K_v(\ell/2)^2/Mr^2\} / K_v/M]^{1/2} = \ell/2r$. Only when the eccentricity e is zero are these frequencies the natural frequencies of the system. If, in addition, the spectral acceleration for a given earthquake at a given "translational period" $(=(M/K_v)^{1/2})$ is S_a, the ratio of the "base shear" to the yield force level of the frames is MS_a/F_y.

The parameters considered independent in this study were:

(i) the "translational period" $(M/K_v)^{1/2}$;
(ii) the "frequency" ratio $\ell/2r$;
(iii) the eccentricity ratio e/ℓ ;
and (iv) the "force" ratio MS_a/F_y .

The information sought was the peak ductility demand μ in the worst situated frame, which was usually that closest to the center of mass.

ANALYSIS PROCEDURE

The computer program STAVROS (incorporating the subroutine BILIN for the bilinear force-deflection behavior of the frames) was used to solve the equations of motion in step-by-step fashion. Four different earthquake records were used, together with four values of the translational period (from 0.5 seconds to 2.0 seconds), six values of the frequency ratio (from 0.5 to 1.75), six values of the eccentricity ratio (from zero to 0.25) and six values of the force ratio (from 2 to 12).

These values were considered to cover a representative and reasonably realistic range of possibilities. Thus, in all, some 3500 cases were considered, which led to more than 100 plots of peak ductility demand as functions of the independent parameters (see reference (1)).

RESULTS AND CONCLUSIONS

By far the most noticeable trend, and one which characterized all the results, was that μ was roughly proportional to MS_a/F_y, the constant of proportionality being not much different from unity. This trend is, of course, well known for symmetric structures that lie in the same period range. Dependence on $\ell/2r$ was not appreciable for smaller values, although there was some scatter in the results when $\ell/2r$ was larger (this scatter could perhaps be associated with buildings with a peripheral frame).

By and large, peak ductility demands were more severe for the frame closest to the center of mass. However, this was not always the case and, as has been predicted in another elastic study (4), more severe demands can be placed on the other frame when $\ell/2r$ is small (such as in a building with an interior core). A simple frequency domain analysis was performed to show that this situation can arise when

$$\frac{\omega}{(K_v/M)^{1/2}} > \frac{\ell}{2r} \quad ,$$

where ω is the frequency of excitation. The present inelastic model exhibited this behavior when the above inequality was approximately met if, in the Fourier amplitude spectrum of the specific earthquake, there was sufficient power below this characteristic frequency.

However, the most interesting conclusion that may be drawn is that there did not appear to be a strong correlation between peak ductility demand and eccentricity. Provided that $e/\ell \lesseqgtr 0.25$, which covers most cases of practical interest, the differences in μ between eccentric and symmetric structures remained small, at least for moderate values of MS_a/F_y. In this range $\ell/2r$ did not significantly affect the results either.

Because this finding is considered to be of interest to those charged with the responsibility of drafting codes of practice for seismic loading, and because such activity in relation to torsional effects is quite widespread at present, a regression analysis was run on the data. The eccentricity ratio was never greater than 0.25 and $MS_a/F_y < 6$, because values of μ much great than 10, say, are difficult to generate in normal structural elements. For these data 50% were found to lie below

$$\mu_{50} \simeq -0.2 + 1.2\ MS_a/F_y \quad ,$$

while 90% lay below

$$\mu_{90} \simeq 2.1 + 1.2\ MS_a/F_y \quad .$$

In the case of symmetric structures the results were

$$\mu_{50} \simeq 0.1 + 0.9\ MS_a/F_y \quad ,$$

and

$$\mu_{90} \simeq 1.7 + 0.9\ MS_a/F_y \quad .$$

As an example, suppose that the design spectral acceleration is $S_{ad} = S_a/4$, and that, with a load factor of 1.5 and a stress reduction factor of 1.33, $F_y = 1.5\ MS_{ad}/1.33 = 5\ MS_a/16$. Then, for eccentric structures $\mu_{90} \simeq 6$, while for symmetric structures $\mu_{90} \simeq 4.6$, which is some 30% less. In fact, overall, the results of this study suggest that the peak ductility demand in the worst "loaded" frame is rarely more than 30% greater than the peak ductility demand in similar symmetric structures. Another finding, of some intuitive appeal, was that the mean ductility demand of both frames was not significantly different from that of similar symmetric structures.

We may close by reiterating that the present model derives its eccentricity from variations in the location of the center of mass of the diaphragm and not, as is more common, by variations in the stiffness of the frames. Strictly speaking, therefore, the present results apply to that case and to no other. Nevertheless, the conclusions drawn can probably be applied more broadly, but the extent to which they apply to multistory structures is obviously a matter for individual judgement. It is, however, unlikely that sufficiently comprehensive parameter studies can be done to resolve that question and, as in the past, reliance will have to be placed on extrapolation of the results from simplified building models.

ACKNOWLEDGMENT

This work was supported by the National Science Foundation under Grant ENV-77-14174.

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