ENGINEERING APPROACH TO MODELING OF PILED SYSTEMS

by

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SUMMARY

Available methods of analysis of piled systems subjected to dynamic excitation invade areas of mathematics usually beyond the reach of a practising engineer.

A simple technique that avoids that conflict is proposed, at least for preliminary studies, and its application, compared with other methods, is shown to be satisfactory.

A corrective factor for parameters currently used to represent transmitting boundaries is derived for a finite strip that models an infinite layer. The influence of internal damping on the dynamic stiffness of the layer and on radiation damping is analysed.

INTRODUCTION

The importance of soil-structure interaction (SSI) increases with the rigidity and degree of embedment of the structures. Such high rigidity and embedment are present in most nuclear installations explaining, together with greater recent demand for safety, the large number of publications that has been devoted to the identification and study of parameters that characterize SSI (e.g. ASCE Specialty and SMIRT Conferences).

The information available on the response of pile supported systems is, however, scarcer. Techniques used nowadays (1980) follow essentially results obtained by Novak et al [1 to 5] and there still is place for further investigations of engineering interest.

The purpose of this note is twofold: (i) introduction of important concepts, frequently masked by mathematical calculations and (ii) presentation of a related simple model for preliminary studies of the response of structures founded on piles.

Mass and damping coefficients are generated, for the low frequency region, under Winkler's type of assumptions and pile group effects are accounted essentially in accordance with Poulos [6] results.

Loading may result from ground motion or from unbalanced machine operation and the analysis of an actual system has to be made accordingly, with kinematic interaction requiring greater attention for low frequency excitation [7] on embedded structures. This subject, however, exceeds the scope of the present study.

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ONE - DIMENSIONAL MODEL

The representation of an infinite medium by a finite model requires accrued care when dynamic behavior is to be considered, especially to avoid the reflection of waves at the artificially created boundaries. This pitfal may be circumvented through the creation of energy transmitting boundaries, a procedure proposed and developed in the relatively recent past [e.g. 8]

Consider the governing equation for compressive waves propagating along a semi-infinite bar of specific mass ρ and wave speed $c = \sqrt{E/\rho}$ (Fig. 1-a)):

$$\ddot{u} - c^2 u'' = 0 \qquad , x > 0 \tag{1}$$

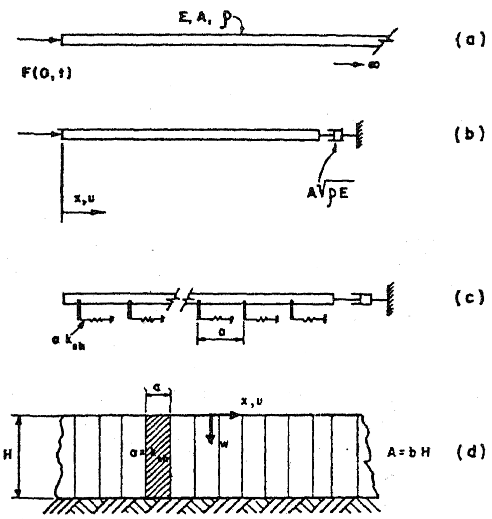

FIG. 1 - SIMPLIFIED MODELS WITH TRANSMITTING BOUNDARIES

Recalling d'Alembert's solution for outgoing waves, $u = f(x-ct)$, and the characteristic relation $\dot{u} = -c\,u'$, the stress coefficient $\sigma_{xx} = E\,u'$ can be expressed by

$$\sigma_{xx} = \rho\ c\ \dot{u} = -\sqrt{\rho E}\ \dot{u} \tag{2}$$

If a finite bar of width b is selected to model the semi-infinite bar, Eq. (2) is respected by adding a dashpot with constant $\eta = \sqrt{\rho E}$ at the artificial boundary, Fig. 1.b).

The equivalent damper of constant $c_d = A\eta$ ($A = bH$ = area of cross--section of bar) absorbs as much energy per cycle as would be required to produce the elastic motion of a wave length.

The finite bar represents a soil layer by including the shear-beam stiffness [4], a purpose achieved by inserting a distributed axial stiffness k_{sh}/u. length, Fig. 1,c). Eq. (1) changes by addition of the term $\omega_s^2 u = (k_{sh}/\rho A)\, u$ to its LHS. A tentative solution of the equation

$$u = \mathrm{Re}\left[u_o\, e^{i(\omega t - \nu x)}\right] \tag{3}$$

is verified if the wave number ν satisfies

$$\omega^2 = c^2 \nu^2 + \omega_s^2 \tag{4}$$

In this case, $\sigma_{xx} = E u' = -(E\nu/\omega)\, \dot{u}$ leads to

$$\eta = \sqrt{\rho E}\ \sqrt{\bar{\Omega}^2 - 1} \tag{5}$$

with $\bar{\Omega} = \omega/\omega_s$. Below the cut-off frequency ω_s it is $\bar{\Omega} < 1$, $\bar{\eta}$ becomes imaginary and, in particular, for $\omega = 0$ the static rigidity $k_{st} = \sqrt{EA\, k_{sh}}$ can be recovered. Eq. (5) shows that no travelling waves exist for $\omega < \omega_s$ and no radiation damping is to be considered.

For low frequencies ($\omega < \omega_s$) the large wave length engages the shear stiffness of the cantilevered soil columns and the static stiffness is approached when $\omega \to 0$. Novak's aforementioned work is valid primarily for the higher (ω/ω_s) region, where the shear beam resistance of the soil is not "excited", while he adjusts his results to avoid large errors for low frequencies.

The insertion of a damping term $c_d\, \dot{u}$ permits the analysis of the effects of internal, hysteretic type of damping. The complex stiffness can then be divided into a real part $k = \bar{k}\sqrt{EAk_{sh}}$ and an imaginary part $D = \bar{\eta}\, A\sqrt{E\rho}$, that define stiffness and damping, as follows

$$\bar{k} = \left[\left(1+\bar{\Omega}^{-4} + (4\xi^2-2)\,\bar{\Omega}^{-2}\right)^{1/2} - 1 + \bar{\Omega}^{-2}\right]^{1/2} \bar{\Omega} \,/\sqrt{2} \tag{7.a}$$

$$\bar{\Omega} = \left[\left(1+\bar{\Omega}^{-4} + (4\xi^2-2)\,\bar{\Omega}^{-2}\right)^{1/2} + 1 - \bar{\Omega}^{-2}\right]^{1/2} /\sqrt{2} \tag{7.b}$$

where ξ is the fraction of critical damping ($2\,\xi\,\omega_s\,\rho = c_d$).

The variation of $\bar{k}$ and $\bar{\eta}$ with $\bar{\Omega}$ is illustrated in Fig. 3 for ξ = 10%, showing no frequency cut-off as predicted by Eq. (7) for $\xi \neq 0$.

If a soil layer is modeled by successive soil columns, discretized at nodal points spaced apart a, and if the points are interconnected by compressive springs k_c and linked to the bedrock by shear springs of stiffness $k_t = k_{sh} a$, the difference equilibrium equation at point n can be written as

$$\left[(k_t/k_c) - (\omega/\omega_c)^2 + 2\right] u_n - u_{n-1} - u_{n+1} = 0 \tag{6}$$

for harmonic excitation e^{iwt}, with $\omega_c^2 = k_c/(\rho A)$ and u_j the displacement of point j. A solution of the type $u_j = A\, e^{i(wt-\theta j)}$ requires the satisfaction of the constraint $\cos\theta = 0.5\,\alpha$, where α is the coefficient of u_n in Eq. (6).

Working on the constraint equation it is found $\omega = (\omega_s^2 + 4\omega_c^2 \sin \theta/2)^{1/2}$, an expression that confirms the low frequency cut-off $\omega = \omega_s$ and shows the high frequency cut-off introduced by the discretization $\omega_{max} = (\omega_s^2 + 4\omega_c^2)^{1/2}$.

In Novak's theoretical development, the layers are considered independent with shear deformation neglected at interface and, as a consequence, the resonance of ω with the soil natural frequencies does not appear.

Piled Layers - Satisfactory results have been obtained by inserting radiation damping devices, as described below, into the model of a pile vibrating in a soil layer.

For vertical radiation damping it is suggested the adoption of coefficient $\mu = 2\pi R.\sqrt{\rho g}$, which corresponds to a shear wave type of dissipation associated to the vertical vibration of the pile. For lateral vibration it is suggested $\mu = 2R\eta$ as the result of adding a contribution due to a compressive-like wave ($\mu_c = 2R\sqrt{\rho E}$) to a shear-like wave term ($\mu_s = 2R\sqrt{\rho G}$). Assuming symmetry, the final, total value for damping/u. length is $\mu = 2 \times 2R\ (\sqrt{E\rho} + \sqrt{G\rho})$ i.e. $\mu = 4R\sqrt{\rho G}\ \sqrt{(3-4\nu)/(1-2\nu)}$ where ν is the Poisson's ratio of the soil.

The proposed treatment of radiation damping can be improved if parametric studies are undertaken to generate corrective factors for η_c and η_s. It is believed that a counterpart to the pioneering work of Lysmer et al [9,10] for the half-space can be generated.

Group Effects - For static loading Poulos obtained factors that measure the effect that a pile r has on displacement (ij) of another pile m, permitting the correction of a preassembled global flexibility matrix.

The dependence of group effects on exciting frequencies is not well known, with few published results for low frequencies [11]. The authors feel that a relationship between the spacing of the piles and the shear wave length is worth investigating.

FORMULATION AND SOLUTION

The condensed stiffness matrix for a finite beam on elastic foundation depends on $\beta L = (k_h bL^4/3E_pI_p)^{1/4}$ where k_h = subgrade modulus and b = width of beam. The value of k_h can be expressed conveniently in terms of soil constants; a comparison of obtained with published results indicates that ($2R\ k_h$) varies approximately between $0.7\ E_s$ and $1.5\ E_s$ where E_s = Young's modulus of soil and R = radius of pile. Interpolation functions g_i, based on the static coordinate functions (βL = constant) permit the calculation of the mass and damping coefficient $m_{ij} = \rho\ A\ d_{ij}$, $c_{ij} = \mu\ d_{ij}$ with $d_{ij} = \int_L g_i\ g_j\ dx$, condensed at the top of the pile.

Fixed or end bearing piles (with zero displacements at the bottom) and friction piles (with zero force resultants) are considered.

Fig. 4 displays dimensionless quantities $\overline{m}_{ij} = \beta^{i+j-1}\ d_{ij}$ that lead to the mass or damping coefficients for each value of (βL).

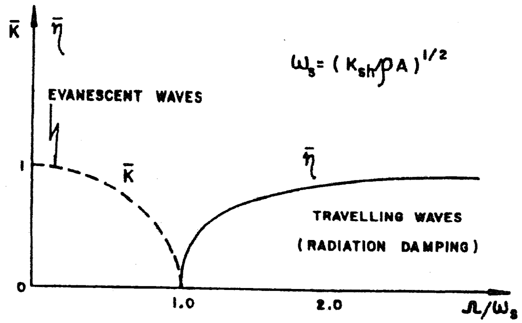

FIG. 2 - BEHAVIOR OF COMPLEX STIFFNESS FOR SIMPLE DISPERSIVE SYSTEMS ($\bar{\eta} = D/\sqrt{E\rho}$, $\bar{K} = K/\sqrt{EK_{sh}}\,A$, $K = \sqrt{1-(\Omega/\omega_s)^2}$)

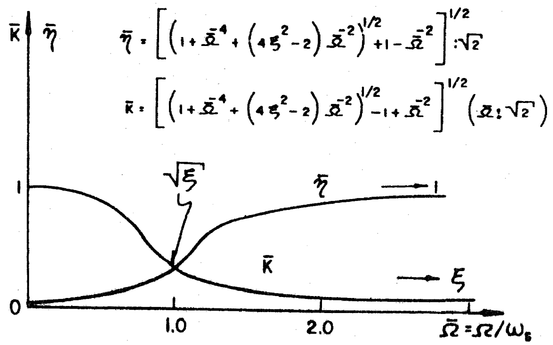

FIG. 3 - STIFFNESS $\bar{K}$ AND RADIATION DAMPING $\bar{\eta}$ WHEN MATERIAL DAMPING $\xi = 10\%\ \xi_c$

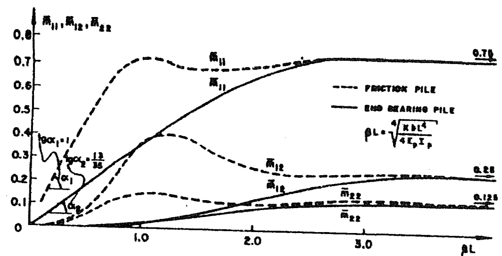

FIG. 4 - INFLUENCE OF PARAMETER βL ON GENERALIZED MASS COEFFICIENTS

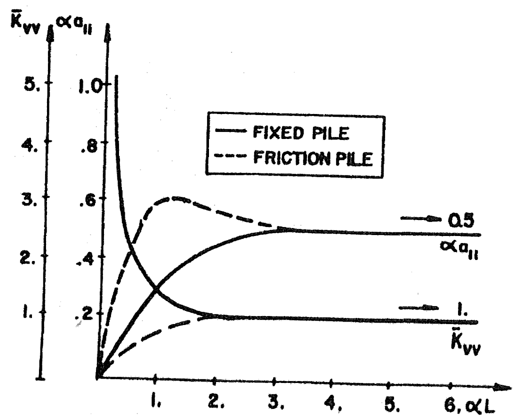

FIG. 5 – STIFFNESS AND DAMPING COEFFICENTS ($\bar{K}_{vv} = K_{vv}/EA\alpha$; $\alpha = \sqrt{K_s/EA}$)

For vertical oscillations, assuming that a uniform distribution of soil resistance k_s with depth is developed along the external surface of the pile, two families of curves can analogously be generated for friction ($u'(L) = 0$) and fixed ($u(L) = 0$) piles. The corresponding condensed stiffness and damping terms a_{11} are shown in Fig. 5. A value of $k_s = 2.5\ G_s$ is reccomended for evaluating these parameters.

COMPARISON WITH NOVAK's PARAMETERS

Parameters S_{u1}, S_{u2} that introduce the Winkler spring and dashpot constants of Novak's theory are shown in [1] and [2]. In the moderate to high frequency range S_{u1} is little dependent on Poisson's ratio and can be estimated as $S_{u1} \approx 4$ (Fig. 2 of [2]), which in turn agrees with usual practice of selecting $k_h d = 4G$ for horizontal stiffness of pile. Su_2 varies linearly with ($GR\ V_s$), V_s = shear velocity, with a slope that depends in ν, like our proposed parameter $4\ \sqrt{(3-4\nu)/(1-2\nu)}$ which, for $\nu = 0.25$, 0.40 leads to values (8.0, 10.5) that practically coincide with the slope of S_{u2} on Fig. 2 of [2]. The value suggested for vertical damping, $2\ \pi\ R\ \sqrt{\rho G}$, also reproduces results found by the much more elaborate theory of Novak.

Example - A tapered tower of 150 m height, founded on 8 piles $\emptyset = 1.0$ m that cross a layer of 36 m to reach fixidity, was studied by different techniques and pile group effects were neglected for the comparison. The piles are headed by a slab of diameter 15.0 m and depth 3.0 m, the layer has $\rho\ g = 20$ KN/m3 and $\nu = 0.4$ and the loading consists on harmonic ground motion of unit acceleration. The units used throughout the calculations are KN, m and second.

It is $k_h\ D \cong E = 2\ G\ (1+\nu) = 4200$, $\beta = 0.29$, $\beta L = 10.44$, $\bar{m}_{11} = 0.75$, $\bar{m}_{12} = 0.25$, $\bar{m}_{22} = 0.125$, $d_{11} = \bar{m}_{11}/ = 2.59$, $d_{12} = 2.97$, $d_{22} = 5.12$.

Frequency Independent - $\mu = 4R\sqrt{3-4\nu}/\sqrt{1-2\nu} = 925.3$, $c_{11} = 2396$, $c_{12} = 2748$, $c_{22} = 4737$. $\alpha = \sqrt{k_s/EA} \cong \sqrt{2.5G/EA} = 0.04$, $\alpha L = 1.44$, $k_{vv} = 105000$, $a_{11} = 0.38$, $c_{vv} = 2\ \pi\ R\sqrt{G\rho}.a_{11} = 5219$. At an earlier stage, $k_{vv} = EA/L = 65400$ (as if $\alpha = 0$), $c_{vv} = 2\ \pi\ R\ \ \rho G\ L/3 = 6592$. Stiffness coefficients from BEF, $k_{11} = 144000$, $k_{12} = 249000$, $k_{22} = 861000$. The response, now, depends solely on the exciting frequency, with above parameters being constant.

Modified Novak's Theory - Utilizing PILAY (a program developed by Novak's group that corrects theoretical results, at low frequencies) the damping and stiffness coefficients are obtained for each frequency. For instance, for $\omega = 8.58$ rad/s, $c_{11} = 2590$, $c_{12} = 2650$, $c_{22} = 4140$, $c_{33} = 7890$, $k_{11} = 179000$, $k_{12} = 280000$, $k_{22} = 908000$, $k_{33} = 963000 \cong k_{vv}$.

Frequency Dependent - Parameters related to horizontal forces are modified by factors read, for each frequency, from Fig. 3. The soil resonant frequency is $\omega_s = 2\ \pi\ V_s/(4H) = 3.74$ rad/sec and the factors are, then, obtained for $\omega/3.74$.

The matrices are set up according to normal procedures [2], inserting thus the contribution due to the eight piles.

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