ANALYSIS OF PILES IN SAND AGAINST EARTHQUAKES

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SUMMARY

A method of analysis of piles against earthquakes, based upon Winkler's springs, has been proposed. Solutions have been obtained for (a) natural frequencies, (b) deflections, (c) rotations, (d) bending moments, (e) shears, and (f) soil reactions along the entire length of the pile. Typical data on piles has been analysed to evaluate the applicability of this analysis and an extremely good tally was obtained.

INTRODUCTION

During an earthquake, a soil-pile system is subjected to a complex set of loads resulting from erratic ground motion. The response of soil-pile system depends upon (a) the nature and type of soil, (b) the restraints offered by the superstructure to the pile top, (c) the loads transferred to the pile from the superstructure and (d) the nature of ground motion.

There are, in general, three techniques to solve problems of soil-pile superstructure interaction (Novak, 1977); (a) to consider soil as a continuum with linear elastic properties, (Novak and Nogami, 1977) (b) the finite element technique, (Kuhlmeyer, 1979), and (c) to represent the soil-pile system by a set of discrete (lumped) masses, springs, and dash pots (Penzien, et al., 1964; Prakash, 1981, Prakash and Chandrasekaran, 1973, 1977, 1980). A simple but reasonably practical solution for soil-pile interaction under dynamic loads has been proposed by Chandrasekaran (1974).

The assumptions of this analysis are: (a) the pile is divided into a convenient number of segments and its mass is concentrated at the center point of the segment, (b) the soil is assumed to act as a linear Winkler's spring. The soil reaction is separated into discrete parts at the center points of the masses, (c) the soil modulus variation is linearly varying with depth, (d) the mass of the superstructure is concentrated at the pile top, (e) the system is one-dimensional in its behavior, and (f) the pile end conditions are either completely free or completely restrained against rotation.

For evaluating the free vibration characteristics, modal analysis is performed by using successive approximations of the natural frequencies of the system with an initially assumed value and related end conditions. The adopted end conditions are also utilized to generate the transfer equations and to evaluate the unknown quantities, either at the pile top or the pile bottom, in terms of the known quantities. These modal quantity values at different station points define the mode shapes. Values at the bottom or top of the piles assists in determining the natural frequencies of

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vibrations in different modes. The details of the analysis are presented elsewhere (Chandrasekaran, 1974; Prakash and Chandrasekaran, 1980). Solutions have been obtained for (1) natural frequency, (2) modal displacements; (3) rotations, (4) bending moments, (5) shear forces, and (6) soil reactions along the lengths of the piles in the first three modes of vibrations. Typical solutions for soil modulus linearly increasing with depth have been discussed in this paper.

NON-DIMENSIONAL SOLUTIONS

It has been shown by Matlock and Reese (1962) that behavior of piles under lateral load is significantly influenced by (a) relative stiffness factor T, Eq. 1 and (b) maximum depth factor Z_{max}, Eq. 2.

$$T = \sqrt[5]{\frac{EI}{n_h}}, \text{ and} \tag{1}$$

$$Z_{max} = \frac{L}{T} \tag{2}$$

in which EI = flexural stiffness of the pile
n_h = constant of horizontal subgrade reaction
L = embedded length of the pile

Z_{max} represents the non-dimensional length of the pile. It has been shown by Chandrasekaran (1974) that these are significant variables in dynamic response also. In his analysis, the range of variables considered are listed in Table 1.

Table 1. Range of Variables

Diameter of Pile	m	0.3, 0.4, 0.5, 0.6 and 0.7
n_h	t/m^3	58.2 — 4634.397
EI	tm^2	4.77×10^2 — 141×10^2
T	m	0.75 — 3.0
Z_{max}	--	1 - 15

One hundred and eighty cases of practical interest were analyzed within the range of variables listed above. Solutions were obtained for the first three modes of vibrations. However, due to space limitations, solutions of only first mode will be discussed herein.

Natural Frequency. In Fig. 1, non-dimensional frequency factors in the first mode of vibration, F_{SL1} or $F_{SL1'}$ defined in Eqs. 3 and 4 are plotted against relative stiffness factor T for Z_{max} of 1, 2, 3, 5, 10 and 15 for free pile top (Fig. 1a) and for pile top restrained against rotation (Fig. 1b).

$$F_{SL1} \text{ (or } F_{SL1'}) = \omega_{n1} \sqrt{\frac{W}{g} \frac{1}{n_h T^2}} \tag{3}$$

and

$$\omega_{n1} = F_{SL1} \text{ (or } F_{SL1'}) \sqrt{\frac{n_h T^2}{M_t}} \tag{4}$$

in which ω_{n1} = circular natural frequency in the first mode,
W = weight of the superstructure assumed concentrated at the top,
g = acceleration due to gravity,
M_t = mass at the pile top (W/g).

Every pile was assigned a vertical load by assuming a suitable value of skin friction and point bearing. In Fig. 2, F_{SL1} and $F_{SL1'}$ versus Z_{max} have been plotted for free pile top and pile top restrained against rotation.

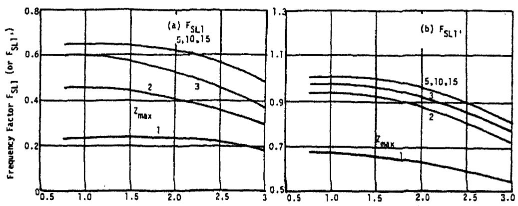

Fig. 1(a) F_{SL1} versus T for Free Pile (b) F_{SL1}' versus T for Pile Top Restrained Against Rotation

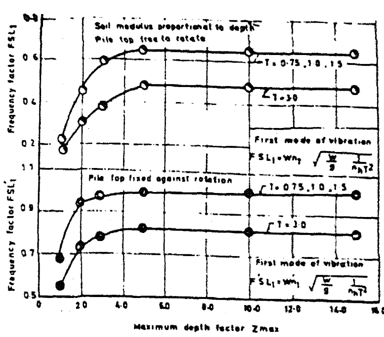

Fig. 2. F_{SL1} (or F_{SL1}') versus Z_{max}

Normalized Modal Quantities. The response of the piles has been studied in terms of non-dimensional modal (a) deflection, (b) rotation, (c) bending moment, and (d) shear force in different modes for all the 180 cases. It was found that for any combination of soil-pile system in Table 1, but with a particular Z_{max} value, unique relationship exists between any one of the non-dimensional modal quantity and the depth factor x/T in which x is the linear dimension along the pile. Non-dimensional deflection coefficient B_{y1} or(B_{y1}') has been plotted against x/T for free pile top in Fig. 3a., and for pile top restrained against rotation in Fig. 3b for Z_{max} of 1, 2, 3, 5, 10 and 15. Similarly, non-dimensional bending moment coefficient B_{m1} or B_{m1}') has been plotted against x/T for free pile top in Fig. 4a and for pile top restrained against rotation in Fig. 4b.

A close examination of Fig. 1 suggests that the frequency factor and hence the natural frequency decreases with increasing 'T'. Fig. 2 shows that for $Z_{max} \geq 5$, the frequency factor F_{SL1} or F_{SL1}' becomes constant. Also, frequency factor for free pile head condition is about two-thirds that for corresponding pile if its head were restrained against rotation. Now, form Eqs. 4 and 1,

$$\omega_{n1} \propto \frac{n_h^{0.3}\ EI^{0.2}}{M_t^{0.5}} \tag{5}$$

Thus for all soil-pile systems, the natural frequency decreases with the superimposed load. Also, for piles in which $Z_{max} > 5$, the natural frequency increases with flexural stiffness EI and soil stiffness (n_h). However, for $Z_{max} < 5$, increase in EI results in increase of 'T' which may offset the increase in ω_{n1} indicated in (5) above. Thus, for such pile lengths, ($Z_{max} < 5$) careful consideration of these effects are necessary. However, in practice, pile lengths are generally large so that $Z_{max} > 5$. From Fig. 3, it is seen that (a) free head piles with $Z_{max} \leq 2$, under go rigid body type displacements (Fig. 3a), (b) for piles with $Z_{max} \geq 5$, the displacements along the length of the pile remain unaltered for all practical purposes for both free head pile and pile with top restrained against rotation. Hence these piles may be treated as infinitely "long" piles and, (c) rotation of the bottom of piles with $Z_{max} < 5$ is greater than that of the "infinitely" long pile.

Displacement y_1 in the first mode of vibrations is given by Chandrasekaran, 1974)

$$y_1 = B_{y1}\ (\text{or } B_{y1}')\times S_{d1} \tag{6}$$

in which S_{d1} = spectral displacement.

Since displacement of the soil-pile system (y_1) is proportional to the spectral displacement (S_{d1}), y_1 depends upon all the factors on which the spectral displacement (S_{d1}) depends. Maximum displacements in all cases, however, occur at the ground level.

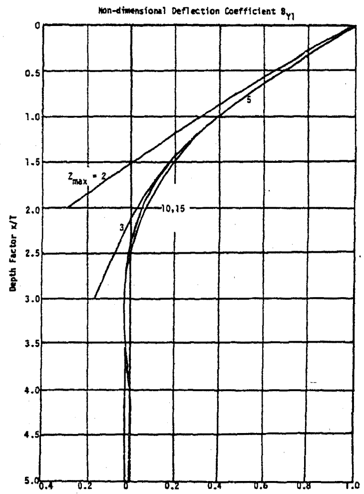

Fig. 3a. B_{Y1} versus x/T for Free Head Pile

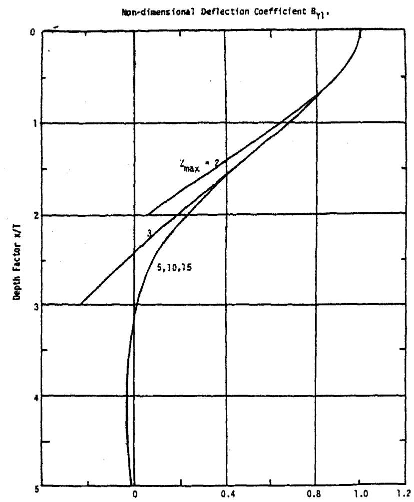

Fig. 3b. $B_{Y1'}$ versus x/T for Pile Head Restrained Against Rotation

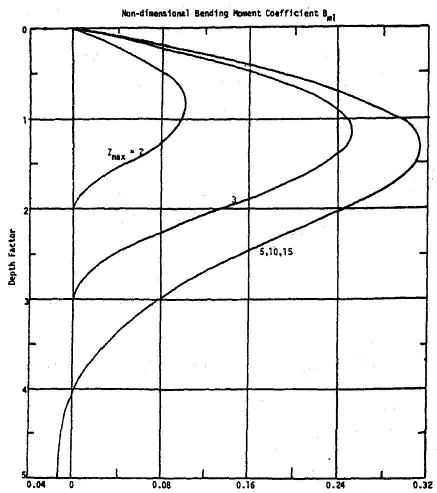

Fig. 4a. B_{m1} versus x/T for Free Pile Head

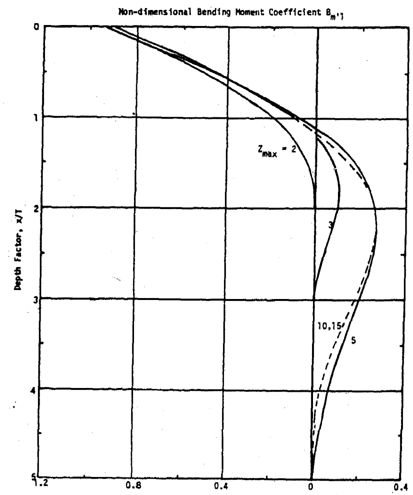

Fig. 4b. $B_{m'1}$ versus x/T for Pile Head Restrained Against Rotation

Bending moment (M_1) in the first mode of vibration is defined by (Chandrasekaran, 1974)

$$M_1 = B_{M1} \ (\text{ro } B_{M'1}) \times n_h T^3 \times S_{d1} \qquad (7)$$

An examination of Figs. 4 show that in a free head pile, the maximum bending moment occurs below the ground level at depths of 0.8 T, 1.15 T and 1.3 T for pile lengths of Z_{max}2, 3, and 5 respectively. For $Z_{max} > 5$, the maximum bending moment does not depend upon Z_{max} value. For pile heads, restrained against rotation, maximum bending moment occurs at the pile top.

Once the displacement of the pile along its length has been determined, soil reaction p_x at depth (x) can be computed as

$$p_x = k_x \cdot y_x = n_h \cdot x \cdot y_x \tag{8}$$

in which k_x = soil modulus at depth x.

It may be noted that soil reaction has zero value at the pile top and increases first with depth and then decreases. Therefore, maximum soil reaction occurs at some depth below the ground level.

SOIL MODULUS DETERMINATION

Very little data on the values of soil moduli are available for use in the dynamic soil-pile-structure interaction problems. On the basis of his experience, Terzaghi recommended rather conservative values for soil moduli and the constant of horizontal subgrade reaction. Davisson (1970) recommended values for analysis under static conditions. Prakash (1981) has proposed corrections for cyclic loading, dynamic loading, and group action.

Prakash and Chandrasekaran (1973) recommended a procedure for determination of soil constant n_h from free and forced vibration tests on piles. The overall stiffness of the soil-pile system as a whole is determined first. The single pile subjected to lateral vibration is treated as a single degree-of-freedom system subjected to forced vibration. For such systems, we have

$$X_{dyn} = \mu \frac{F_o}{k}, \tag{9}$$

in which F_o = dynamic force $(me\omega^2)$

m = mass placed at an eccentricity, (e)

$$\mu = \frac{1}{\left\{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\frac{\omega}{\omega_n}\right)^2\right\}^{1/2}},$$

ω = forcing frequency

ω_n = natural frequency

k = stiffness of the soil-pile system.

The quantity k' defined as X_{dyn}/F_o is referred to as the "stiffness influence coefficient".

From the free vibration test results, the damping factor, ξ, and the natural frequency, ω_n, can be determined. It has been observed that the ratios of the amplification factor at any two frequencies are equal to the ratios of the stiffness influence coefficients at the same frequencies, (Chandrasekaran, 1974). This relationship can be maintained only when the value of the overall stiffness, k, of the soil pile system is frequency independent. Therefore, the value of k can be computed using Eq. 9. This value is not altered materially by the variation of ξ, with a forcing frequency, and different initially assumed values of the natural frequency, ω_n, of the system.

The separate soil stiffness values must be known to take into account the interaction effects. For the linear variation of soil stiffness with depth the solution for deflection, Y_g, under a lateral load, Q_{hg}, is

$$\frac{Y_g}{Q_{hg}} = \frac{A_y T^3}{EI} = \frac{A_y}{EI}\left(\frac{EI}{n_h}\right)^{3/5} \tag{10}$$

in which A_y = 2.435 at ground level (Matlock and Reese, 1962). The quantity Q_{hg}/Y_g is called the overall stiffness of the system (k), Eq. 10. Now k, EI and Q_{hg} are known, the value of n_h can be determined (Eq. 10), as illustrated next.

There is a need to collect extensive data on the soil modulus values under dynamic condition.

TESTS ON PILES

Static lateral load, lateral free vibration and forced vibration tests were performed on piles of different sizes and placed in two types of soils, Table 2.

Table 2. Details of Piles Tested

Serial No.	Pile Identification	Pile Section and Length Diameter cm,	Length m	Soil Type	Remarks
1	VTP1	40	25	Soft clay	Franki pile
2	VTP2	40	24	Soft clay	Franki pile
3	VTP3	40	22	Medium clay	Franki pile
4	VTP4	50	16	Stiff clay	Simplex pile
5	VTP5	12.6	6	Silty sand	Steel pipe pile
6	VTP6	12.6	6	Medium silty sand	Steel pipe pile

In static test, lateral load was applied by jacking two adjacent piles and lateral displacements were recorded when the rate of movement of the pile was 0.002 mm per hour or less. In free vibration test, the pile was excited by pulling the two piles towards each other and then suddenly releasing the same by a suitable clutch designed for the purpose. In lateral vibration test, the piles were excited with a Lazan* type mechanical oscillator driven by a motor, the speed of which was controlled by a suitable device. The motion of the piles in both of the tests was sensed by acceleration pickups, securely attached to the pile. The time-history of motion was recorded on an oscillograph.

Results of piles No. 5 and 6 have been analysed here since in silty sands the soil modulus is assumed linearly increasing with depth. Test data on piles No. 1-4 in clays has been presented elsewhere (Prakash and Chandrasekaran, 1980).

Static Tests. Load deflection plots were obtained. For every point on this curve, the value 'T' and n_h were determined from Eq. 9. For typical data, these values are listed in Col. 1-6 of Table 3.

Table 3. Analysis of Data on Pile VTP5 and VTP6 of EI = 6.21×10^8 kg cm^2

Lateral Static Test						Free Vibration Tests*		Forced Vibration Test		Computed	
1	2	3	4	5	6	7	8	9	10	11	12
No.	Pile	Load kg	yg mm	T cm	n_h kg/cm^3	f_n* cps	ξ(%) Damping Factor	K kg/cm	n_h kg/cm^3	cm	f_n* cps
5	VTP5	750	10	69.8	0.375	2.38	12	185	0.036	111	2.42
6	VTP6	950	6.5	56	1.135	4.8	10	537	0.215	78	4.14

*Sustained vertical load - 800 kg.

*Trade name

<u>Free Vibration Tests</u>. Data was analysed to determine the natural frequency and damping of the systems. These values are listed in Col. 7 and 8 in Table 3.

<u>Forced Vibration Tests</u>. Amplitude in the forced vibration test was computed from measured acceleration and frequency. Dynamic force versus dynamic amplitude of motion plot for pile VTP5 is shown in Fig. 5. It was possible to vary the dynamic force at any one frequency by changing the eccentric settings of the two sets of masses in the mechanical oscillator Thus knowing the amplitude-frequency-dynamic force relationship and ξ and ω_n from the free vibration tests, δ'_{st} (equivalent static deflections) was

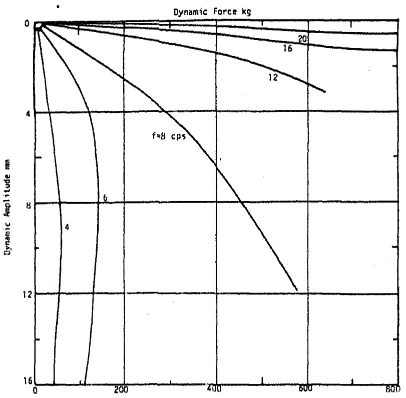

Fig. 5. Dynamic Force versus Dynamic Amplitude for Pile VTP5

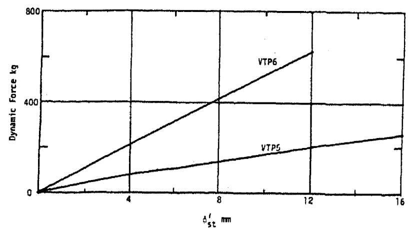

Fig. 6. Dynamic Force versus δ'_{st} for Determination of Overall Stiffness of Pile VTP5 and VTP6.

computed (Eq. 9) and plots of F_o and δ'_{st} drawn, (Fig. 6). For all practical purposes, these plots are unique irrespective of variation in the forcing frequency ω and the natural frequency ω_n. The tangent modulus of this plot is overall stiffness $K(FL^{-1})$ of the soil pile system <u>under dynamic conditions</u>. With this K, the value of the soil-constant under dynamic conditions was determined (Eq. 10) and is listed in Col. 9 of Table 3. These values of dynamic constants were used in the analysis and natural frequencies were determined (Col. 12, Table 3).

The tally of the observed and computed natural frequencies is extremely well. However, the data on realistic dynamic soil constants is lacking. Therefore, every effort needs to be made to compile soil data from field tests.

The maximum bending moment in the pile, its deflection and soil reaction along its length can be checked with the help of Eqs. 6, 7 and 8 respectively.

Based upon this analysis, a tentative method of design of piles under earthquakes has been proposed (Prakash, <u>et al</u>., 1979 and Prakash, 1981).

CONCLUSION

1. A simple method for analysis of piles in sand under earthquake condition has been presented for determination of (a) natural frequency of the system, (b) deflection, (c) bending moments, and (d) soil reaction.

2. The efficacy of the method has been demonstrated by comparing the natural frequency of the pile with the predicted frequency, where an excellent tally was found.

3. There is a need to determine relevant dynamic soil constants for use in designs of piles.

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