INELASTIC DYNAMIC RESPONSE OF SATURATED SOILS

by Chris A. Katsikas[I], and E. Benjamin Wylie [II]

SUMMARY

A one-dimensional numerical model for soil liquefaction under dynamic excitation during an earthquake is presented. The soil is modeled as an inelastic shear beam, and the pore-water motion is described by a Darcian law. The shear and pressure waves are coupled through an incremental law which takes the total volumetric deformation as the sum of an elastic and an inelastic increment. Both increments are represented by empirical equations. The method has been applied in several case studies. Results obtained for the 1964 Niigata earthquake compare favorably with the observed behavior of the soil during the earthquake.

INTRODUCTION

Extensive research during the past fifteen years has been devoted towards the prediction of liquefaction of soils induced by earthquake motion. In cohesionless soils, liquefaction is defined as the transformation from a solid state to a liquefied state as a consequence of increased pore-water pressure and reduced effective stress. Liquefaction results in a temporary loss of shear resistance but does not always produce a final loss of shear strength. The change of state during liquefaction is independent of the initiating disturbance which may be monotonic straining, vibratory, shock loading, etc.

A numerical model for soil liquefaction, which provides interaction between shearing deformation and transient pore-water pressure development, is presented. One-dimensional shear wave propagation through the solid matrix of the soil and one-dimensional pressure wave propagation through the pore-water constitute the principal parts of the model. This model includes some unique features: it treats the soil mass as a continuous medium; it introduces pore-water pressure development and seepage through the fundamental equations of motion and continuity of the water; it couples the shear and pressure waves; and it associates the dynamic pore-water pressure changes with the inelastic volumetric deformation of the soil solid matrix.

VOLUMETRIC CYCLIC SOIL DEFORMATION CONCEPT

The nature of volumetric deformation of a soil element under cyclic shear straining is determined by considering the following two states of its particle group:

a) A dry, partially saturated or saturated freely drained state.

[I]Engineer II, Giffels Assoc., Inc., Southfield, Michigan.

[II]Professor of Civil Engineering, Univ. of Michigan, Ann Arbor, Mich.

In this case the application of the cyclic straining will not change the effective stress of the structure nor the pore-water pressure, but the solid matrix will reduce its volume as the soil particles rearrange themselves and water is expelled from the voids. Hence the nature of deformation is irreversible densification.

b) An undrained saturated state. In this case particle rearrangement will result, accompanied by a transformation of intergranular forces to the pore-water. The transformation of forces increases the pore-water pressure, relaxes the solid matrix and reduces the effective stress. The relaxation of the particle structure will release reversible volumetric strain stored at the points of particle contact. Hence the net deformation of the solid matrix is the sum of a recoverable volumetric rebound and a non-recoverable volumetric densification.

The two states mentioned are two extreme conditions for an in-situ element of a saturated sand deposit. An element cannot drain instantly during the short period of an earthquake, and even for low soil permeabilities, completely undrained conditions may not exist.

Based on the behavior of a contractive sand during straining as outlined above, the following formulation of the deformation of its solid matrix is proposed:

$$\Delta\varepsilon_{v} = \Delta\varepsilon_{vs} + \Delta\varepsilon_{vr} \qquad (1)$$

where: $\Delta\varepsilon_{v}$ = Net volumetric deformation
$\Delta\varepsilon_{vs}$ = Inelastic densification induced by particle rearrangement of the solid structure
$\Delta\varepsilon_{vr}$ = Elastic rebound induced by the relaxation of the solid structure.

For the evaluation of the recoverable volumetric deformation, $\Delta\varepsilon_{vr}$, the following empirical equation is adopted:

$$\varepsilon_{vr} = S(\sigma'_{0})^{m} \ \% \qquad (2)$$

where σ'_{0} is the mean effective stress in kN/m^2, ε_{vr} the corresponding volumetric deformation and S,m are constants. In general the constant S and m can be evaluated from two unloading tests for a particular sand. According to the experimental findings of El-Sohby (1969) the parameter S (elastic compressibility factor) and m (elastic power exponent) may depend only on relative density. In this study the elastic power exponent m is taken as constant equal to 0.47, while the elastic compressibility factor S is given by the following equation (Katsikas, 1979):

$$S = 0.041 + 0.18\eta_{0} \qquad (3)$$

where η_{0} is the initial porosity.

The inelastic deformation increment, $\Delta\varepsilon_{vs}$, is modeled with an empirical equation which takes into account the following factors affecting densification (Silver and Seed, 1969): number of cycles, straining amplitude, and relative density. The equation used is:

$$\Delta\varepsilon_{vs} = -A_1 |\gamma|^{A_2} |\Delta\gamma| / (1 + A_3 (N_c)^{A_4}) \ \% \qquad (4)$$

where N_c = number of cycles from the begining of straining; $|\gamma|$ = instantaneous shear strain at the end of ΔT (%); $|\Delta\gamma|$ = change in shear strain over time ΔT (%); A_1, A_2, A_3, A_4 = parameters which may depend on relative density for a particular sand. The minus sign and the absolute values of γ and $\Delta\gamma$ indicate that densification is a volume reduction.

The numerical values for parameters A_1, A_2, A_3 and A_4 may be obtained from simple shear tests of a sand. Crystal silica #20 sand was tested in simple shear under uniform cyclic straining by Silver and Seed (1969). The test results include information for the influence of straining amplitude, number of cycles, and relative density, D_r, on the densification of this sand. For any relative density between 40% and 80% of the silica sand the four parameters A_1, A_2, A_3 and A_4 are chosen as follows:

$$A_1 = A_{10} \left(\frac{D_{r0}}{D_r}\right)^2; \quad A_{10} = 0.20 \qquad (5)$$

$$A_3 = A_{30} \left(\frac{D_r}{D_{r0}}\right); \quad A_{30} = 0.80 \qquad (6)$$

$$A_2 = 0.10; \quad A_4 = 0.85 \qquad (7)$$

where D_{r0} = 60%.

Results using Eq. 4 compare favorably with experimental findings of Silver and Seed (1969). Values of A_1, A_2, A_3 and A_4 in the absence of simple shear tests for a particular sand, are determined qualitatively (Katsikas, 1979). For any sand the parameter A_1 may be determined from:

$$A_1 = (0.24 - 0.06\, D_{50})(60/D_r)^2 \qquad (8)$$

where D_{50} is the grain diameter in mm.

Equation 4 is used for both uniform and nonuniform cyclic straining. In cases of earthquake excitation an equivalent period T_{eq}, representative of the particular earthquake is chosen. The number of cycles N_c in Eq. 4 is then replaced by the ratio T/T_{eq} where T is the time from the beginning of the earthquake.

THE GOVERNING EQUATIONS

In the one-dimensional shear wave propagation through level or near level soil deposits, shearing stresses are generated by the base motion and they are propagated through the soil column. The soil is treated as an inelastic shear beam. The dynamic equation of motion and the inelastic shearing stress-shearing strain equation for the shear wave are:

$$\frac{\partial\tau}{\partial z} - \rho\frac{\partial V}{\partial t} - \rho g \sin\theta = 0 \qquad (9)$$

$$\frac{\partial \tau}{\partial t} - G\frac{\partial V}{\partial z} = 0 \qquad (10)$$

where τ is the shearing stress, ρ is the mass density of the soil, θ is the slope of the deposit, g is the acceleration of gravity, V is the horizontal shearing particle velocity and G is the tangent shear modulus at a given stress level. The nonlinear behavior of the soil is modeled with a strain softening type shear stress-shear strain equation first proposed by Ramberg and Osgood (1943). The method of characteristics is used for the numerical solution of Eqs. 9 and 10 (Streeter et al, 1974). The equations of continuity and motion of the pore-water are examined next for one-dimensional pressure wave propagation.

The continuity equation of pore-water is:

$$\eta\rho_w \frac{\partial U_z}{\partial z} + \eta\rho_w \beta \frac{\partial p}{\partial t} + \rho_w \frac{\partial \eta}{\partial t} = 0 \qquad (11)$$

where ρ_w is water density, η is porosity, U_z is vertical water seepage velocity, p is water pressure and β is water compressibility. In Eq. 11 the soil is considered isotropic and the water density varies only with the water pressure. For a soil element of bulk volume $\forall_b$ and for incompressible solid particles, the time rate of change of its void volume, $\partial\eta/\partial t$, is:

$$\frac{\partial \eta}{\partial t} = (1-\eta)\frac{1}{\forall_b}\frac{\partial \forall_b}{\partial t} \qquad (12)$$

and the net volumetric deformation, $\Delta\varepsilon_v$, of the element under cyclic straining is:

$$\Delta\varepsilon_v = \frac{\Delta\forall_b}{\forall_b} \qquad (13)$$

Combining Eqs. 1, 11, 12 and 13, the continuity equation becomes:

$$\frac{\partial U_z}{\partial z} + \beta\frac{\partial p}{\partial t} + \frac{1-\eta}{\eta}\left(\frac{\partial \varepsilon_{vs}}{\partial t} + \frac{\partial \varepsilon_{vr}}{\partial t}\right) = 0 \qquad (14)$$

The total macroscopic volumetric stress, σ_t is assumed to be only a function of depth:

$$\sigma_t = \sigma'_z - p; \quad \frac{\partial \sigma'_z}{\partial t} = \frac{\partial p}{\partial t} \qquad (15)$$

where σ'_z is the vertical effective stress. Defining the rebound modulus of the solid matrix, K_r, as the rate of change of σ'_z with respect to the elastic deformation ε_{vr}:

$$\frac{\partial \varepsilon_{vr}}{\partial t} = \frac{1}{K_r}\frac{\partial \sigma'_z}{\partial t} = \frac{1}{K_r}\frac{\partial p}{\partial t} \qquad (16)$$

equation 14 takes the form:

$$\frac{\partial U_z}{\partial z} + \left(\frac{1}{K_w} + \frac{1 - \eta}{\eta}\frac{1}{K_r}\right)\frac{\partial p}{\partial t} + \frac{1 - \eta}{\eta}\frac{\partial \varepsilon_{vs}}{\partial t} = 0 \qquad (17)$$

and relates the pore-water pressure p and the seepage velocity U_z with the inelastic deformation ε_{vs} of the solid matrix. $K_w = 1/\beta$ is the bulk modulus of water.

The equation of motion for the water, modified to include the inertia multiplier, α^2, in the acceleration term is:

$$\frac{\partial p}{\partial z} + \rho_w \alpha^2 \frac{\partial U_z}{\partial t} + \frac{\eta \rho_w g}{k_z} U_z - \rho_w g = 0 \qquad (18)$$

The method of characteristics is used for the numerical solution of Eqs. 17 and 18. The pressure wave through the pore-water propagates with a wave velocity given by (Katsikas, 1979):

$$V_p = \left(\frac{K_w/\rho_w}{1 + \frac{1 - \eta}{\eta}\frac{K_w}{\rho_w}}\right)^{0.5} \qquad (19)$$

which is the sound speed through the pore-water modified to account for the volumetric deformation of the solid matrix.

THE LIQUEFACTION MODEL WITH APPLICATIONS

The liquefaction model is formulated by coupling the shear and pressure wave propagation. Equation 1 provides the basic means for the coupling which is achieved as follows: during an earthquake, shear waves propagate through a soil column with shear wave speed $V_s = (G/\rho)^{1/2}$ inducing dynamic shearing stresses and strains. The induced strains generate volumetric deformation (Eq. 1) which alters the pore-water pressure and initiates pressure waves which propagate with a wave speed V_p (Eq. 19). A change in pressure changes the vertical effective stress of the soil affecting the strength characteristics of the soil and the pressure wave speed. The strength characteristics of the soil are the maximum shear modulus G_o and the maximum shear strength τ_m. Their dependence on the vertical effective stress has been described by Hardin and Drnevich (1972). The new conditions of the soil affect the shear wave speed V_s and generate additional shearing stresses and strains. This sequence of events constitutes one cycle of the dynamic response of the soil during the earthquake. The cycle is repeated until motion ceases. Thus, the liquefaction model provides a complete analysis of the behavior of the soil column during the earthquake.

The Niigata earthquake is an excellent case study for the model because the conditions that led to failure are known. The response of a soil deposit with properties corresponding to those of the heavier damaged zone C and the lightly damaged zone B is examined (Katsikas, 1979). The depth of the soil deposit is taken equal to 21.3 m. No liquefaction is expected below this depth. The water table in zones B and C is 1 m below the ground surface and the remaining 20.3 m of the saturated deposit consist of 7 layers with uniform properties as shown in Table 1. The 1 m of top soil is represented by a 15.8 kN/m^2 overburden pressure at the water table.

TABLE 1

Layering and Relative Densities of each Layer for Zones B and C

Layer #	Layer Depth (m)	Relative Density Zone B	Relative Density Zone C
1	2.14	44.5	44.5
2	3.05	48.5	41.5
3	3.04	61.5	44.5
4	3.05	75.0	59.5
5	3.05	82.5	72.0
6	3.05	86.5	77.5
7	3.04	89.0	81.5

According to Seed and Idriss (1967) the ground acceleration recorded during the Taft earthquake of July 21, 1952 in California, is a reasonable estimate of the Niigata earthquake in accordance with recognized seismological measurements and observations. The velocity-time history obtained by integration of the Taft S21W accelerogram is used as input excitation. The input velocities are scaled down by a factor of 0.70. Figure 1 shows the first 10 seconds of Taft S21W accelerogram, and the modified input velocity at the base of the deposit.

The soil response to the imposed excitation is shown in Figs. 2, 3, 4 and 5 for both Zones B and C. Reach #2, which is the most vulnerable, remains strong in Zone B but fails in Zone C, as indicated in Figs. 3-5.

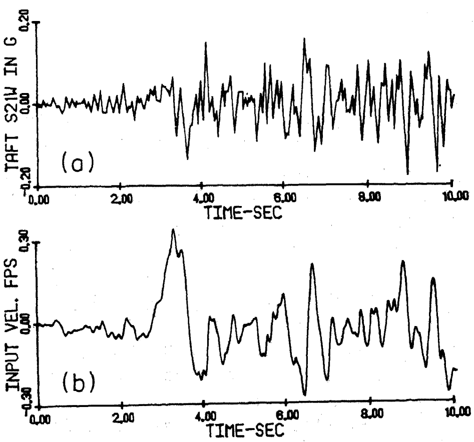

Figure 1 (a) Taft S21 W Accelerogram.
(b) Modified Input Velocity.

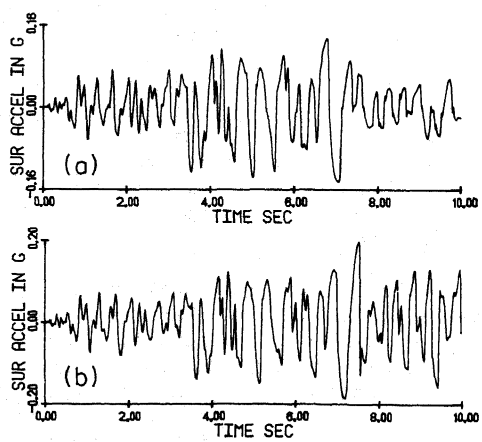

Figure 2 Developed Surface Acceleration. (a) Zone C
(b) Zone B

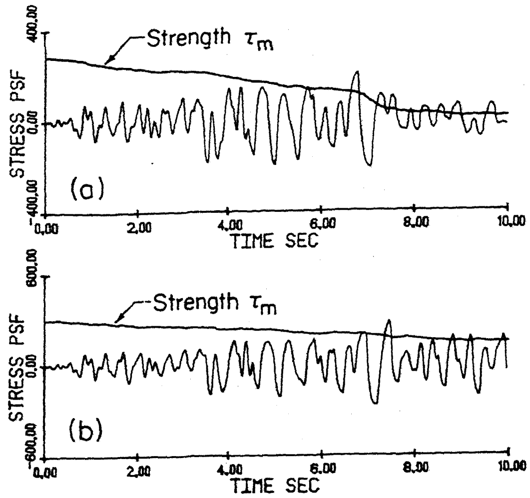

Figure 3 Maximum Shear Strength and Shear Stress.
(a) Zone C
(b) Zone B

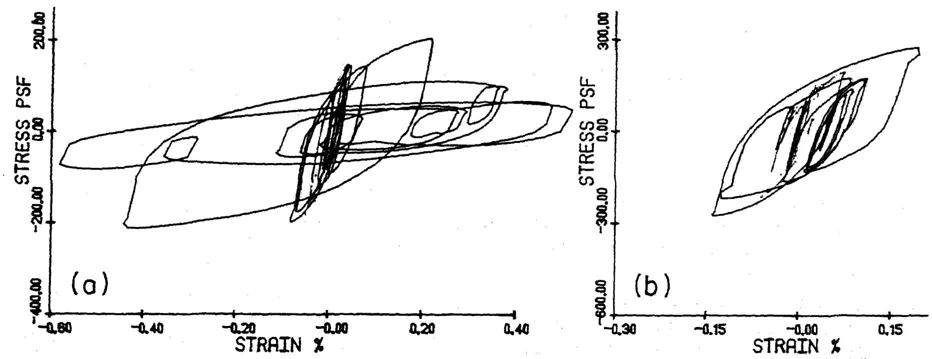

Figure 4 Hysteretic Behavior.
(a) Zone C
(b) Zone B

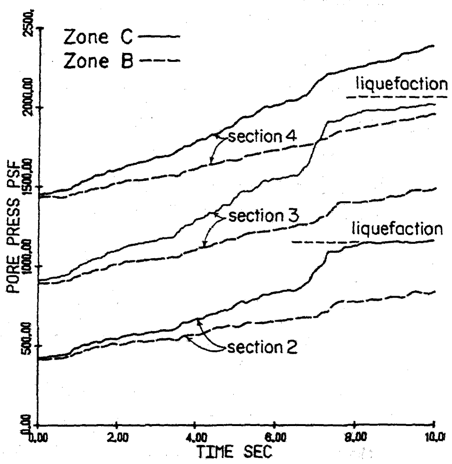

Figure 5 Pore-water Pressure History in Zone C.

The analysis for Zone B predicted no excessive pore-water pressure development during the motion. The deposit retained most of its strength, and its high resistance to liquefaction is in agreement with the field behavior of the soil in Zone B during the Niigata earthquake. On the contrary the analysis for Zone C predicted the occurrence of liquefaction starting at t = 8.0 sec. at a depth of about 3 m below the ground surface and extending to a depth of 5.4 m below the ground surface. This behavior simulates closely the observed soil behavior of Niigata City during the earthquake of June 16, 1964. The difference in response between Zones B and C (Ohsaki, 1966) which was attributed to the looser layers between 3 and 6 m for Zone C was correctly simulated by the model in this case study.

CONCLUSIONS

A one-dimensional numerical model, based upon the method of characteristics, has been presented to study the inelastic dynamic response of fully saturated cohesionless soils. The effective stress model provides results that compare favorably with observed behavior during the 1964 Niigata earthquake.

ACKNOWLEDGEMENTS

The investigation described herein was supported by the National Science Foundation Grant PFR 77-22219 to The University of Michigan.

REFERENCES

1. El-Sohby, M.A., (1969) "Elastic Behavior of Sand," Jour. of the Soil Mechanics and Foundation Engineering Division. Proc. ASCE, Vol. 95, No. SM6, Nov., pp. 1393-1409.

2. Hardin, B. O., and Drnevich, V. P. (1972) "Shear Modulus and Damping in Soils: Design Equations and Curves," Jour. of the Soil Mechanics and Foundation Engineering Division. Proc. ASCE, Vol. 98, No. SM7, July, pp. 667-692.

3. Katsikas, C. A., (1979) "Numerical Models for Soil Liquefaction," Ph.D Dissertation, The University of Michigan, Ann Arbor, Michigan.

4. Ohsaki, Y. (1966) "Niigata Earthquake 1964, Building Damage and Soil Condition," Soils and Foundations, JSSMFE, Vol. 6, No. 2, pp. 14-36.

5. Ramberg, W. and Osgood, W. T. (1943) "Description of Stress-Strain Curves by Three Parameters," Technical Note, 902 NASA.

6. Seed, H. B., and Idriss, I. M. (1967) "Analysis of Soil Liquefaction: Niigata Earthquake," Jour. of the Soil Mechanics and Foundation Div., ASCE, Vol. 93, No. SM3, May 1967, pp. 83-108

7. Silver, M. L., and Seed, B. H. (1969) "The Behavior of Sands under Seismic Loading Conditions," Earthquake Engineering Research Center. Report No. EERC 69-16, University of California, Berkeley, Dec.

8. Streeter, V. L., Wylie, E. B. and Richart, F.E., Jr. (1974) "Soil Motion Computations by Characteristics Method," Jour of Geotechnical Engr. Div. Proc. ASCE, Vol. 100, No. GT3, pp. 247-263.