THE ANALYSIS OF LIQUEFACTION POTENTIAL BASED ON PROBABILISTIC GROUND MOTIONS

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SUMMARY

A new approach to the analysis of the load history in cyclic liquefaction based on the theory of extremal statistics and the Palmgren-Miner hypothesis is presented. This methodology allows us to assess the liquefaction potential based on the RMS parameter $1/\lambda$.

INTRODUCTION

Liquefaction of cohesionless soils has been one of the most thoroughly researched problems in geotechnical engineering during the last fifteen (15) years. Many works on this subject have been published, generally dealing with a deterministic analysis of liquefaction. This paper addresses the question of load characterization in liquefaction analysis presenting a new method through which the number of cycles at some stress level required to cause initial liquefaction can be tied in with ground motions that are probabilistically described. We will use and extend the methodology developed by (Zsutty and DeHerrera, 1979) together with the Palmgren-Miner hypotheses to study this interesting geotechnical problem.

The primary cause of cyclic liquefaction of saturated cohesionless soils during earthquakes is the buildup of excess hydrostatic pressure due to the application of cyclic shear stress (Seed, Arango and Chan, 1975). It has been necessary to distinguish between various states of liquefaction depending on the deformations that are developed. We shall deal exclusively with initial liquefaction, which is defined as the "condition where, during the course of cyclic stress applications the residual pore water pressure on completion of any full stress cycle becomes equal to the applied confining pressure" (Seed, 1976). The occurrence of initial liquefaction gives no indication as to what the subsequent deformations will be.

The liquefaction characteristics of soils are affected by several factors (Seed, 1976). Among these are:

1. The nature of the soil, where by "nature" we mean grain size distribution and grain characteristics
2. The structure of the soil
3. The relative density of the soil
4. The magnitude of the stress cycles applied to the soil
5. The number of stress cycles applied to the soil.

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Factors 4 and 5 are especially relevant to the scope of this paper.

REVIEW OF THE EXPONENTIAL HALF TAIL MODEL

We define a peak as the asbolute value of the maximum between two zero crossings in an earthquake time history. For a given time history, let X_1 = largest peak value, X_2 = second largest peak value, etc., so that $\{X\} = \{X_1, X_2, \ldots X_m\}$ form the set of M peak values. Furthermore, let $Y_1 = (X_1)^k$, $Y_2 = (X_2)^k$, $(k>0)$ ... be the transformed set of peaks $\{Y\}$. The X_i and Y_i are random variables sampled from a population with some unknown probability density function (PDF). Assuming statistical regularity, order statistics tell us that the estimates for the exceedance probabilities $P(X > X_i)$, or $P(Y > Y_i)$ are functions of $i/(M+1)$, where i is the position number of X_i of Y_i on a list sorted in descending order; "i" is generally referred to as the "rank" of the observation.

The exponential half tail (EHT) model uses the exponential PDF to probabilistically describe the upper median peaks. The choice of this distribution allows the use of several interesting results from the theory of extremal statistics (Gumbel, 1958, Galambos, 1979). At this point we refer the reader to (Zsutty and DeHerrera, 1979, 1980) for the derivation of this method.

APPLICATION TO INITIAL LIQUEFACTION

An inherent assumption in the analysis of liquefaction is that one can represent an earthquake response time history by a combination of equivalent uniform stress level cycles. This assumption has been verified by (Annaki and Lee, 1977), and is based on the well-known Palmgren-Miner hypothesis. This hypothesis, whose functional form is given by

$$D = \sum_i \frac{n_i}{N_{if}} \tag{1}$$

has been widely used for fatigue life prediction of metal structures. Equation (1) states that the damage D is a function of the number of cycles n_i at stress level i given that the number of cycles to failure at stress level i is N_{if}. Failure occurs when $D \geq 1$.

The number of cycles to failure at stress level i is usually determined from an "S-N diagram." S is a suitably normalized log of a stress ratio and N is the log of the number of cycles to failure (see Figure 1). Figure 2 shows a plot of the number of cycles to initial liquefaction as a function of the corrected stress ratio and as such it is a form of S-N diagram. In Figure 3, the y-intercept ξ is a function of the relative density of the soil and the effective overburden pressure before the earthquake. The equation of the line in Figure 3 is

$$\log \frac{Y_i}{Y_o} = \xi + \psi \log N_{if} \tag{2}$$

The present state of the art in load characterization consists of assigning an arbitrary value for the number of load cycles at a site, this number being a function of the Richter Magnitude of the "design" earthquake. The methodology presented in this paper is an improvement over the state of the art in that we can work with the expected value of the largest peaks, thus obtaining a higher measure of reliability in this load characterization. Given that we have a value of $1/\psi$ which considers the effect of site condition and response, we can easily find the expected value of the k^{th} shear stress peak ($k = 1,N$). With a set of ordered peaks Y_i normalized by some suitable stress Y_o, we can obtain the ratios Y_i/Y_o and substitute them into the Palmgren-Miner equation. Now $n_i = 1$ for all i because of the way we have ordered the peaks. Therefore

$$N_{if} = e^{-\xi/\psi} \left[\frac{Y_i}{Y_o}\right]^{1/\psi} \tag{3}$$

$$D = \sum_i e^{\xi/\psi} \left[\frac{Y_i}{Y_o}\right]^{-1/\psi} \tag{4}$$

SUMMARY AND CONCLUSIONS

A new approach to the analysis of the load history in cyclic liquefaction based on the theory of extremal statistics and the Palmgren-Miner hypothesis has been presented. This methodology allows us to assess the liquefaction potential based on the RMS parameter $1/\lambda$.

ACKNOWLEDGEMENTS

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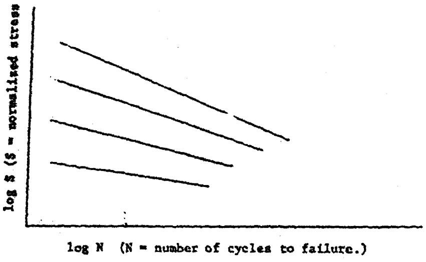

Figure 1: S – N Diagram

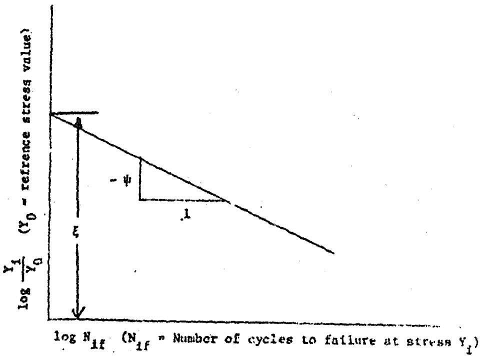

Figure 3: Typical S – N Diagram for cohesionless soil

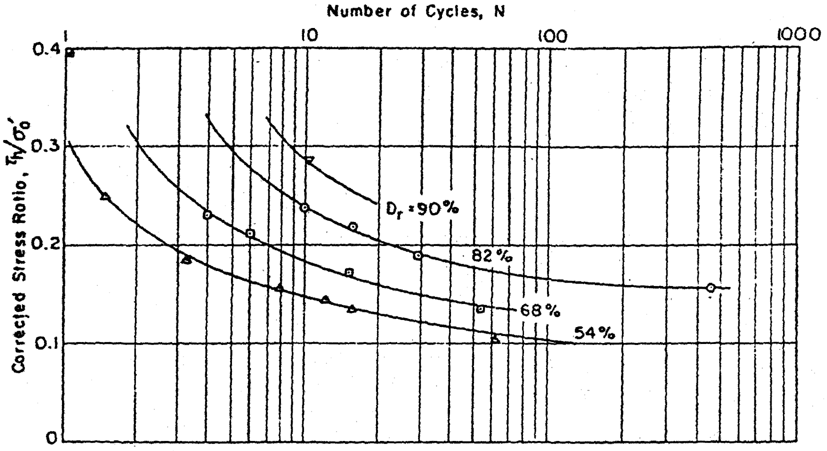

FIGURE 2 : CORRECTED τ_h/σ_0' VS. N FOR INITIAL LIQUEFACTION
(Ref. 3)