ENDOCHRONIC THEORY OF SAND LIQUEFACTION

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SUMMARY

Endochronic theory is used to describe the volume changes and pore-water pressures generated in saturated sands by earthquake motions. Advantages and disadvantages of the endochronic approach are discussed. Procedures and conclusions are based on a great amount of data from six different sands.

INTRODUCTION

Endochronic theory was developed by Valanis (1971) to describe non-linear material response. The non-linearity is described by means of a parameter which describes the sequence of events leading to successive states of the material. Although not the time variable, the parameter functions as a kind of intrinsic time, hence the term endochronic. The endochronic parameters are mathematical transformations of real physical variables of the problem but may, themselves, have no direct physical significance. For the liquefaction problem we will see that the parameter depends on a transformation of deformation increments.

Bazant and Krizek (1976) developed an endochronic constitutive law for the liquefaction of sand. They used an endochronic description of the stress-strain relations and represented the densification or volumetric strains caused by cyclic shearing in terms of endochronic variables. Zienkiewicz, Chang and Hinton (1978) also used endochronic variables to describe the volumetric strains caused by cyclic loading. In both cases, the volumetric strains were then incorporated into a pore-water pressure model which gave the increments of pore-water pressure in terms of the volumetric strains and the bulk moduli of the soil skeleton and water. Both approaches were based on very limited test data.

In this paper we describe the application of endochronic methods to describing the volume changes and pore-water pressures generated by cyclic loading in two North American and four Japanese sands. The procedures for generating the endochronic variables are described and the general applicability of the endochronic approach is assessed. Finally, the advantages and disadvantages of using the approach for relating pore-water pressures and volume changes to parameters of dynamic response in total or effective stress dynamic analyses are discussed.

The brief description of the Martin, Finn and Seed model for volume change and pore-water pressures given below, is useful as a foundation for

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deriving the endochronic variables and appreciating the advantages of the endochronic approach.

CURRENT PORE-WATER PRESSURE MODEL

The pore-water pressures that develop in saturated undrained sands during seismic loading are caused by plastic volumetric strains generated by slips at grain contacts (Martin, Finn and Seed, 1975). The plastic volumetric strains ε_v are prevented from occurring immediately by the presence of the more rigid water. Thus, load is transferred to the water creating an increase in pore-water pressure. This pressure reduces the effective stress-regime in the sand and allows the sand skeleton to rebound elastically. The pore-pressure increase will be big enough to allow an elastic rebound equal to the total plastic volumetric strains if we assume that the bulk modulus of the water is an order of magnitude stiffer than that of the soil skeleton. Otherwise, it will be sufficient to cause a rebound equal to the difference between the compressive volumetric strains in the pore-fluid (water or water and gas) and the plastic volumetric strains (Martin, Finn and Seed, 1975, 1978). This pore-pressure model is described by the following incremental equations.

$$\Delta\varepsilon_v = C_1(\gamma - C_2\varepsilon_v) + C_3\varepsilon_v^2 \,/\, (\gamma + C_4\varepsilon_v) \tag{1}$$

in which $\Delta\varepsilon_v$ = increment in volumetric strain, γ = current shear strain amplitude, and C_1, C_2, C_3 and C_4 are constants for a given sand at a given density. $\Delta\varepsilon_v$, ε_v and γ are expressed in percentages.

$$\Delta u = \Delta\varepsilon_v \,/\, \left(\frac{1}{\bar{E}_r} + \frac{n}{K_w}\right) \tag{2}$$

in which Δu = pore-water pressure increment, $\Delta\varepsilon_v$ = increment in volumetric strain, $\bar{E}_r$ = one-dimensional rebound modulus of sand skeleton, n = porosity and K_w = bulk modulus of water.

$$\bar{E}_r = (\sigma_v')^{1-m} \,/\, mK_2(\sigma_{vo}')^{n-m} \tag{3}$$

in which σ_{vo}', σ_v' = initial value and current value of effective stress, respectively and n,m and K_2 are constants for a sand at a given density. These equations are based on very extensive test data. It will be noted that the pore-water pressure model requires the evaluation of seven constants. Furthermore, it has been shown that $\bar{E}_r$ cannot be measured in oedometer tests as was considered adequate previously because the rebound response of sand under cyclic loading conditions is different from that under static unloading (Finn and Bhatia, 1980). The measurement of $\bar{E}_r$ under cyclic loading conditions requires sophisticated equipment and is very time consuming. Therefore, there is considerable incentive to develop a direct link between pore-water pressure and the dynamic response parameters of the sand-water system that obviates the need to measure the rebound modulus of the sand skeleton under cyclic shear conditions.

The endochronic method is a "black-box" approach that seeks to express important parameters of response such as pore-water pressure as monotonically increasing functions of suitable transformed variables. It does not attempt to model the actual physical process involved and therefore is a good prospect for eliminating the need to measure the rebound modulus.

DERIVATION OF ENDOCHRONIC FUNCTIONS

It is clear from the experimental data in Fig. 1 and from Eq. 1 that, in drained constant strain cyclic loading tests in simple shear, the plastic volumetric strains ε_v at any time are a function of the strain amplitude γ and the number of shear strain cycles N. An alternative to N, which allows generalization to cycles of non-uniform loading is the length of the strain path ξ, defined as $d\xi = \frac{1}{2}(d\varepsilon_{12}, d\varepsilon_{12})^{\frac{1}{2}}$ in which ε_{12} are the deviatoric strains in the plane. Thus, for simple shear $d\xi = \frac{1}{8}|d\gamma|$. Therefore, we may write

$$\varepsilon_v = f(\gamma, \xi) \qquad (4)$$

Similarly, for pore-water pressure, u we may write (based on data in Fig. 5 and Eqs. 1 and 2)

$$u/\sigma'_{vo} = g(\gamma, \xi) \qquad (5)$$

in which σ'_{vo} is the initial vertical effective stress. We would like to express ε_v and u as monotonically increasing functions of a single variable κ. This can be done if a transformation T exists such that for $\kappa = T\xi$.

$$\varepsilon_v = F(\kappa) \qquad (6)$$

$$u/\sigma'_{vo} = G(\kappa) \qquad (7)$$

The parameter κ is called the damage parameter. The transformation T and the functions F and G will be obtained using the data in Figs. 1 and 5.

The volumetric strains generated in cyclic simple shear at various constant strain amplitudes ranging from γ=0.05% to γ=0.3% are shown in Fig. 1 versus the number of load cycles N. The same data is shown in Fig. 2 plotted against the logarithm of the length of the strain path, ξ. We now seek a transformation T which when applied to ξ will collapse the curves for different strain amplitudes into one curve. It is reasonable to assume that if such a transformation exists it will be a function of γ.

Consider a volumetric strain ε_{v1} occurring at ξ_1 for a shear strain amplitude γ_1 and at ξ_2 for a shear strain γ_2. Can ε_{v1} be associated with the value κ_1 of a variable κ such that

$$\kappa_1 = T\xi_1 = T\xi_2? \qquad (8)$$

Consider

$$T = e^{\lambda\gamma} \qquad (9)$$

$$\kappa_1 = \xi_1 e^{\lambda\gamma_1} = \xi_2 e^{\lambda\gamma_2}$$

$$e^{\lambda(\gamma_1 - \gamma_2)} = \xi_2/\xi_1$$

or

$$\lambda = \ln(\xi_2/\xi_1) \;/\; (\gamma_1 - \gamma_2) \qquad (10)$$

For F and G to exist for the assumed form of T the constant λ must have a unique value for all corresponding ε_v, γ and ξ. When Eq. 10 is applied to the data in Fig. 2 a range of values for λ was determined with a mean-value of $\lambda=5.7$. Using this value of λ the data points in Fig. 2 were transformed to κ-space in Fig. 3. It may be seen that the points do not define a unique curve but a narrow band. A non-linear least squares curve fitting method was used to define the curve in Fig. 3, describing the relationship between volumetric strain and the logarithm of κ. The relationship is given in Fig. 4 to a natural scale and defines the function $F(\kappa)$ as

$$F(\kappa) = \kappa(D\kappa + C) \;/\; (A\kappa + B)$$

with

$$A = 0.07,\; B = 0.004,\; C = 0.071 \text{ and } D = 0.138$$

A similar procedure is applied to the pore-water pressure data in Fig. 5. The transformation to ξ-space is shown in Fig. 6 and the transformation to κ-space in Fig. 7 (logarithmic scale) and Fig. 8 (natural scale). The function $G(\kappa)$ defining pore-water pressure is the same as for F except that

$$A = 0.066,\; B = 0.00066,\; C = 0.062 \text{ and } D = 0.154$$

The transformation constant λ in this case is $\lambda=3.56$.

A mean-value of λ that will very closely transform the test data to a curve in κ-space without further manipulation is obtained by determining many values of λ over the whole range of likely interest before taking the mean. We determine λ in what we consider the most relevant range of earthquake strains: $0.05 \leq \gamma < 0.3\%$ and $0 \leq \varepsilon_v \leq 1\%$. The broader the strain range the more difficult to select a single satisfactory value of λ by the above procedure. If adjustments in λ are required a useful procedure is to perform the inverse transformation to ξ-space and note the regions of greatest divergence from experimental data. In these regions determine additional values of λ. These additional values will then weight the new mean value of λ in a way that will improve the representation in the areas of inferior definition.

DISCUSSION AND CONCLUSION

The data discussed above is representative of data from six different sands. Therefore, it seems clear that plastic volumetric strains and pore-water pressures generated in saturated sands due to seismic excitation can be expressed as continuous monotonically increasing functions of a damage parameter κ which is linked with the dynamic response of the ground through shear strain γ and the accumulated length of the shear strain path ξ. Thus, the increments in pore-water pressure or volumetric strain associated with the current value of κ can be readily included in a method of dynamic effective analysis such as that presented by Finn, Lee and Martin (1977).

The determination of pore-water pressure at any point in the dynamic analysis does not require a knowledge of the highly non-linear rebound modulus of the sand skeleton. This obviates the need to measure this parameter and reduces the number of constants that must be determined from seven to four. However, a price must be paid for these advantages; the endochronic formulation is computationally somewhat less efficient.

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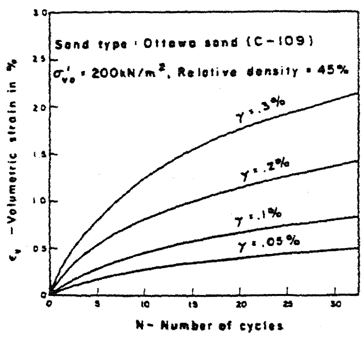

Fig. 1 Experimental data on volumetric strain.

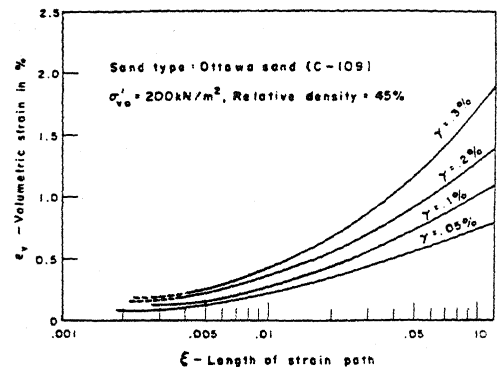

Fig. 2 Volumetric strains vs. length of strain path.

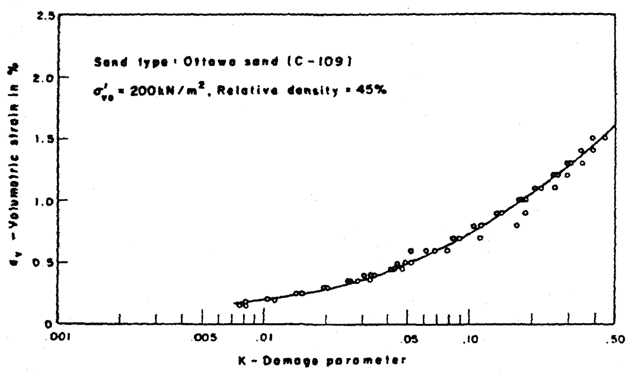

Fig. 3 The volumetric strain function, $\varepsilon_v = F(\ell n\ \kappa)$.

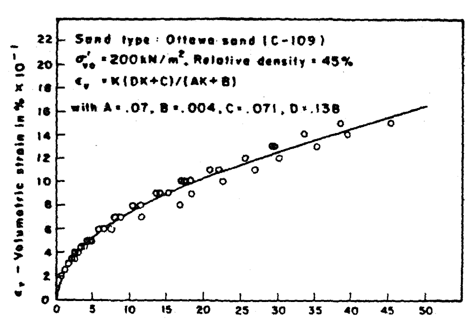

Fig. 4 The volumetric strain function, $\varepsilon_v = F(\kappa)$.

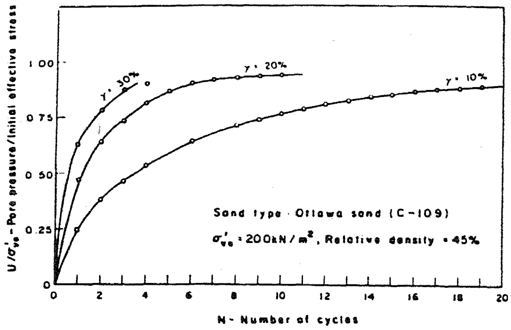

Fig. 5 Experimental data on pore-water pressures.

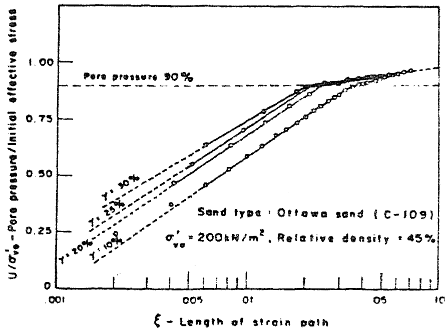

Fig. 6 Pore-water pressures vs. length of strain path.

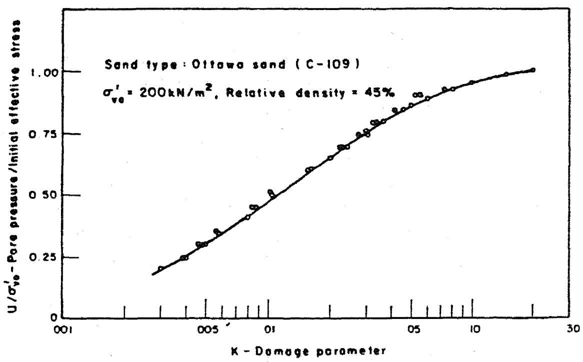

Fig. 7 The pore-water pressure function, $\frac{u}{\sigma'_{vo}} = G(\ell n\ \kappa)$.

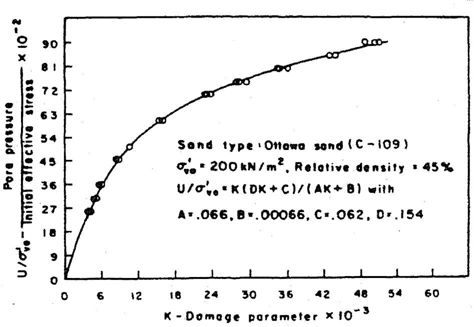

Fig. 8 The pore-water pressure function, $\frac{u}{\sigma'_{vo}} = G(\kappa)$.