

LIQUEFACTION ANALYSIS OF HORIZONTALLY LAYERED SAND
CONSIDERING A Laterally Confined Condition

by
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SUMMARY

The dynamic response of horizontally layered soil is numerically analyzed, using a stress-strain relation of sand and the equation of motion of two phase mixture. The saturated soil is treated as an elastic-plastic solid-fluid mixture. As the pore water and the soil particle are assumed to be incompressible, the effective stress analysis becomes useful. Since the laterally confined condition of level ground is taken into account, the stress path which is peculiar to liquefaction can be obtained.

INTRODUCTION

Many experimental studies of liquefaction were carried out after Niigata earthquake of 1964. The Seed et al. proposed the method to estimate the condition when liquefaction occurs, based on the experimental results. But, Seed's method has a defect that the reliability of the analysis to get the shear stress is low. To improve this defect, it becomes popular to analyze the liquefaction using the stress-strain relation of soil directly. In the following study, the liquefaction phenomenon is analyzed, using the stress-strain relation of sand and the equation of motion for two phase mixture.

STRESS-STRAIN RELATION OF SAND

The elastic layers are treated as a linear elastic body. The saturated sand layers are modelled by the modified Nishi et al.'s¹⁾ elastic-plastic constitutive equation of sand. According to Ishihara et al.²⁾, there exists a critical value of stress ratio. Once the stress ratio more than this critical value is applied to the sand sample under the undrained condition, the excessive pore water pressure increases during the subsequent unloading and decreases during the loading. The angle defined by the critical value is called an angle of phase transformation. In order to express this behavior mentioned above, Nishi et al.'s equation is modified.

Nishi et al.'s original elastic-plastic constitutive equation of sand is as follows.

$$d\epsilon_{ij} = \frac{\kappa}{1+e} \frac{1}{\sigma'_m} d\sigma'_m + \frac{1}{2G} ds_{ij} + h_s \frac{\partial g_s}{\partial \sigma'_{ij}} df_s + h_c \frac{\partial g_c}{\partial \sigma'_{ij}} df_c \quad (1)$$

$$h_s = \frac{3\sigma'_m}{2G'} \left(\frac{M_f - \tau_{oct}/\sigma'_m}{M_f - \tau_{oct}/\sigma'_m} \right)^2, \quad h_c = \frac{\lambda - \kappa}{1+e} \frac{1}{\sigma'_m}$$

Yield Condition

For shear deformation $f_s - f_{sy} = \tau_{oct}/\sigma'_m - (\tau_{oct}/\sigma'_m)_y = 0 \quad (2)$

For consolidation $f_c - f_{cy} = \sigma'_m - \sigma'_{my} = 0 \quad (3)$

Plastic Potential

For shear deformation $g_s = \tau_{oct}/\sigma'_m + M \ln \sigma'_m \quad (4)$

For consolidation $g_c = \ln \sigma'_m \quad (5)$

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where σ'_m : effective mean stress, γ_{oct} : octahedral shear stress, s_{ij} : deviatoric stress tensor, κ : swelling index, λ : consolidation index, e_{ij}^v : void ratio, G : elastic shear modulus, γ_{oct}^p : plastic octahedral shear strain.

In order to introduce the behavior after the phase transformation as discussed above, the following assumption is made. The stress ratio defined by the angle of phase transformation is equal to M_m . Once the stress ratio reaches M_m , the stress path during the subsequent unloading is expressed by the straight line $\tau_{oct}/\sigma'_m = M'$. Then, under the undrained condition,

$$d\sigma'_m = (1/M') d\tau_{oct} \quad (6)$$

Resulting from Eq.(6), the following relation should hold.

$$d\epsilon_{kk}^s = \frac{-K}{(1+e)\sigma'_m} d\tau_{oct} \quad (7)$$

where $d\epsilon_{kk}^s$ is a plastic volumetric strain increment due to the shear deformation.

The following failure conditions are introduced.

$$\tau_{oct}/\sigma'_m = M_f, \quad |\epsilon_{12}^p| \geq 0.05, \quad \sigma'_m \leq 0.05\sigma'_m(0) \quad (8)$$

After failure, the constitutive equation of soil is replaced by the following bi-linear stress-strain relation.

$$\sigma'_{12} = 2G\epsilon_{12} \quad (|\sigma'_{12}| < \sigma'_{12y}) \quad \sigma'_{12} = 2G\epsilon_{12} \quad (|\sigma'_{12}| \geq \sigma'_{12y}) \quad (9)$$

LIQUEFACTION ANALYSIS OF GROUND

The one-dimensional approximated governing equation are given as follows, which was reported by one of the authors³⁾

$$\frac{\partial \sigma'_{21}}{\partial x_1} = \rho \frac{\partial v_2}{\partial t} \quad (10) \quad \frac{1}{2} \frac{\partial U_2}{\partial x_1} = -\epsilon_{12} \quad (11)$$

where v_i is a component of velocity vector, ρ is a density and U_i is a component of displacement vector.

From the equation of mass conservation and the equation of motion for fluid phase, neglecting the acceleration term, the following equation is obtained.

$$\frac{\partial^2 u}{\partial x_1^2} = -\frac{\rho f g}{k} \frac{d\epsilon_{kk}}{dt} \quad (12)$$

where u is a pore water pressure, ρ^f is a specific density of water, k is a permeability coefficient and g is a gravitational acceleration.

From the lateral deformation confined condition, $d\epsilon_{22}^p = d\epsilon_{33}^p = 0$.

$$\text{Therefore, } \frac{d\sigma'_{11}}{dt} = -K \left(\frac{k}{\rho f g} \frac{\partial^2 u}{\partial x_1^2} + \frac{d\epsilon_{kk}^s}{dt} \right) - 4G \left(\frac{1}{3\rho f g} \frac{\partial^2 u}{\partial x_1^2} - \frac{d\epsilon_{33}^p}{dt} \right) \quad (13)$$

$$\frac{d\sigma'_{33}}{dt} = -K \left(\frac{k}{\rho f g} \frac{\partial^2 u}{\partial x_1^2} + \frac{d\epsilon_{kk}^s}{dt} \right) + 2G \left(\frac{1}{3\rho f g} \frac{\partial^2 u}{\partial x_1^2} - \frac{d\epsilon_{33}^p}{dt} \right) \quad (14)$$

where $d\epsilon_{ij}^p$ is a plastic deviatoric stress tensor and $K = \frac{(1+e)\sigma'_m}{m}$.

From the above discussions, Eqs.(1),(7),(9),(10),(11),(12),(13) and (14) must be solved simultaneously in the calculations. The finite difference method is used for calculation combined with the method of characteristics.

NUMERICAL RESULTS

The soil properties for computation is similar to Niigata site where the hard damage occurred, used by Seed et al.⁴⁾ The distribution of relative density and G are shown in Fig.1. The ground water level is at the depth of 2 m. The base rock under the depth of 30 m is treated as the elastic body. The geometrical dissipation is also taken into account. k is determined by the formula $k=Ce^3/(1+e)$ (C :constant). The shear modulus G is proportional to $\sqrt{\sigma'_m}$. The fixed parameters used for the calculation are as follows.

$$k=0.0001 \text{ m/sec}, M_m=0.64, M_f=0.739, M'=1.279, \kappa=0.003, e_{\min}=0.634, \\ e_{\max}=0.991, \Delta x_1=1 \text{ m}, \Delta t=0.0025 \text{ sec}, \bar{q}=5 \text{ kg/cm}^2, \sigma'_{12y}=0.05 \text{ kg/cm}^2.$$

The first 18 sec of the S69E component of accelerogram at Taft during 1952 is used as a vertically incident wave by transforming into the velocity record. The amplitude of velocity used for computation is 0.5 times as that of the original velocity record.

Fig.2 shows the distribution of the excessive pore water pressure for $G'=400$ (case A) and $G'=500$ (case B). The developed excessive pore water pressure in case A is more than that in case B. The shear failure zone in case A is between -9 m and -16 m. In Fig.3, the response of shear stress, shear strain and pore water pressure at a depth of 10 m in case A is presented. In Fig.3, the liquefaction occurs at 9.8 sec, and after that the magnitude of shear stress becomes small. The shear strain increases rapidly when the liquefaction occurs and converges to a certain value, whereas, the excessive pore water pressure gradually increases as time goes on and converges to a constant value. The effective stress path at a depth of 10 m is designated in Fig.4. The figure denotes the pattern which is peculiar to liquefaction, where σ'_m decreases with the decrease of τ_{oct} and the stress path turns to the origin. Fig.5 shows the variation of σ'_{11} and σ'_{33} with time for case A. From this figure, it is seen that $\sigma'_{11}/\sigma'_{33}$ increases during liquefaction by introducing the laterally confined condition.

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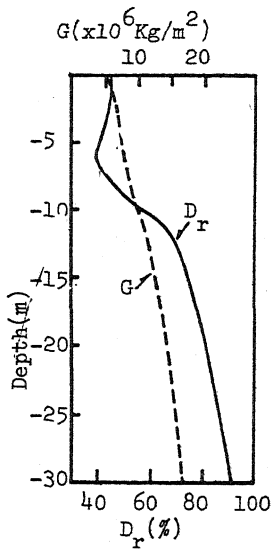


Fig.1 Distribution of G and D_r

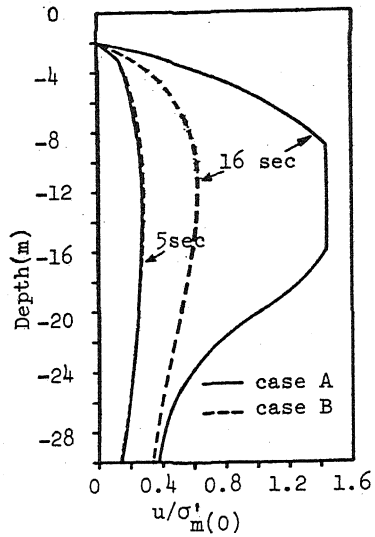


Fig.2 Variation of Pore Water Pressure

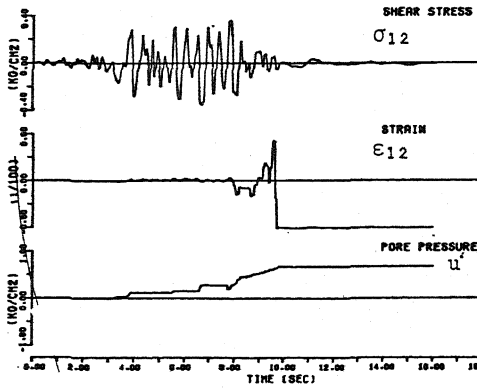


Fig.3 Time Histories of σ_{12} , ϵ_{12} and u

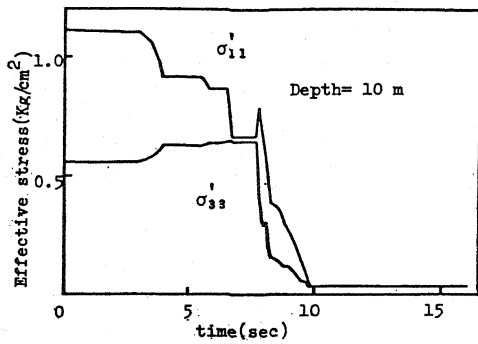


Fig.5 Time histories of σ'_{11} and σ'_{33}

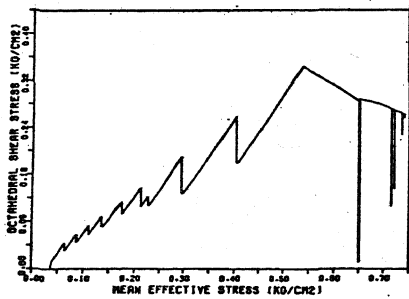


Fig.4 Stress path