A METHODOLOGY FOR LOCATING STRONG MOTION ARRAYS

by

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ABSTRACT

This paper investigates a methodology for locating strong motion accelerographs in a seismically active region. Starting with the probability density of earthquakes in a given region, the paper attempts, within the framework of optimization theory, to formulate the following two questions: (1) given N accelerographs, where should they be located in a seismically active zone, and (2) having fixed these N accelerographs, where should the next M be located?

Specific application of the method to the locationing of twenty strong motion instruments in a seismically active area has been carried out.

INTRODUCTION

The increase in information obtained by laying out a strong motion network can be thought of as being accomplished in two possible ways. The first is by an appropriate design and distribution of the various sensors and the second is by the use of improved data processing techniques. Whereas a large amount of effort to date [1] has been invested in the latter activity, there are very few studies that have been made on the design of seismic networks [2].

In this investigation into the development of a methodology, we have concentrated on the information gathering function of acquiring records through the development of three simple cost functions. Optimization techniques are used to determine configurations of the arrays which minimize these cost functions. The first cost function requires that the sum of the distances of the sensors, on an average, from the epicenters be as small as possible. The second requires that the configuration be so selected as to maximize the average value of the recorded peak acceleration for the whole array. The third strategy lays out the arrays so as to maximize the probability of exceeding a given acceleration level, in a given period of time, at at least one sensor location.

These cost functions are but a few which could be used in arriving at optimal sensor locations for seismic arrays. The reason for these choices can be heuristically put forth as follows: the first approach attempts to acquire data on close-in ground shaking -- data which is particularly scarce in present day strong motion data banks. This criterion may also serve for maximizing information (through parametric identification) on the source mechanism, provided, of course, that the instrument locations so arrived at are not so close to the epicenter that nonlinear effects (for which adequate analytical models do not exist) dominate. The second approach attempts to acquire a high yield of accelerograms from a seismic instrumentation network, while the third may be used to focus on specifically large amplitude ground motion records by adjustment of the acceleration level chosen. The three criteria so developed are used to illustrate the optimal sensor patterns in a local region, under the inposition of

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realistic areal and intersensor distance constraints.

FORMULATION OF THE MODEL

Before one can settle on the "optimal" configuration of the various sensors, one needs to decide exactly what it is that makes one configuration better than another. This would involve (1) ascribing a cost function which serves as an index of the performance of any two alternative configurations, and (2) settling on a criterion for obtaining the "best" configuration. The ultimate configuration arrived at would naturally depend on the nature of the information that one would want to extract from the records obtained at those specific locations [4].

In this section, the three cost functions mentioned earlier will be developed in a form suitable for the application of optimization techniques.

(a) Criterion I: Placement of Sensors to Minimize the Average Epicentral Distance: Consider a two-dimensional region in which the probability of occurrence of an earthquake epicenter at location (x,y) is defined as p(x,y). Let (x_e,y_e) be the epicentral location of a given earthquake, (x_{s_i},y_{s_i}) the coordinates of sensors placed on the ground surface and, r_{e,s_i} the distance between the earthquake epicenter and the sensor location s_i. The expected value of this distance r_{e,s_i} for sensor s_i is given by

$$c_i = \int_\Omega p_e(x,y) r_{e,s_i} \, d\Omega \tag{1}$$

Furthermore, if we have N sensors, s_i, $i=1,\ldots,N$, then the sum of the expected values of the distance to the epicenters from all the sensors becomes $C = \Sigma c_i$. The function C is the sum of the average distance that each sensor would be from an epicenter. Thus, in order to locate the sensors as close as possible to the epicenters, we need to minimize the function C over all possible coordinate positions of the N sensors s_i. The optimal configuration would then be obtained by finding the minimum value of C for all pairs (x_{s_i},y_{s_i}), $i=1,\ldots,N$. The minimization that is indicated would try and find the locations of sensors that would be closest to the epicenters in an "average" sense.

However, we know, on physical ground, that two sensors when placed extremely close to each other provide redundant data. Though it is not entirely obvious that redundancy in data is necessarily disadvantageous (especially in view of possible instrument malfunctions during earthquakes), we shall assume that with the recent advances in the reliability of instruments it would not be cost effective to place more than one instrument in the same location. Furthermore, if one is interested basically in acquiring information up to frequencies f_u involving wavelengths which are greater than λ_L, then the placement of instruments much closer than $\lambda_L/10$ would, in good measure, yield redundant data. For frequencies of interest (up to 20-30 cps) and wave speeds of the order of 1-5 km/sec, λ_L could range from tens of meters to tens of kilometers. Thus, in addition to the minimization criterion, a minimum distance between any two sensors, predicated on the highest wavelength of interest, may be stipulated. Thus, the optimal configuration would correspond to minimizing C, $\forall(x_{s_i},y_{s_i})$, $i=1,\ldots,N$, under the constraints $r_{s_i,s_j} \geq \Delta$, i, $j,\ldots,N$. The constraint inequalities provide $\binom{N}{2}$ independent relations that must be satisfied during the minimization of C.

On assuming that the earthquake epicenters occur only along some known, J, lineal features (faults), then relation (1) reduces to

$$c_i = \sum_{j=1}^{J} c_{i_j} = \sum_j \int_{f_j} P_{f_j}(s) r_{f_j, s_i} \, ds \quad (2)$$

where s is a dummy variable indicating position on the fault f_j, and $P_{f_j}(s)$ is the weighting function relating to the probability of occurrence of an event on fault f_j at location, s, along it. If the J faults, can each be thought of as being straight lines and the function $P_{f_j}(s)$ on each fault f_j to be linear in s, then the c_{ij}'s can be calculated in closed form [3].

The minimization problem can then be stated as minimizing Σc_{i_j} for all (x_{s_i}, y_{s_i}) under the constraint $r_{s_i,s_j} \geq \Delta$, i,j=1,...,N where c_{ij} is defined in Eq. 2.

(b) Criterion II: Placement of Sensors so as to Maximize the Average Peak Acceleration: Realizing that instrument triggering is dependent upon the level of acceleration recorded at a station, we require, in this criterion, to put instruments at such stations that the expected value of the accelerations recorded thereat is a maximum. Several researchers [4] have empirically tried to fit different attenuation functions to the data recorded during strong ground shaking. A commonly used expression for the peak acceleration, a, is

$$\ell na = \ell nA + \alpha M - \gamma \ell n(R + \beta) \quad (3)$$

where A, α, γ and β are constants, M is the earthquake magnitude and R is the epicentral distance to the recording station. Typical values found for the parameters A, α, γ and β are in the vicinity of 12,000, .5, 25 and 1.2 to yield peak accelerations, a, in mm/sec^2. The uncertainties involved, in the generation and transmission of the seismic energy from source to a specific receiver station placed within a certain area, would imply that these parameters could perhaps best be thought of as being random variables. We assume that each of them is a uniformly distributed independent random variable and that, for simplicity, the variances of these random variables depend only on the location of the sensor s_i and the epicenter (x_e, y_e). The probability densities $P_A(A)$, $P_\alpha(\alpha)$, $P_\gamma(\gamma)$, and $P_\beta(\beta)$ are assumed uniform between A_1 and A_2, α_1 and α_2, β_1 and β_2, γ_1 and γ_2, respectively. Also, the probability density of occurrence of an event of magnitude m given that it occurs within an infinitesimal area $d\Omega_n$ surrounding a point (x_n, y_n) may be expressed as

$$P_M(m; x_n, y_n) = k(x_n, y_n) \exp[-\delta(x_n, y_n) m] \quad (4)$$

The constant $k(x_n, y_n)$ is a normalization factor chosen so that the area under the density is unity. This yields $k(x_n,y_n)=\delta[\exp(-\delta m_L(x_n,y_n)) - \exp(-\delta m_u(x_n,y_n))]^{-1}$ where $m_L(x_n,y_n)$ and $m_u(x_n,y_n)$ constitute the upper and lower magnitude bounds for events occurring in $d\Omega_n$.

With these parameters, the expected value of the logarithm of the peak acceleration recorded at sensor s_i given that an earthquake has occurred at (x_n, y_n) in $d\Omega_n$ is obtained:

$$E[\ell na \text{ at } s_i | \text{earthquake of magnitude m occurs in } d\Omega_n]$$
$$= \bar{A} + 0.5(\alpha_1 + \alpha_2) - 0.5(\gamma_1 + \gamma_2)\{[F(R,\beta_1) - F(R,\beta_2)]/(\beta_2 - \beta_1)\}$$

where

$$\bar{A}(x_n, y_n; s_i) = [(A_2 \ell nA_2 - A_1 \ell nA_1) - (A_2 - A_1)]/(A_2 - A_1)$$

and

$$F(R, \beta_k) = (R + \beta_k)\ell n(R + \beta_k) - (R + \beta_k); \quad k = 1, 2. \quad (5)$$

We note that A_k, α_k, β_k, and γ_k, k=1,2, are all functions of s_i, x_n, and

y_n. Averaging over all magnitudes m from $m_u(x_n,y_n)$ to $m_L(x_n,y_n)$ we get the expected value of $\ell n(a)$ at sensor location s_i, given that earthquakes occur in $d\Omega_n$ as:

$$e(x_n,y_n;s_i) = \bar{A} - 0.5\{[F(R,\beta_2) - F(R,\beta_1)](\gamma_1 - \gamma_2)/(\beta_2 - \beta_1)\}$$
$$- 0.5(\alpha_1 + \alpha_2)k\delta^{-1}[(m_L + \delta^{-1})\exp(-\delta m_L) - (m_u + \delta^{-1})\exp(-\delta m_u)] \quad (6)$$

Thus, the expected value of the logarithm of the peak acceleration at recording sensor s_i is

$$e_i = \int_\Omega e(x,y;s_i)p(x,y)d\Omega. \quad (7)$$

If the earthquake epicenters can be assumed to occur only along some known faults, then relation (7) can be further simplified in a manner similar to that done before, and the integration done over the various faults. Furthermore, to acquire records which have an acceptable signal to noise ratio, one possible criterion for locating sensor s_i would be to maximize the ratio of the expected value of the recorded accelerations relative to the minimum sensitivity of the instrument. In this sequel, this minimum acceleration sensitivity level, a_t, is, for brevity, referred to as "the triggering threshold acceleration level." As $\ell n(a)$ is a monotonic increasing function of a, we may consider the maximization of the expected value of the logarithm of the ratio of 'a' to 'a'_t'. Thus e_i could be scaled with respect to the triggering level a_t to give $e_i^s = e_i - \ell n(a_t)$. And hence, the sensor s_i could be located to maximize e_i^s. If all N sensors of the same variety are to be deployed, then $I = \sum_{i=1}^{N} e_i^s$ needs to be maximized under the constraint that no two sensors are closer than a predetermined minimum distance. The minimization problem can then be stated as minimizing $[-I]$, $\forall(x_{s_i},y_{s_i})$, under the constraints $r_{s_i,s_j} \geq \Delta$, $i,j=1,\ldots,N$.

(c) Criterion III: Locating Instruments so that the Probability of the Recorded Acceleration Exceeding a Given Threshold is a Maximum in a Given Period, T, of Time: Let us assume that the instrument array has a functioning life of T years and that it is to be deployed in a region Ω where the number of earthquakes per T years is ν_o, and the spatial probability of earthquake occurrence in the elemental areas, $d\Omega_n$, located at (x_n,y_n) is $p(x_n,y_n)d\Omega_n$. The probability that the logarithm of the peak acceleration exceed a threshold value L_t at a recording site $s_i(x_{s_i},y_{s_i})$ when an earthquake occurs in an infinitesimal area $d\Omega_n$, centered around the point (x_n,y_n) be given by: $P[\ell n(a)>L_t \text{ at } s_i | \text{an event occurs at location } (x_n,y_n) \text{ in } d\Omega_n] = q(x_n,y_n)d\Omega_n$. The function q may be thought of as either obtained on the basis of empirical data or derived from suitable sormulation.

If $\nu(x_n,y_n)d\Omega_n$ is the average rate of occurrence of earthquakes in the infinitesimal area $d\Omega_n$ around the point (x_n,y_n), then under the assumption that the events occur in time according to a Poisson distribution,

$$P[\ell n(a)>L_t \text{ at si at least once in T years} | \text{events occur in } d\Omega_n]$$
$$= 1 - \exp[-q(x_n,y_n;s_i)\nu(x_n,y_n)Td\Omega]. \quad (8)$$

This gives

$$P[\ell n(a)>L_t \text{ at } s_i \text{ at least once in T years} | \text{events occur in } \Omega]$$
$$= t_i = 1 - \exp[-\int_\Omega q(x,y;s_i)\nu(x,y)Td\Omega] \triangleq 1 - \exp(-h_i) \quad (9)$$

Thus, the sensor location s_i needs to be so chosen that for a given L_t, t_i is maximized. This would require the maximization of h_i over all possible sensor locations.

The above integral may be physically interpreted as the average number of times that $\ell n(a)>L_t$, in the time period T, given that earthquakes occur in Ω and that the occurrence rate per unit area at point (x,y) is $\nu(x,y)$.

If several sensors are to be located, it would be reasonable to place the sensors in such a configuration that the sum of the average number of times that $\ell n(a)>L_t$, in the time period T, taken over all the instrument locations is a maximum [3]. The minimization problem, if N sensors are deployed in Ω, then becomes

$$-\sum_{i=1}^{N} h_i, \ \forall(x_{s_i},y_{s_i}), \ i=1,2,\ldots,N \tag{10}$$

under the constraints $r_{s_i,s_j}\geq\Delta$, $i,j=1,2,\ldots,N$ where $h_i=\int_\Omega q(x,y;s_i)\nu(x,y)d\Omega$.

In the following analysis, mainly for illustration, a simple form of $q(x,y;s_i)$ will be chosen. Using the attenuation relation (3), assuming that A, α, β and γ are constants and Eq. 4, h_i can be evaluated [3].

CONSTRAINT CONDITIONS

In the preceding discussion, it was assumed that prior to the laying out of the instrument array in the region Ω, no instruments were deployed therein. More often, one is faced with the problem of optimally locating a set of N instruments in an area where M instruments have already been installed. The minimization that leads up to the optimal locations of the N instruments, in that case, remains the same except that the "minimum distance constraints" increase in number.

Another constraint which one may foresee, arises from the practical difficulty of locating instruments in certain areas which for some reason are not suitable for their placement. Additional inequality constraints would be required of the form, $g_i(x,y)\geq 0$ $i=1,\ldots,N_a$ where N_a is the number of such areal constraints. For convenience in formulation, g_i has been chosen to be the form $g_i(x,y)=a_ix+b_iy+c_i$.

PROBLEM FORMULATION

Under these constraints, the optimal sensor location problem posed by either of criteria I, II, or III can now be generally formulated.

Let $\vec{X}$ denote the row vector of all sensor locations so that $\vec{X}=\{x_1,y_1,x_2,y_2,\ldots x_N,y_N|x_{N+1},y_{N+1},\ldots x_{N+M},y_{N+M}\}=\{\vec{X}^m|\vec{X}^f\}$ where $\vec{X}^m$ is the vector of position coordinates of all the instruments that have to be optimally located and $\vec{X}^f$ the vector containing the position coordinates of all the fixed sensors.

Then we need, for a suitably defined functional $F(\vec{X})$, to find the $\mathrm{Min}\{F(\vec{X})\}$, $\forall\vec{X}^m$ under the constraints $u_k(\vec{X})\geq 0$, $k=1,2,\ldots,N_d$, $g_k(\vec{X})\geq 0$, $k=1,2,\ldots,N_a$ where $u_k(\vec{X})\geq 0$ are the "distance constraints" which have the form $r_{s_i,s_j}-\Delta\geq 0$ and, $g_k(X)\geq 0$ are the "areal constraints" which have the form $a_kx_i+b_ky_i+c_k\geq 0$ for all sensor locations (x_i,y_i). The number N_d, of distance constraints with M fixed sensors is $\binom{N+M}{2}-\binom{N}{2}=N(N-1)/2+MN$.

AN EXAMPLE STUDY FOR MONITORING LOCAL EARTHQUAKES

The pattern of sensor locations obtained by minimizing each of three abovementioned cost functions is next illustrated through an application to a small region. Nonlinear programming techniques were used to arrive at the optimal array designs [3]. Fig. 1(a) is a map of a region showing some of the major faults together with a small rectangular box (areal constraints) wherein it shall be assumed that sensors are required to be placed. Furthermore, it shall be assumed that future events will occur on these faults and that the probability of earthquake occurrence on each of these faults is related to the average slip rate that has occurred in the last, say 10^3 to 10^7 years, depending on the individual fault [5]. For convenience, the values of the A's, α's, β's, and γ's are taken to be constants over the entire rectangular area of concern. These constants together with a short description of each fault have been shown in Table I. All the faults have been idealized by straight line segments. Twenty sensors are assumed to be located with a minimum intersensor distance of 15 km between each of them. Starting with the initial "guess" locations shown in Fig. 1(a), the three objective functions were used to arrive at the optimal locations, using the SUMT algorithm. The triggering threshold acceleration for the instruments was taken to be 100 mm/sec^2. Figures 1(b), and 1(c) show the emergent patterns. The resulting patterns for the first two criteria look similar and were found to be stable (almost independent of initial "guess" locations). A markedly regular pattern seems to emerge. Each three adjacent sensors form almost a triangle. The pattern created by Criterion III, however, shows the sensors to be more nearly centered along the faults themselves, as opposed to those created by Criteria I and II.

For comparison purposes, a "random array" has also been shown (Fig. 1(c)). The sensor coordinates are taken to be uniformly distributed independent random variables, each sensor location being a minimum of Δ = 15 km from every other location. The emergent pattern of such a random array is quite different from the pattern produced through the minimization of any of the three cost functions.

Lastly, a situation where ten 'additional' sensors are to be located, in the presence of ten extant ones, is illustrated. Using the data of Table I, optimal sensor locations are obtained using the fixed sensor locations (indicated by ◇) and the initial guess locations shown in Fig. 2(a). The results are shown in Figs. 2(b) - (c). The minimum intersensor distance used is 15 km. The presence of the fixed sensors creates a more even spread of the moveable sensors over the rectangular area.

CONCLUDING REMARKS

This paper represents only an initial effort in the development of a methodology for optimal "free field" instrument deployment. The method utilizes the minimization of the expected value (over the entire areal domain) of a suitably defined functional, F, that characterizes ground shaking at a point.

The optimization techniques used, systematize the solution of the optimal sensor locationing problem vis-a-vis the objective function which is to be minimized. The methods utilized here do not constitute a replacement for intuition and good judgement in locating sensors in large seismic arrays. When the number of sensors increase, the nonuniqueness in the problem increases; different initial sensor location guesses lead to different optimal locations. It is in the selection of these initially guesses sensor location that good judgement needs to be brought to bear. Studies such as these lead to an appreciation of the effect of choosing various functionals, F, on the optimal sensor locations, thus developing an "intuitive feel" for the array design problem. Perhaps one of the major advantages of this formulation is its ability of showing the effect of changes in various parameters on the optimal sensor location patterns.

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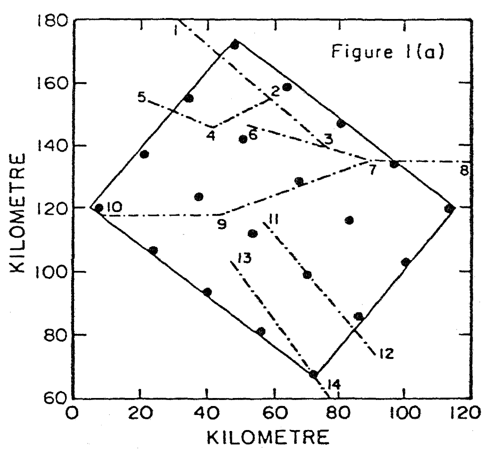
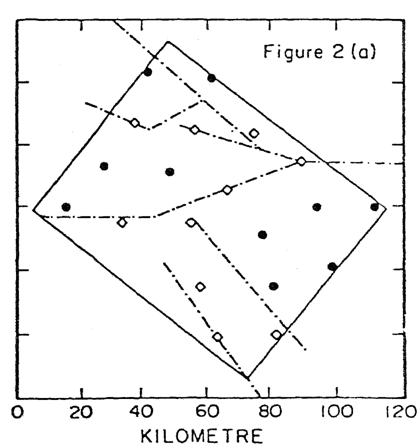
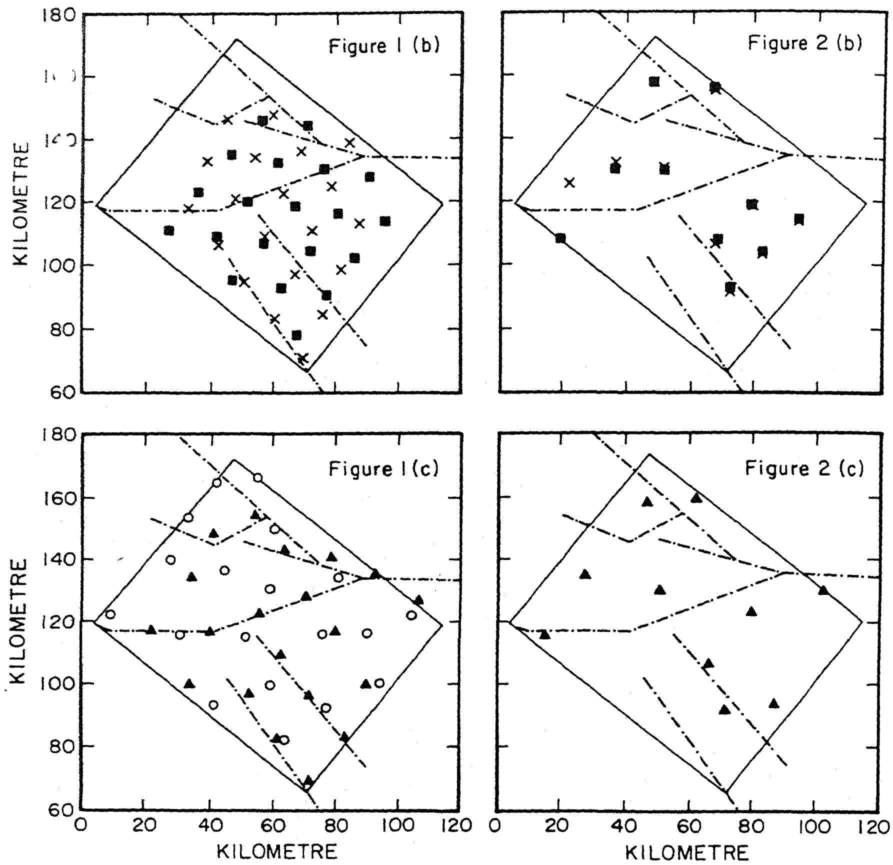
TABLE I

Fault No.	Segment	Weighting	M_L	M_u	δ
I	1-2-3	0.02	4.0	6.5	2.0
II	2-4	1.00	4.0	6.5	2.0
III	4-5	1.00	4.0	6.5	2.0
IV	6-7	0.60	4.0	6.5	2.0
V	7-8	0.60	4.0	6.5	2.0
VI	7-9	0.50	4.0	6.5	2.0
VII	9-10	0.50	4.0	6.5	2.0
VIII	11-12	0.60	4.0	6.5	2.0
IX	13-14	1.00	4.0	6.5	2.0

$A = 12500.0$, $\alpha = 0.5$, $\beta = 25.0$, $\gamma = 2.0$
$A_1 = 12000.0$, $\alpha_1 = 0.4$, $\beta_1 = 20.0$, $\gamma_1 = 1.8$
$A_2 = 13000.0$, $\alpha_2 = 0.6$, $\beta_2 = 30.0$, $\gamma_2 = 2.2$







Legend: Initial guess locations ●

Fixed sensors ◇

Optional locations using:

Criterion I ■

Criterion II ×

Criterion III ▲

Random placing ○