# STUDY OF THE CHARACTERISTICS OF STRONG MOTION FOURIER SPECTRA ON BEDROCK

by

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## SUMMARY

In order to qualitatively study the Fourier spectral characteristics of strong motion earthquake records on bedrock, the Fourier spectra of 17 accelerograms ($4.3 \leq M \leq 7.1$), which were recorded in Japan, were analyzed.

The result of this study suggests that the averaged acceleration Fourier amplitude spectra on bedrock level can be considered to be nearly flat statistically during the periods 0.1 sec and $T_m$ (= $10^{0.39M-1.7}$) sec. This qualitative result agrees well with the characteristics of theoretical spectra calculated by propagating fault model. The period $T_m$ and the corner frequency (Savage, 1972) [3] also agree well for magnitude $M \geq 6.0$.

These results also indicate that the averaged Fourier spectra of accelerograms observed at a given site should be similar in shape to the transfer function of that site.

## INTRODUCTION

The evaluation of characteristics of strong motion Fourier spectra is an important problem for earthquake-resistant design of structures. The methodologies of studying the spectrum can be classified into two groups; (1) the theoretical studies based on propagating fault model and (ii) the empirical studies based on the analysis of observed strong motion accelerograms.

In order to recognize the general characteristics of strong motion spectrum in the short period range ($T < 1.0 \sim 2.0$ sec), it is necessary to study the averaged strong motion acceleration spectrum on a bedrock level and then comparing them with theoretically derived fault plane solution.

In this paper, the spectra of 17 accelerograms were analyzed and compared with the theoretical Fourier spectrum (Savage, 1972) (Fig. 1).

## EARTHQUAKE DATA

The earthquake data of 17 events analyzed are shown in Table 1. The set of records obtained at two sites in Japan show a wide range of magnitude ($4.3 \leq M \leq 7.1$), focal depth ($0 \leq h \leq 130$ Km), epicentral distance ($30 \leq R \leq 505$ Km) and site conditions ($V_s$ = 1.3 Km/sec, for A1; $V_s$ = 1.6 km/sec, for B1). The two stations; A1 and B1, and the locations of epicenters are shown in Fig. 2.

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## METHODOLOGY FOR ESTIMATING THE FOURIER SPECTRA ON BEDROCK

In order to get the Fourier amplitude spectra on bedrock level the effect of surface layer should be filtered. To estimate the individual Fourier spectrum of events, the following procedure was used in this paper.

The frequency domain relationship between input and output for a stationary signal and linear system is given by

$$Y_i(w) = H(w) \cdot X_i(w) \tag{1}$$

where, $Y_i(w)$ and $X_i(w)$ are the Fourier spectra of output and input of the system and $H(w)$ is the transfer function of the system.

The averaged spectrum obtained at the same site is

$$\frac{1}{n}\sum_{i=1}^{n} Y_i(w) = H(w) \cdot \frac{1}{n}\sum_{i=1}^{n} X_i(w) \tag{2}$$

If the term, $\frac{1}{n}\sum_{i=1}^{n} X_i(w)$, is statistically constant, the left hand side term, $\frac{1}{n}\sum_{i=1}^{n} Y_i(w)$ should be similar in shape to the $H(w)$. Under the assumption mentioned above, the spectrum on bedrock level of an individual earthquake can be estimated by the following equation.

$$\begin{aligned} \tilde{X}_i(w) &= Y_i(w) \Big/ \frac{1}{n}\sum_{i=1}^{n} Y_i(w) \\ \tilde{X}_i(w) &\propto X_i(w) \end{aligned} \tag{3}$$

In order to examine whether the average of $< \tilde{X}_i(w) >$ is statistically constant or not, the period range are restricted to the intervals 0.1 sec $< T < T_m$ sec.

Where, $T_m = 10^{0.39M - 1.7}$ (Kanai, 1958) (4)

Figure 3 shows the relationship between $T_m$ and the corner frequency, $T_1^c$ and $T_2^c$, of theoretical spectrum (Savage, 1972). Considering the difficulty in identifying $T_1^c$ and $T_2^c$ from the records, it can be concluded that $T_m$ agrees reasonably well with the corner frequencies for $M > 6.0$, but disagree for $M < 6.0$. For $M < 6$, $T_1^c$ can be taken as $T_m$.

## RESULTS OF ANALYSIS

In order to estimate the overall characteristics ofspectra $\tilde{X}_i(w)$ of each event, the linear equation (eq. 5) is fitted to the peaks of spectra indicated by circles in Figure 4 as an example.

$$\mathrm{Amp} = a + b \cdot T \tag{5}$$

In Table 2, the values of $\{b\}$ of each spectrum are indicated. The confidence interval of the mean value is indicated by the following equation according to t - distribution;

$$P(-t_{\alpha} < T < T_{\alpha}) = \alpha \tag{6}$$

Accordingly, for $\alpha = 0.95$ the confidence interval for the population mean of $\{b\}$ should be

$$P(-0.11 < \mu < 0.17) = 0.95 \tag{7}$$

It can be concluded that from the above spectral analysis, the population mean of the sample $\{b\}$ is close to zero. Hence it can be said that the averaged acceleration Fourier spectra at the bedrock level should be reasonably constant in the period range mentioned above. Also, the averaged Fourier spectraum, $\frac{1}{n}\sum_{i=1}^{n} Y_i(w)$, of accelerograms obtained at the same site is similar in shape to the transfer function, $H(w)$, of the site.

## ACKNOWLEDGEMENT

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## REFERENCES

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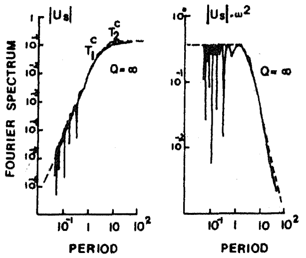

Fig.1 Theoretical Fourier amplitude spectra. Left, the displacement; right, acceleration spectrum.

Table 1 The earthquake data.

| EARTH.NO. | MAGNITUDE | FOCAL DEPTH | HYPOCENTRAL DISTANCE |
|---|---|---|---|
| A1-1 | 5.2 | 0 | 175 |
| -2 | 7.0 | 60 | 330 |
| -3 | 6.4 | 80 | 270 |
| -4 | 5.4 | 60 | 240 |
| -6 | 7.1 | 30 | 505 |
| -7 | 6.6 | 30 | 500 |
| B1-1 | 4.8 | 40 | 60 |
| -2 | 4.8 | 60 | 30 |
| -3 | 4.3 | 60 | 90 |
| -4 | 5.2 | 50 | 120 |
| -5 | 4.7 | 50 | 150 |
| -7 | 6.2 | 0 | 110 |
| -8 | 5.5 | 40 | 60 |
| -9 | 6.0 | 50 | 60 |
| -10 | 5.7 | 40 | 60 |
| -11 | 5.5 | 50 | 50 |
| -12 | 5.2 | 50 | 40 |

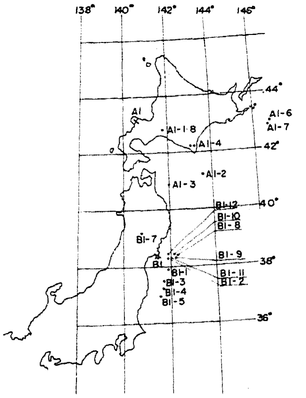

Fig.2 The location of two stations(A1,B1). and the epicenters used to analysis.

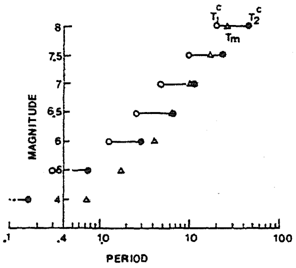

Fig.3 Relationship between $T_m$ and two corner frequencies (Savege, 1972).

Table 2 Values of {b} in eq.(5).

| EARTH.NO. | {b}VALUE |
|---|---|
| A1-1 | 0.91 |
| -2 | -0.35 |
| -3 | 0.14 |
| -4 | -0.25 |
| -6 | -0.01 |
| -7 | -0.02 |
| B1-1 | 0.22 |
| -2 | -0.11 |
| -3 | 0.07 |
| -4 | -0.10 |
| -5 | 0.02 |
| -7 | -0.30 |
| -8 | -0.28 |
| -9 | 0.22 |
| -10 | 0.05 |
| -11 | 0.13 |
| -12 | 0.14 |

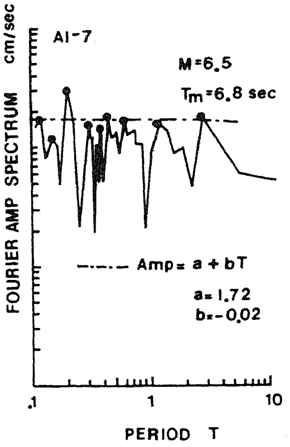

Fig.4 Fourier spectra on the bedrock($\tilde{X}(\omega)$) The chain line in the figure indicates the eq. (5) plotted by log-log scale.