AN ANALYSIS OF ACCELEROGRAM AND RESPONSE PEAKS USING THE EXPONENTIAL DISTRIBUTION

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INTRODUCTION

It is generally agreed that earthquake ground motions are representative of the output from a random process. Because of this randomness, the methods of probabilistic analysis are well suited for extracting relevant information. This paper presents several results from an ongoing research effort on the study of seismic risk at Stanford University. It has been found that histograms of ranked absolute peak values from certain robust accelerograms (such as El Centro 1940, Taft 1952, Pacoima 1971) are well described by an exponentially decaying function with parameter λ. In addition, for a certain period range, the distribution of the absolute response peaks has been found to be close to that of an exponential distribution, even for accelerograms that are not initially exponential. This exponential property makes it possible to describe the family of peaks contained in a given record in terms of λ and N, the effective number of peak values. This is similar to the use of Root-Mean-Square (RMS) acceleration and related time duration to provide a generalized description of a given strong motion record. Taking advantage of this "exponentiality," it is proposed that the parameter $1/\lambda$ at a zero or nonzero period be used as a normalization constant for response spectra in place of PGA.

VERIFICATION OF EXPONENTIAL BEHAVIOR FOR A RANDOM SAMPLE OF SIZE M

Given M observations X_i, arranged so that $X_1 > X_2 > \ldots > X_m$, the estimate of the exceedance probability is given by

$$P(X>X_i) = \frac{i}{M+1} \qquad (1)$$

where i is the <u>rank</u> of the observation X_i. For an exponential distribution, the exceedance probability is given by $G_X^i(W) = \exp(-\lambda W)$; assuming that i/(M+1) estimates $G_x(X_i)$ results in

$$\lambda X_i = -\ln \frac{i}{M+1} \qquad (2)$$

If the X_i were drawn from an exponential population, a plot of X_i vs $-\ln\{i/(M+1)\}$ would be scattered about a straight line with slope λ. Further verification of exponential behavior would occur if the median value of X_i at $i=M/2$ were to fall at $-\ln(0.5) = 0.693$. Lack of "exponentiality" would cause a marked nonlinear X_i vs. $-\ln\{(i/(M+1)\}$ plot. The next section will summarize a methodology that considers only those values X_i that exceed the median of an exponential set of size N, where in general $M \neq N$ and N is found from some simple mathematical relationships.

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THE EXPONENTIAL HALF TAIL DISTRIBUTION AND ESTIMATION OF ITS PARAMETERS

In an earthquake accelerogram there exists a multitude of peaks comprising the whole record. Engineers are more concerned with the large values of these peaks X_i than with the smaller values, and therefore it is necessary to develop a model that best describes the larger X_i. Recall that, for an exponential probability density function (PDF), the nth moment of the random variable X is given by

$$E\{X^n\} = \int_0^\infty x^n \lambda e^{-\lambda x} \, dx \quad (3)$$

If instead of zero, one were to have a lower limit of integration equal to the median m_x, then one would have the <u>half tail moments</u>

$$E'\{X^n\} = \int_{m_x}^\infty x^n \lambda e^{-\lambda x} \, dx \quad (4)$$

Specifically, it is desired to compute $E'\{X\}$ and $E'\{X^2\}$, where the primes denote half tail moments. It was shown in (Zsutty and DeHerrera, 1979) that $m_x = \ln 2/\lambda$ and

$$E'\{X\} = 0.847/\lambda \quad (5)$$

$$E'\{X^2\} = 1.933/\lambda^2 \quad (6)$$

$$RMS_x = 1.39/\lambda \quad (7)$$

The half tail moments can be used to find the exponential dsitribution that can probabilistically describe the behavior of the largest peaks above the median m_x. To accomplish this task, we define the exponential half tail (EHT) model for the PDF. It is given by

$$f_x(X) = \lambda e^{-\lambda x} \quad m_x \leq x < \infty \; ; = \text{undefined } x < m_x \quad (8)$$

with excess distribution function (EDF)

$$G_x(X) = e^{-\lambda w} \quad \text{for } x \geq m_x \; ; = \text{undefined for } x < m_x \quad (9)$$

The EHT model is indifferent to what the exact PDF or EDF is below the value $X = m_x$, since no probability statements are made in that region. Since $m_x = \ln 2/\lambda$, Eqs. (6) and (7) become

$$m_x = 0.819 \; E'\{X\} \quad (10)$$

$$m_x = 0.499 \sqrt{E'\{X^2\}} \quad (11)$$

ESTIMATION OF $E'\{X\}$ AND m_x

When estimating $E'\{X\}$ from ranked data, one is faced with the problem of not knowing what the median of the exponential population m_x is. Given a sample of size M sorted in descending order ($X_i \geq X_j$ for $j>i$), it is required to seek that X_i which will estimate the median of an EHT population in accordance with Eq. (10). This is achieved through a trial and error procedure that assumes that each successive X_i, starting from the largest peak X_1, is in fact the median value and computing the statistical estimates of Eq. (10). The median of the population with an exponential upper half tail is identified when, and the half sample size R is found from

$$X_s = X(i{=}S) \simeq 0.819\ \overline{X}(S) = 0.819 * \frac{1}{2S} \sum_{i=1}^{S} X_i \quad \underline{1} \leq \underline{S} \leq \underline{M} \tag{12}$$

In a manner analogous to that of the full exponential distribution, one can develop a probability paper in which data drawn from the exponential half tail model will exhibit a straight-line behavior. The excess distribution for the EHT model is given by exp $(-\lambda X_i)$ and the corresponding order statistic sample estimate for the exceedance probability is $i/(N+1)$.

THE EXTREMAL STATISTICS OF AN EXPONENTIAL SAMPLE

Given a large sample of N peaks X_i drawn from an exponential distribution, the largest value W_1 has an expected value and variance given by

$$W_1 = \frac{1}{\lambda}(\ln N{+}\gamma), \quad (\sigma_1)^2 = \frac{\pi^2}{6\lambda^2} \tag{13a,b}$$

where γ=0.577 and λ is the parameter of the exponential distribution (Gumbel, 1958). The expected mean and variance of the kth peak are given by

$$W_k = \frac{1}{\lambda} \sum_{z=k}^{N} (z)^{-1} \;;\; (\sigma_k)^2 = \frac{1}{\lambda^2} \sum_{z=k}^{N} (z)^{-2} \tag{14a,b}$$

Equations (13)-(14) are important in that they give what the expected value of the kth peak should be after several repetitions of an event with given Richter Magnitude and hypocentral distance at a given site.

From the results of the last section, it is noted that each record has a unique value of N. For purposes of practical applications and prediction of future values, it would appear that both λ and N must be predicted in analogy with RMS and duration prediction. The analysis also indicates a further simplification as follows: It was found that the value ln N+γ could be replaced by a constant ε with very little error introduced. This is analogous to saying that all those earthquakes behave as if they came from a sample size N* from an exponential population, where N* is found from N*= exp $(\varepsilon-\gamma)$. Note that this is <u>not</u> the same as saying that all of those earthquakes have the same number of peaks. The appropriate mean and variance of the kth peak are then given by

$$W_k \simeq \frac{1}{\lambda} \sum_{z=k}^{N^*} (z)^{-1}; \; (\sigma_k)^2 \simeq \frac{1}{\lambda^2} \sum_{z=k}^{N^*} (z)^{-2} \tag{15a,b}$$

The series in Eqs. (13)-(15) can be easily computed.

ANALYSIS OF RESPONSE

Using the methodology developed previously, the absolute response peaks of a single degree of freedom oscillator were analyzed to observe the variation of their PDF with frequency for small values of the damping ratio ξ. It was found that, in general, "exponential" records held their exponentiality within the natural period range T= 0.1 to T = 1.0 seconds, and degenerated into non-exponentiality at T > 2 second range. Figures 1-2 show probability plots for two earthquakes for various periods. As one might expect, this suggests an influence of the site conditions on the form of the distribution of the largest peaks. Research is being conducted at Stanford to examine not only the effect of site conditions on the distribution of peaks but also the effects of certain geophysical parameters, notably directivity, wave travel path and fault parameters (fault geometry, rupture length, etc.), keeping in mind the limitations due to inhomogeneous media and poor availability of geophysical data.

A major advantage of using $1/\lambda$ is that response spectra can be normalized with a parameter that is more stable than PGA. It is expected that using this normalization will lead to narrower values of the coefficient of variation of the response spectra.

ACKNOWLEDGEMENTS

This research was supported by the National Science Foundation under Grant Number ENV 77-17834.

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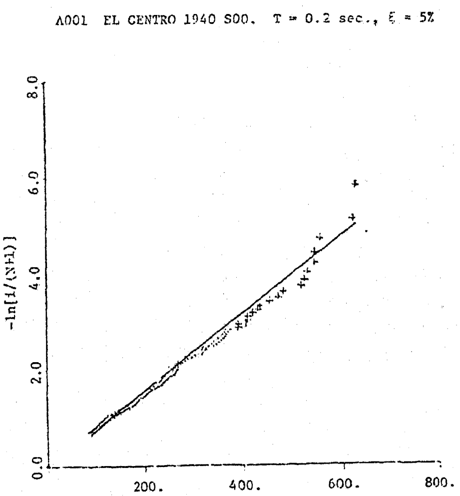

Fig. 1 x_i, cm/sec^2

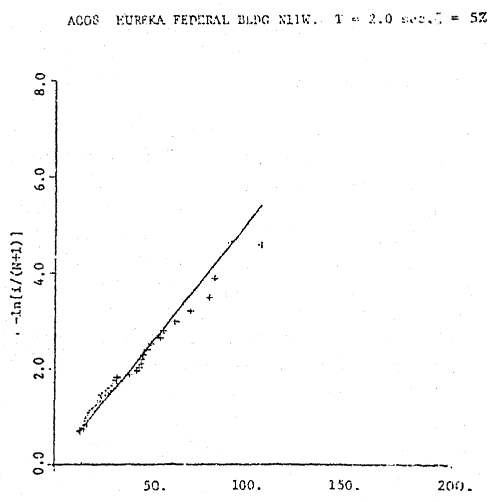

Fig. 2 x_i, cm/sec^2