RESPONSE SPECTRA OF EVOLUTIONARY EARTHQUAKE MODELS

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SUMMARY

A method for generating seismic spectra by approximating the probability density function of seismic structural responses by a time dependent Rayleigh distribution is presented. The structure is lightly damped, and the power spectrum of the stochastic seismic model is evolutionary. The approximate method is based on a combination of deterministic and stochastic averaging procedures. An example of application is given using a stochastic model with a deterministic modulating envelope which consists of a constant and an exponentially decaying segment.

INTRODUCTION

Recently, considerable attention has been given to statistical models of earthquake motion which have evolutionary spectra, as for example in ref. [1,4]. The advantages of this kind of earthquake models lie on the fact that they account for the duration, intensity, peak value, and time variation of the frequency content of recorded earthquake motions. Unfortunately, the exact mathematical expressions for the statistics of structural responses to these models are extremely complicated and cumbersome to derive. A method of approximate analysis applicable for lightly damped structures is presented in this article.

FORMULATION

Consider the equation of motion

$$\ddot{x} + 2\zeta\omega_o\dot{x} + \omega_o^2 x = w(t); \quad x(0) = \dot{x}(0) = 0 \tag{1}$$

of a linear structure of angular frequency ω_o and ratio of critical damping $\zeta \ll 1$ subjected to a stochastic seismic excitation $w(t)$. The spectrum of $w(t)$ is evolutionary and given by the equation

$$S_w(\omega,t) = |A(\omega,t)|^2 \, S(\omega) \; . \tag{2}$$

Properties of the functions $A(\omega,t)$ and $S(\omega)$ regarding general evolutionary spectra can be found in reference such [2]. For the present analysis it suffices to state that the process $w(t)$ is broad-band in the sense that $S_w(\omega,t)$ has, for all values of t, significant values over a band of frequencies which is roughly of the same order of magnitude as the center frequency of the band. It is assumed that the problem has been normalized so that

$$\pi S_w(\omega_o,t)_{max}/2\zeta\omega_o^3 = O(1) \quad \text{as} \quad \zeta \to 0 \; . \tag{3}$$

Due to the small magnitude of ζ it is appropriate to assume that the response amplitude $a(t)$ defined by the equation

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$$a^2(t) = \omega_o^2 x^2 + \dot{x}^2 \tag{4}$$

and the response phase $\phi(t)$ defined by the equation

$$\phi(t) = -\omega_o t - \tan^{-1}(\dot{x}/\omega_o x) \tag{5}$$

vary slowly with respect to time. Differentiating Eq. (4) and Eq. (5) and taking into consideration Eq. (1) yields

$$\dot{a}(t) = -2\zeta\omega_o a \sin^2(\omega_o t + \phi) - w(t)\sin(\omega_o t + \phi)/\omega_o a \ , \tag{6}$$

$$\dot{\phi}(t) = -2\zeta\omega_o \sin(\omega_o t + \phi)\cos(\omega_o t + \phi) - w(t)\cos(\omega_o t + \phi)/\omega_o a \ . \tag{7}$$

AVERAGING

Based on the slow variation of $a(t)$ and $\phi(t)$ it is logical to approximate the terms $\sin^2(\omega_o t + \phi)$ and $\sin(\omega_o t + \phi)\cos(\omega_o t + \phi)$ which appear in Eq. (6) and Eq. (7) by their average values over one period $[0, 2\pi/\omega_o]$. This procedure yields

$$\dot{a}(t) = -\zeta\omega_o a - w(t)\sin(\omega_o t + \phi)/\omega_o \ , \tag{8}$$

$$\dot{\phi}(t) = -w(t)\cos(\omega_o t + \phi)/\omega_o a \ . \tag{9}$$

Clearly, the values of the amplitude $a(t)$ and the phase $\phi(t)$ are correlated with the values of the excitation. However, due to the fact that the spectrum $S(\omega,t)$ is broad and the response parameters $a(t)$ and $\phi(t)$ are slowly varying, the correlation time of $w(t)$ is much shorter than the relaxation time of $a(t)$ and $\phi(t)$. It is logical, therefore, to assume that the values of $w(t)$ are statistically independent from the values of $a(t)$ and $\phi(t)$ which correspond to a slightly shifted time $t-\Delta t$. Based on these properties, Eq. (8) and Eq. (9) can be used to approximate the term $-w(t)\sin(\omega_o t + \phi)$ by the equation

$$-w(t)\sin(\omega_o t + \phi) = \pi S(\omega_o,t)/2a\omega_o + \sqrt{\pi S(\omega_o,t)}\ \eta(t) \ , \tag{10}$$

where $\delta(t)$ is a zero-mean and delta-correlated process of intensity one. That is,

$$\langle \eta(t) \rangle = 0 \ ; \ \langle \eta(t)\eta(t+\tau) \rangle = \delta(\tau) \ , \tag{11}$$

where $\delta(\tau)$ is the Dirac delta function. Substituting Eq. (11) in Eq. (8) yields

$$\dot{a}(t) = -\zeta\omega_o a + \pi S(\omega_o,t)/2a\omega_o^2 + \sqrt{\pi S(\omega_o,t)}\ \eta(t)/\omega_o \tag{12}$$

PROBABILITY DENSITY FUNCTION

Eq. (12) allows the modeling of the response amplitude by a Markov process. The associated Fokker-Planck equation is

$$\frac{\partial p(a,t)}{\partial \tau} = \frac{\partial}{\partial a}\left[\left(\zeta\omega_o a - \frac{\pi S(\omega_o,t)}{2a\omega_o^2}\right)p\right] + \frac{\pi S(\omega_o,t)}{2\omega_o^2}\frac{\partial^2 p}{\partial a^2}, \tag{13}$$

where $p(a,t)$ is the time dependent probability density function of the response amplitude. It can be shown that the solution of Eq. (13) which is compatible with the initial conditions of Eq. (1) and the restriction $0 \leq a < \infty$ is the following Rayleigh distribution

$$p(a,t) = a \exp[-a^2/2c^2(t)]/c^2(t) , \tag{14}$$

$$c^2(t) = \pi\exp(-2\zeta\omega_o t)\int_o^t \exp(2\zeta\omega_o z)S(\omega_o,z)dz/\omega_o^2 \tag{15}$$

Using Eq. (15) it can be readily proved that the statistical moments of $a(t)$ are given by the equation

$$\langle a(t)^{2k}\rangle = k\cdot 2^k\left[c(t)\right]^{2k} ; \langle a(t)^{2k+1}\rangle = \sqrt{\frac{\pi}{2}}\ (1)(3)\ \ldots\ (2k+1)\left[c(t)\right]^{2k+1} \tag{16}$$

Furthermore, Equation (14) can be used to construct seismic response spectra with a specified confidence level C_L. For example, for $C_L = 1\%$ it can be readily proved that

$$a(t) \leq 3.03485\ c(t) \equiv a_B(t)\ ;\ a(t)_{max} \leq 3.03485\ c(t)_{max} \equiv a_{B,max} \tag{17}$$

over the entire duration of the seismic structural response.

NUMERICAL APPLICATION

Consider the separable evolutionary spectrum given in ref. [3]

$$S_w(\omega,t) \equiv g^2(t)\bar{S}(\omega) = g^2(t)[1 + 2.4(\omega/5\pi)^2]\Big/\left\{[1-(\omega/5\pi)^2]^2 + 1.44(\omega/5\pi)^2\right\} \tag{18}$$

$$g(t) = u(t)-u(t-11.5)[1-\exp(-0.155t)];\ u(t) \equiv \text{Heaviside function} . \tag{19}$$

Substituting Eq. (18) into Eq. (15) yields

$$c^2(t) = \begin{cases} \pi\bar{S}(\omega_o)[1-\exp(-2\zeta\omega_o t)/2\zeta\omega_o^3\ ;\ 0 \leq t \leq 11.5 \\ \pi\bar{S}(\omega_o)\{\exp[-0.31(t-11.5) - d\exp[-2\zeta\omega_o(t-11.5)]\}\ ;\ 11.5 \leq t \end{cases}$$

$$d = 1 - (\zeta\omega_o - 0.155)[1-\exp(-0.31\zeta\omega_o)] . \tag{21}$$

Using Eq. (20) and Eq. (21) it can be readily proved that

$$c^2(t)\Big|_{max} = \frac{\pi\bar{S}(\omega_o)}{2\zeta\omega_o^3}\left[6.451\ \zeta\omega_o\right]^{6.451\ (0.155\ -\zeta\omega_o)} \tag{22}$$

In Fig. (1) the function $a_B(t)$ has been plotted for $\omega_o = 2\pi$ rad·sec^{-1} and $\zeta = 0.0000\overset{B}{,}$ 0.05, 0.01, 0.02. It is noted that for every value t, smaller values of $a_B(t)$ corresponds to larger values of ζ. This trend is compatible with the energy dissipation capacity of the structures considered.

In Fig. (2) the function $a_B(t)$ has been plotted for $\zeta = 0.01$ and $\omega_o = 2\pi, 4\pi, 6\pi$, and 8π rad·sec^{-1}. It is observed that for every value of t, smaller values of $a_B(t)$ correspond to larger values of ω_o. This trend is compatible with the capacity of the structures to resist deflection.

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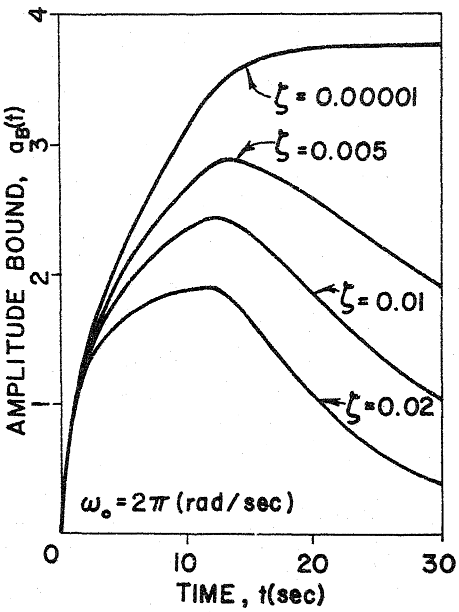

Figure 1

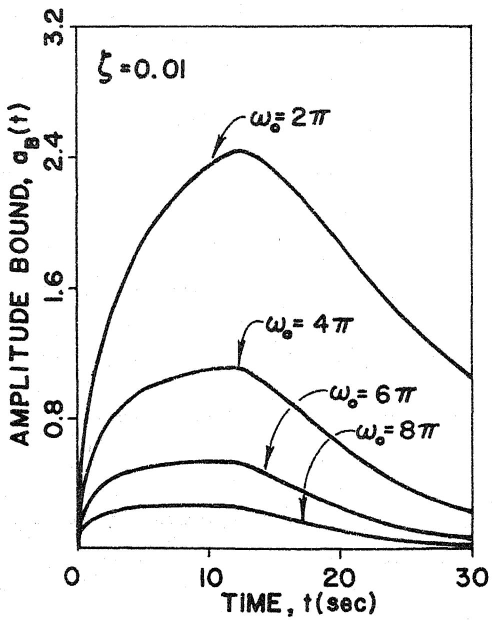

Figure 2