DESTRUCTIVE CAPABILITY OF EXTREME EARTHQUAKE MOTIONS EXAMINED IN TWO DIMENSIONS OF HORIZONTAL PLANE

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SUMMARY

Results of the destructive ratings for an ensemble of important strong-motion records are shown by reflecting totally the effects of two-dimensional shaking and response on the horizontal plane. The structural systems used account for most of the significant trends in the dynamics of failure of flexural reinforced concrete as well as the biaxial interaction of yielding and hysteretic restoring forces. Two specific criteria relating the severity of motion to serious damage and ultimate collapse are then characterized by use of a bare-minimum measure of four parameters specifying the total oscillational energy, its time and frequency-domain accumulations, and its planar sprawl.

INTRODUCTION

In a series of previous investigations by the author,[1,2,3] an extensive set of quantitative examinations has been made of the capability of intense ground shakings in causing damage and collapse of R/C buildings. Restricting the scope of formulations and analyses within one-dimensional dynamics, the study was intended to identify prominent trends in the structural failure under gravity loading, and to interpret these in relation to both the fundamental properties of structure and the gross characteristics of earthquake excitation. By applying the "equivalent" modelling of the fully nonlinear behaviour of total structure, role of basic system parameters was shown to differ clearly at the two different response levels of damage and collapse. Use of the "damage and collapse accelerations" has led directly to the destructive ratings of strong-motion records, while a three-parameter measure in terms of amplitude, duration and frequency content was newly introduced in order to characterize the individual motion. A sufficient scaling of the severities by this measure has emphasized the significantly different influences of duration and frequency content upon the particular aspects of structural response. The present study, which investigates the destructive potentials in two dimensions of horizontal plane, lies along an extension of these preceding works.

A mathematical formulation has been developed for modelling the restoring force characteristics of biaxial sidesway.[4,5,6] This is a two-dimensional interpretation of various nonlinear systems for one-dimensional response analyses, including the bilinear, trilinear and quadrilinear yielding models combined with the nondegrading or degrading properties during cyclic loading. For instance, the degrading quadrilinear combination can embody most of the marked features exhibited by flexural-failure reinforced concrete: cracking, yielding, crushing and spalling, associated stiffness reduction and ductility deterioration, hysteresis-loop degradation, and so on. Validity of the present formulation has been reported in part;[5,7] this permits to reflect the serious effects of biaxial response to expedite the deterio-

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ration and the complicated restoring force relationship which is no longer piecewise linear. With this biaxial model incorporated into a two-dimensional version of the "equivalent 1-DOF" structure, the current examination also contains the effects of nonlinear geometry in the deformed configuration, which are mainly associated with the influence of gravity to destabilize the system with increasingly large drift.

INTENSE GROUND MOTIONS EXAMINED AND FOUR-PARAMETER REPRESENTATION OF THEIR PROPERTIES

The earthquake motions examined in the current presentation are listed in Table 1. This ensemble is intended to cover many of the accelerograms of engineering importance, which includes impulsive type shakings featured by markedly high peak accelerations in addition to the more typical records of still longer duration. Deleting the vertical component because of its insignificant role during the destabilizing action of gravity, this provides 23 samples of planar motion.

Importance of the total energy, or RS value, of motion as a primitive single parameter reflecting its dynamic effects has been strongly suggested by the mathematical equalities recently noticed (A. Arias, 1970; G. W. Housner and P. C. Jennings, 1977). This has been shown to measure the work done by an excitation upon ensemble of simple oscillators; moreover, spectral representation of the work for damped oscillators corresponds to a smoothed Fourier amplitude spectrum of the excitation.[8] For the purposes of specifying the variations seen in the fundamental properties of motion, a leading role is therefore assigned, in the following, to the factor of total energy.

Table 1

Vernon [Long Beach EQ in Calif.]	3/10-'33	VER
El Centro [Lower Calif. EQ]	12/30-'34	E34
Helena [Helena, Montana EQ]	10/31-'35	HEL
El Centro [Imperial Valley EQ in Calif.]	5/18-'40	E40
Olympia [Western Washington EQ]	4/13-'49	049
Taft [Kern County EQ in Calif.]	7/21-'52	TAF
Eureka [Eureka EQ in Calif.]	12/21-'54	EUR
Golden Gate Park [San Francisco EQ in Calif.]	3/22-'57	GOL
Kushiro [Hiroo-Oki EQ in Japan]	4/23-'62	KUS
Olympia [Puget Sound, Washington EQ]	4/29-'65	065
Hoshina [Matsushiro EQ Swarm in Japan]	4/5-'66 -	HOS
Cholame Shandon No. 5 [Parkfield EQ in Calif.]	6/27-'66	CH5
Cholame Shandon No. 8 [Parkfield EQ in Calif.]	6/27-'66	CH8
Temblor [Parkfield EQ in Calif.]	6/27-'66	TEM
Lima [Peru EQ]	10/17-'66	LIM
Koyna Dam [Koyna EQ in India]	12/11-'67	KOY
Hachinohe Harbor [Tokachi-Oki EQ in Japan]	5/16-'68	HAC
Hiroo [Hidaka-Sankei EQ in Japan]	1/21-'70	HIR
Pacoima Dam [San Fernando EQ in Calif.]	2/9-'71	PAC
Holiday Inn [San Fernando EQ in Calif.]	2/9-'71	HOL
Rocca [Ancona EQ Swarm in Italy]	6/14-'72	ROC
Melendy Ranch [Stone Canyon EQ in Calif.]	9/4-'72	MEL
Esso Refinery [Managua, Nicaragua EQ]	12/23-'72	MAN

(in chronological order)

Table 2

$$\begin{Bmatrix} x(t) \\ y(t) \end{Bmatrix} : \text{time-history of a planar motion, measured along an arbitrary set of mutually orthogonal axes} \quad \cdots\cdots(1)$$

$$[T(t)] = \begin{Bmatrix} x(t) \\ y(t) \end{Bmatrix} \lfloor x(t) \;\; y(t) \rfloor : \text{real and symmetric} \quad \cdots\cdots(2)$$

$$[E(t)] = \int_{-\infty}^{t} [T(t_1)]\, dt_1 : \text{real and symmetric} \quad \cdots\cdots(3)$$

$$E(t) = \mathrm{tr}\,[E(t)] = \int_{-\infty}^{t} \{|x(t_1)|^2 + |y(t_1)|^2\}\, dt_1 \quad \cdots\cdots(4)$$

$$\begin{Bmatrix} \underset{\sim}{x}(\Omega) \\ \underset{\sim}{y}(\Omega) \end{Bmatrix} = \int_{-\infty}^{+\infty} \begin{Bmatrix} x(t) \\ y(t) \end{Bmatrix} e^{-j\Omega t}\, dt : \text{Fourier transform of } \begin{Bmatrix} x(t) \\ y(t) \end{Bmatrix} \quad \cdots\cdots(5)$$

$$[\underset{\sim}{T}(\Omega)] = \begin{Bmatrix} \overline{\underset{\sim}{x}(\Omega)} \\ \overline{\underset{\sim}{y}(\Omega)} \end{Bmatrix} \lfloor \underset{\sim}{x}(\Omega) \;\; \underset{\sim}{y}(\Omega) \rfloor : \text{complex and Hermitian, and } [\underset{\sim}{T}(-\Omega)] = [\underset{\sim}{T}(\Omega)]^t \; (= \overline{[\underset{\sim}{T}(\Omega)]}) \quad \cdots\cdots(6)$$

$$[\underset{\sim}{E}(\Omega)] = \frac{1}{2\pi} \int_{-\Omega}^{+\Omega} [\underset{\sim}{T}(\Omega_1)]\, d\Omega_1 : \text{real and symmetric} \quad \cdots\cdots(7)$$

$$\underset{\sim}{E}(\Omega) = \mathrm{tr}\,[\underset{\sim}{E}(\Omega)] = \frac{1}{\pi} \int_{0}^{\Omega} \{|\underset{\sim}{x}(\Omega_1)|^2 + |\underset{\sim}{y}(\Omega_1)|^2\}\, d\Omega_1 \quad \cdots\cdots(8)$$

$$[E(\infty)] = [\underset{\sim}{E}(\infty)] \quad \cdots\cdots(9)$$

$$E = \int_{-\infty}^{+\infty} \{|x(t)|^2 + |y(t)|^2\}\, dt = \frac{1}{\pi} \int_{0}^{\infty} \{|\underset{\sim}{x}(\Omega)|^2 + |\underset{\sim}{y}(\Omega)|^2\}\, d\Omega \quad \cdots\cdots(10)$$

$$P = \sqrt{0.7 \frac{E}{1+r^2} \frac{1}{t_e}} \quad \cdots\cdots(11)$$

Consider a two-dimensional motion in the time domain, measured along an arbitrary set of mutually orthogonal axes [(1) in Table 2]. As is easily seen, a symmetric matrix $[E(t)]$ in Eq. (3), by way of the tensor product of vectors in Eq. (2), is subjected to tensorial transformation and its trace, $E(t)$ in Eq. (4), remains invariant when rotating the measuring axes. On the other hand, the counterpart relationships in the frequency domain are given by Eqs. (5), (6), (7) and (8); a real and symmetric matrix $[\underset{\sim}{E}(\Omega)]$ and its trace $\underset{\sim}{E}(\Omega)$ have the same properties during the rotation. Even though the orientation of principal axes for $[E(t)]$ and $[\underset{\sim}{E}(\Omega)]$ is separately time or frequency-dependent, their terminal values of $[E(\infty)]$ and $[\underset{\sim}{E}(\infty)]$ can be simultaneously diagonalized according to the Parseval's equality indicated by Eq. (9). The last property permits to define the major and minor axes of motion over the entire domains of time and frequency, and the common trace of matrix therein, denoted by E in Eq. (10), is henceforth referred to as the total energy in two-dimensional cases. Furthermore, the two functions of $E(t)$ and $\underset{\sim}{E}(\Omega)$, rendered independent of a particular choice of reference frame, turn out to describe the patterns of energy accumulation toward E in the time and frequency domains, respectively. These are an extension of the concept of "intensity tensor" proposed by A. Arias in 1970.

Together with use of E as the measure of absolute intensity, characteristic parameters of r, t_0 and T_0 are introduced which embody its distributions in the three domains of space, time and frequency. The parameter r is defined by the ratio of RS values between the two one-dimensional motions along minor versus major axes. Following the tensor property noted above, this specifies uniquely the planar sprawl of E. On the other hand, the parameter t_0 stands for the time interval which comprises the middle 70 % of E on the curve of $E(t)$. The interval can be referred to as duration, although this may tend to be shorter than its ordinary understanding. In a similar vein, the remaining parameter T_0 is the period of 50 % transmission of E on the curve of $\underset{\sim}{E}(\Omega)$, which coincides, in some cases, with the conventional notion of predominant period. The latter two parameters have been somewhat arbitrarily chosen; however, these can play a remarkably eminent

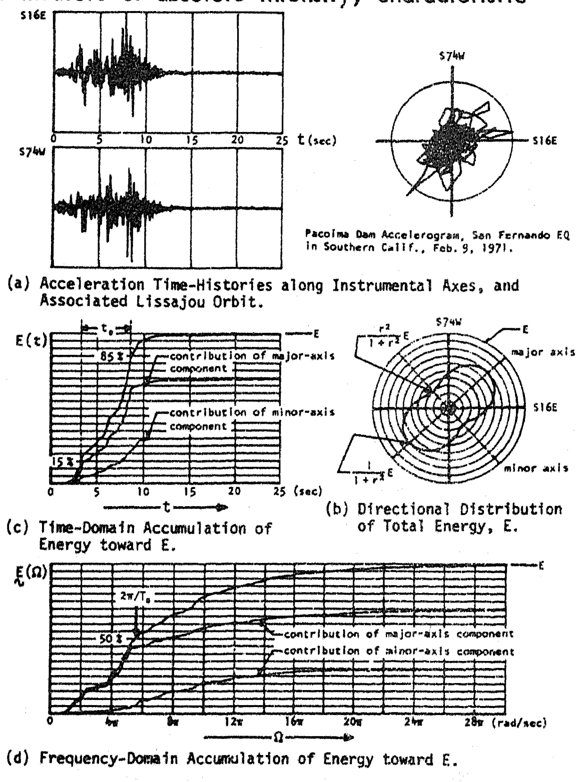
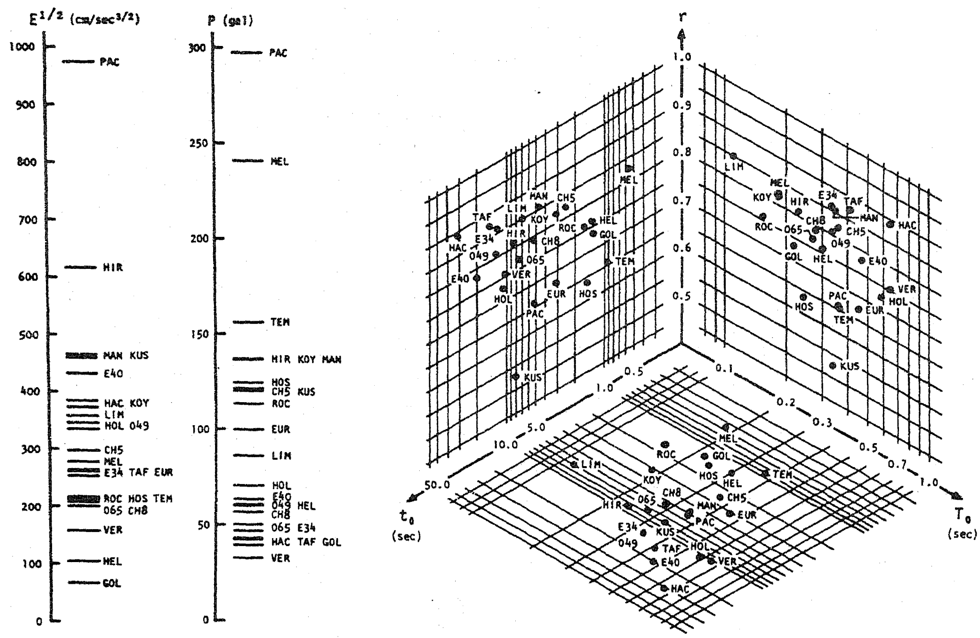


Fig. 1 Illustration of the Planar Sprawl, and the Time and Frequency-Domain Accumulations, of Total Energy.



(a) Intensity Scales of $E^{1/2}$ and P.

(b) Tripartite Plot of the Distribution Range of Characteristic Parameters (r , t_0 , T_0).

Fig. 2 Four-Parameter Representation of Fundamental Properties for the Intense Earthquake Motions Examined.

part as enunciated in the foregoing characterization of one-dimensional destructive capabilities.[3] Fig. 1 illustrates evaluation procedures of all the parameters.

Let the amplitude parameter, P, be then defined by converting E to the RMS value over t_0 and on the major axis [vide Eq. (11) in Table 2]. The resulting set of four parameters (P , r , t_0 , T_0) is used, in the later study, to characterize grossly the features of motion individually complicated. For comparison, results of their evaluation are summarized in Fig. 2. Besides the intensity scales of $\sqrt{E}$ and P, the figure displays the distribution ranges of r, t_0 and T_0. Definition of r leads to its range between one and zero; however, it is interesting to note the limited distribution ranging from 0.65 to 0.9 with an exception of 0.55 for KUS. Also, t_0 and T_0 are shown to distribute from 0.6 to 30 seconds, and from 0.1 to 0.7 seconds, respectively. The fact that HAC is given the largest values in any measure of r, t_0 and T_0 is worthy of a special notice.

CRITERIA OF SERIOUS DAMAGE AND ULTIMATE COLLAPSE, AND ASSOCIATED DESTRUCTIVE RATINGS OF MOTION

Within the context of one-dimensional dynamics, the structural models used herein are the equivalent 1-DOF system characterized by degrading trilinear properties of restoring force, ideally ductile beyond yielding. The system is correspondingly interpreted in two dimensions of horizontal plane according to the formulation referred to in the introduction. Properties of structure are assumed to be identical along its principal axes, and the response drift in the radial direction is examined. Under these conditions and following the same reasonings as given in the preceding study,[1,2] the ductility-factor response, μ_{max}, is shown to be governed by elastic period, T_e, viscous damping factor, ζ_e, crack versus yield capacities, a_c, reduction factor of secant stiffness at yield point, α_y, relative strength of excitation versus structure, I ($=A_{max}/A_y$; A_{max}: peak acceleration in the major-axis component, A_y: yield strength in acceleration), and gravity-effect parameter, α_g, as well as the two-dimensional time-history of unit-intensity excitation, $\{e(t)\}$. Then, use of the fixed parameters of $\zeta_e=0.05$, $a_c=0.5$ and $\alpha_y=0.2$ leads to associating μ_{max} with I under the specification of T_e, α_g and $\{e(t)\}$. Furthermore, only the two representative values are taken for α_g (6×10^{-3} for soft frames, and 1×10^{-3} for stiff frames), and total number of stories, n, is alternatively referred to by the use of its dependent relation to T_e and α_g.

Two specific criteria of the severities of motion, I_d and I_c, are then determined by associating the relative intensity I with serious damage (represented by $\mu_{max}=3$) and ultimate collapse of the equivalent structures. An example of the results of evaluation is given in Fig. 3, while Fig. 4 summarizes them totally by limiting the display to the boundary cases of $n=1$ and 10. In these figures, the trend toward continuity in the two curves of I_d reflects the elementary fact that the response at the low level cannot be noticeably affected by α_g, thus allowing its relation to a single factor T_e. Also, examinations of the two curves of I_c indicate a strong tendency to overlap together when n is employed as the common abscissa. Consistent with the previous findings, the latter feature suggests the virtually insignificant role of T_e; eminent factors governing the extremely nonlinear response are A_y and the absolute magnitude of gravity effect. The representations of I_d and I_c emphasize, in addition, markedly differing severities of individual motions under the same ampli-

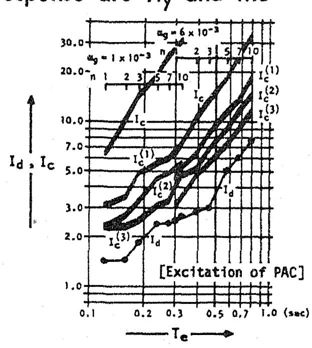


Fig. 3 Illustration of the Relative Intensities of Damage and Collapse.

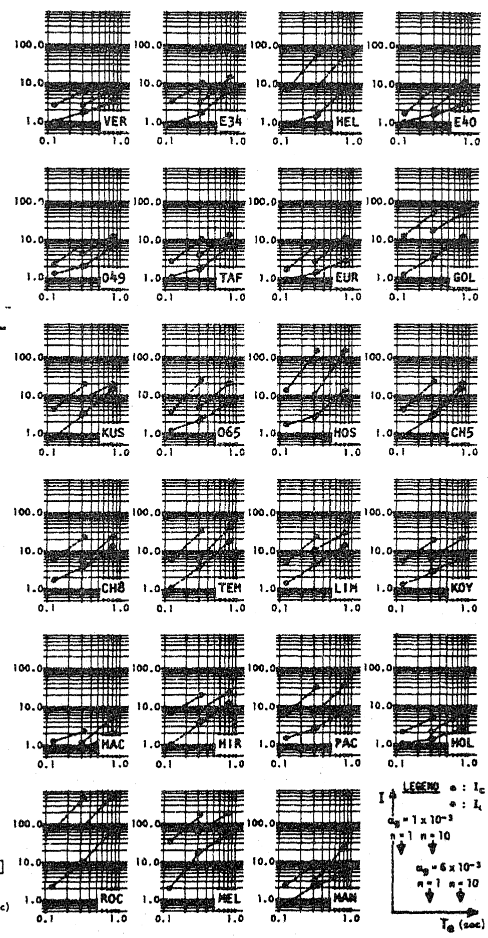


Fig. 4 Relative Intensities of Damage and Collapse for the Intense Earthquake Motions Examined.

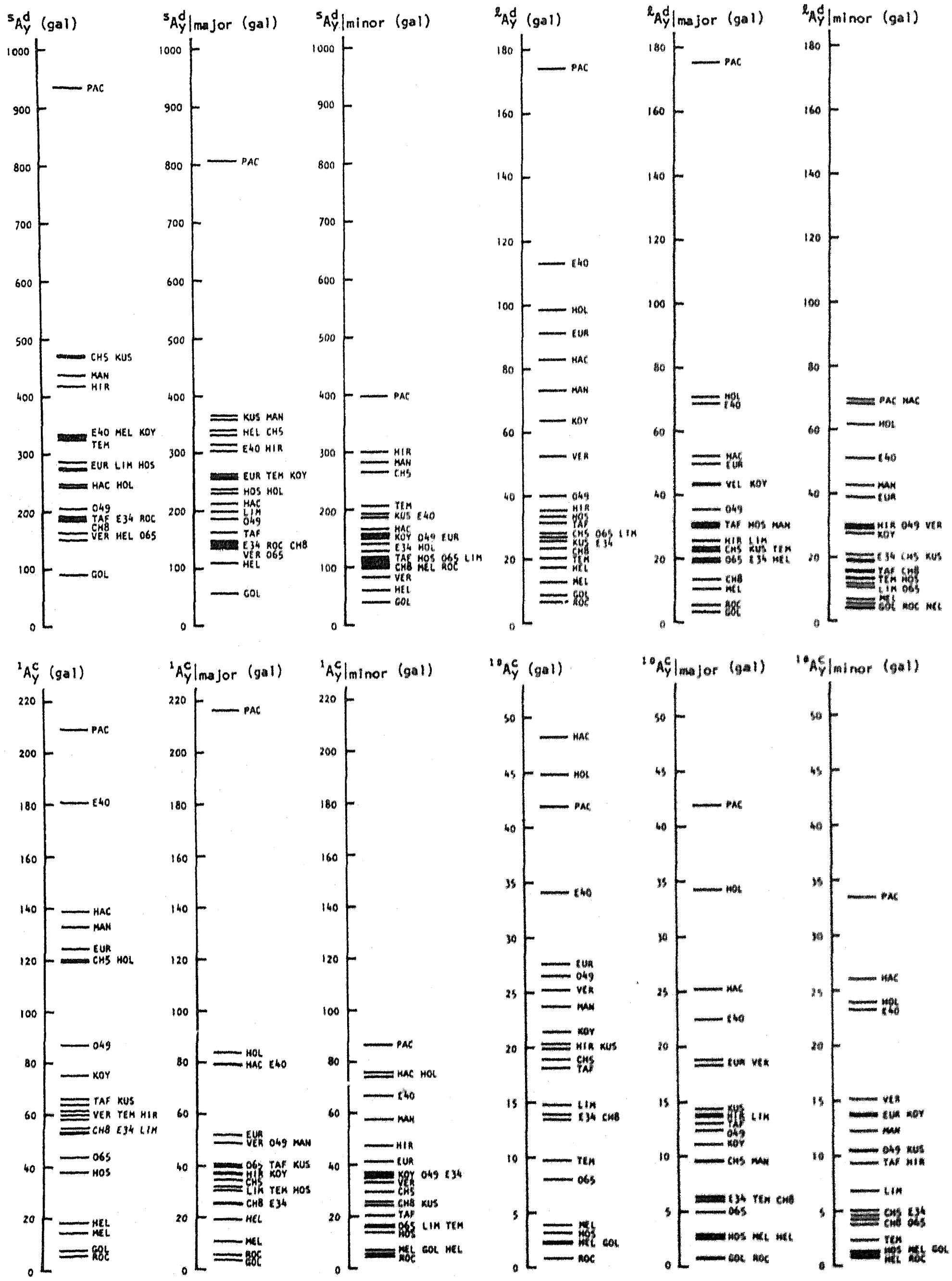


Fig. 5 Destructive Ratings in Terms of Damage and Collapse Accelerations.

tude of peak acceleration, as was the case in the one-dimensional study.[2]

Let the unreliable measure of A_{max} be eliminated by substitution of the recorded value, which results in introducing the "damage and collapse accelerations", $A_y^d (= A_{max}/I_d)$ and $A_y^c (= A_{max}/I_c)$. These stand for the minimum yield strength of structure, required for resisting the motion without sustaining the serious damage and ultimate collapse, respectively. On the basis of the general trends noted above, two damage accelerations of ${}^{s}A_y^d$ and ${}^{\ell}A_y^d$ are specifically evaluated from the boundary cases of $[n=1,\ \alpha_g=1\times10^{-3}]$ and $[n=10,\ \alpha_g=6\times10^{-3}]$. Also, the collapse accelerations in the two cases of n=1 and 10, ${}^{1}A_y^c$ and ${}^{10}A_y^c$, are used to specify the latter severity by the application of geometric average. The resulting scales of intensity for ${}^{s}A_y^d$, ${}^{\ell}A_y^d$, ${}^{1}A_y^c$ and ${}^{10}A_y^c$ are shown in Fig. 5, which demonstrates the range of severities from the different viewpoints. The capability of PAC far superior to others is particularly obvious in A_y^d; however, its ratings by A_y^c are seen to depreciate comparatively, losing the highest rank in ${}^{10}A_y^c$. For identifying the two-dimensional effects, the figure includes additionally the corresponding intensity scales for the major and minor-axis motions. The increase ratios observed in A_y^d and A_y^c are plotted in Fig. 6; the two-dimensional effects are apparently more complex than simply embodied by the sprawling parameter of r.

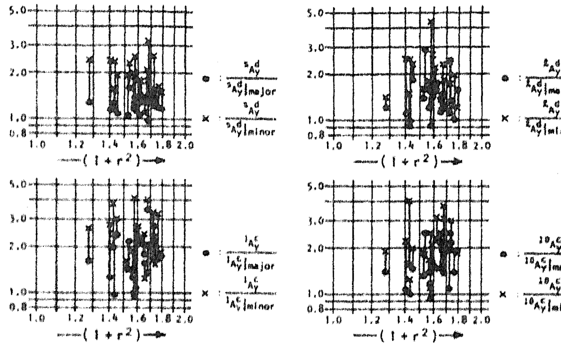


Fig. 6 Two-Dimensional Effects in Damage and Collapse Accelerations.

Table 3

$\log(X/P) = \log\mu + \pi_1\log t_0 + \pi_2\log T_0 + \pi_3\log(1+r^2) \pm \log\nu$

I : two-dimensional response data (23 samples)
II : one-dimensional response data (23 x 2 samples)
III : simple combination of I and II (23 x 3 samples)

	X	μ	π_1	π_2	π_3	ν
I	${}^{s}A_y^d$	3.1	-	-	-	1.43
		2.0	0.29	-	-	1.18
		2.8	0.24	0.24	-	1.14
		2.7	0.24	0.25	0.15	1.14
	${}^{\ell}A_y^d$	0.42	-	-	-	2.67
		0.14	0.72	-	-	1.78
		0.54	0.53	0.92	-	1.58
		0.45	0.50	0.95	0.53	1.58
	${}^{1}A_y^c$	0.67	-	-	-	2.89
		0.19	0.82	-	-	1.82
		0.84	0.60	1.01	-	1.58
		0.61	0.57	1.07	0.98	1.57
	${}^{10}A_y^c$	0.15	-	-	-	3.50
		0.031	1.02	-	-	1.88
		0.13	0.81	0.97	-	1.66
		0.083	0.76	1.06	1.42	1.64
II	${}^{s}A_y^d$	2.3	-	-	-	1.58
		1.3	0.36	-	-	1.26
		2.3	0.28	0.36	-	1.19
	${}^{\ell}A_y^d$	0.30	-	-	-	2.66
		0.097	0.74	-	-	1.72
		0.39	0.54	0.91	-	1.48
	${}^{1}A_y^c$	0.39	-	-	-	2.77
		0.11	0.81	-	-	1.64
		0.38	0.63	0.82	-	1.44
	${}^{10}A_y^c$	0.096	-	-	-	3.49
		0.020	1.00	-	-	1.81
		0.066	0.83	0.77	-	1.65
III	${}^{s}A_y^d$	2.6	-	-	-	1.56
		1.5	0.34	-	-	1.29
		2.5	0.27	0.33	-	1.24
		2.3	0.26	0.34	0.57	1.18
	${}^{\ell}A_y^d$	0.34	-	-	-	2.66
		0.11	0.73	-	-	1.77
		0.44	0.53	0.92	-	1.54
		0.40	0.53	0.93	0.63	1.51
	${}^{1}A_y^c$	0.46	-	-	-	2.88
		0.13	0.81	-	-	1.80
		0.51	0.62	0.89	-	1.60
		0.44	0.61	0.90	1.11	1.49
	${}^{10}A_y^c$	0.11	-	-	-	3.53
		0.024	1.00	-	-	1.89
		0.084	0.82	0.84	-	1.72
		0.074	0.81	0.85	0.95	1.65

FOUR-PARAMETER CHARACTERIZATION OF DESTRUCTIVE POTENTIALS INCLUDING THE INFLUENCE OF DUCTILITY DETERIORATION

An adequate scaling of the destructive capabilities represented by ${}^{s}A_y^d$, ${}^{\ell}A_y^d$, ${}^{1}A_y^c$ and ${}^{10}A_y^c$ is then examined with respect to the gross parameters of motion, P, t_0, T_0 and r. To this end, results of three groups of regression analyses are shown in Table 3, where *μ, π_1, π_2 and π_3 are correlation constants with ν indicating* the associated bound of variance. A similar type of correlation was used in the preceding study;[3] the new parameter of π_3 intends to clarify the effect of the current interest with relation to E and P. The group I correlations sample all the data of two-dimensional response. Even though ν decreases considerably according to the subsequent inclusion of the factors of t_0 and T_0, little improvement in the degree of correlation is associated with the third factor of r. This is partly because of the narrowly limited range of $(1+r^2)$ as compared to those of t_0 and T_0. The correlations are also flawed by the more notable increase of ν for collapse and for longer-period systems. However, the significance of t_0 and T_0 is very clear in any correlation, emphasizing their differing degrees of influence upon the different aspects as noted previously.[3] This consequence is again obvious in the group II correlations, in which the one-dimensional response data along major and minor axes are sampled by regarding the excitations as particular cases of two-dimensional motion. Then, the results in the group III correlations are obtained by amalgamating simply the data of I and II. Adding the data with r identically zero, the fundamental role of r has become clearer; $\pi_3 \simeq 0.5$ for A_y^d, and $\pi_3 \simeq 1.0$ for A_y^c, as a rule of thumb. This highlights an insufficient characterization of the two-dimensional effect upon structural collapse by the use of total energy.

In closing this presentation, the effect of ductility deterioration to accelerate structural collapse is noted briefly. Its examinations can be done by applying the two-dimensional extension of the quadrilinear restoring force concept; the additional system parameters are crush versus yield capacities, a_u, ductility factor at crush point, μ_u, and negative gradient beyond crush relative to elastic stiffness, p, all related to the specification of uniaxial skeleton curve. A few examples ($a_u=1.0$, $\mu_u=3.0$) are included in the foregoing Fig. 3, where the different cases of $p=-0.005$, -0.010 and -0.020 are discriminated by the superscripts of (1), (2) and (3) on I_c. Reduction of I_c accompanies a trend of continuity between the corresponding two curves, which reflects the declining importance of gravity effect. The data are then replotted in Fig. 7; the abscissa and ordinate represent the equivalent total height and the relative intensity of collapse, when converted to the instance of ideal ductility on the basis of the effective restoring capacity on uniaxial skeleton. Convergence to a single curve is clearly seen in this illustration, indicating the reduction of I_c approximated well by the coefficient of $\{a_u+(-p)\mu_u/\alpha_y\}/\{1+(-p)/\alpha_g\}^{1/2}$.

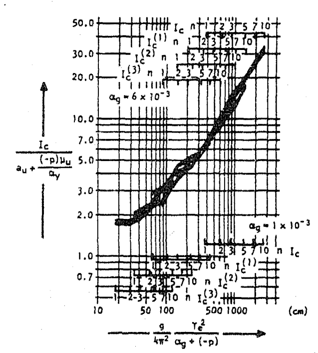


Fig. 7 Rearrangement of the Data for I_c Included in Fig. 3.

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