THE GENERATION OF ARTIFICIAL ACCELEROGRAMS BY SUPERPOSITION OF SINUSOIDAL WAVES OF RANDOM DURATION

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SUMMARY

A simple approach to the generation of artificial accelerograms is discussed. The basis of the method is to obtain an initial wave form by superposition of sinusoidal wave components starting and terminating at randomly determined instants. Utilizing the good agreement between the Fourier amplitude spectrum and the undamped velocity response spectrum at peak ordinates, the amplitudes of the sinusoidal wave components are determined. Through an iterative procedure, the initial wave form is modified in frequency domain to match a target velocity response spectrum.

INTRODUCTION

In the simulation of engineering structures during an earthquake, the acceleration time-history of probable strong-motion earthquakes at a particular site is needed. However, the existing earthquake records are, in general, inadequate in two respects:

1- Dynamic charecteristics of the ground at a site under consideration are different from the ground where the record was obtained,
2- Generally, the frequency content of natural earthquakes has irregularities, which may result in the underestimation of seismic forces.

There are still many problems related to the modification of a rather simple wave generated from the fault movement at the hypocenter, to create complex seismograms at a particular site. However, we have evidence to believe that earthquake engineering has advanced to estimate at least some important characteristics of future earthquakes as spectral density or response spectra. Using this information, it is theoretically possible to generate artificial strong-motion time histories with the same statistical properties as natural earthquakes. These accelerograms may be so generated that they are uniformly rich in a wide spectrum of frequencies.

The object of the method presented in this paper is to start with an initial wave form, having non-zero values only at the strong portion of the accelerogram, and to improve this wave form to match a target spectrum. No care is taken for the envelope function.

SELECTION OF THE INITIAL WAVE FORM

A component wave of circular frequency ω_j starts at time $t_1=(m_1-1)\Delta t$ and terminates at time $t_2=(m_2-1)\Delta t$. N, the total number of discrete values of

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acceleration-time history is assumed to be the power of two in order to apply the fast Fourier transform during the subsequent computations. T_d, the duration of the record is $(N-1)\Delta t$. Clearly, the duration of this component wave is t_2-t_1.

By definition, the Fourier transform of this component wave is

$$C_{kj} = \frac{1}{N} \sum_{m=m_1}^{m_2} \sin\left[(m-m_1)\omega_j \Delta t\right] e^{-2i\pi(k-1)(m-1)} \qquad (k=1,\ 2,\ldots,\ N) \tag{1}$$

where

$$\omega_j = \frac{2\pi}{N\Delta t}(j-1)$$
$$i = \sqrt{-1}$$

By evaluating the finite trigonometric series of Eq.1, the following closed form expressions are obtained for C_{kj}:

$$\left.\begin{aligned}
C_{kj} &= \frac{1}{2Ni}\, e^{-\alpha} \left[\frac{e^{-\mu\beta}-1}{e^{-\beta}-1} - \frac{e^{-\mu\gamma}-1}{e^{-\gamma}-1}\right] && (k=1,2,..N \;\; k\neq j,\; k\neq N-j+2)\\
C_{kj} &= \frac{1}{2Ni}\, e^{-\alpha} \left[\mu - \frac{e^{-\mu\gamma}-1}{e^{-\gamma}-1}\right] && (k=j)\\
C_{kj} &= \frac{1}{2Ni}\, e^{-\alpha} \left[\frac{e^{-\mu\beta}-1}{e^{-\beta}-1} - \mu\right] && (k=N-j+2)
\end{aligned}\right\} \tag{2}$$

where

$$\left.\begin{aligned}
\alpha &= \frac{2\pi i}{N}(k-1)(m_1-1), & \gamma &= \frac{2\pi i}{N}(k+j-2)\\
\beta &= \frac{2\pi i}{N}(k-j), & \mu &= m_2-m_1+1
\end{aligned}\right\} \tag{3}$$

If we have N_w component waves with different circular frequencies and different m_1 and m_2, the Fourier transform of the resultant wave is the sum of the Fourier transforms of the component waves.

The parameters m_1 and m_2 may be generated in a random manner, satisfying the conditions

$$N_1 \leqslant m_1 \leqslant N_2$$
$$N_3 \leqslant m_2 \leqslant N_4$$

where N_1, N_2, N_3 and N_4 are to be supplied as input data.

For programming purposes, it is convenient to generate and store m_1 and m_2 values in two integer arrays, so that they may be used many times in subsequent computations

$$\left.\begin{aligned} m_1 &= M_1(\ell) \\ m_2 &= M_2(\ell) \end{aligned}\right\} \qquad (\ell = 1,2. \ \ldots \ N_\omega)$$

By generating new M_1 and M_2 arrays, infinitely many initial wave forms can be obtained.

Numerical applications indicate that more satisfactory results are obtained if the frequencies are selected at equal intervals. Therefore, when j_{min} and j_{max} are supplied, the integer array

$$J(\ell) = j_{min} + \frac{j_{max} - j_{min}}{N_\omega - 1}\,(\ell - 1) \quad (\ell=1,2, \ \ldots \ N_\omega)$$

can be used to calculate the circular frequency ω_ℓ

$$\left.\begin{aligned} j &= J(\ell) \\ \omega_\ell &= \frac{2\pi}{N\Delta t}\,(j-1) \end{aligned}\right\} \quad (\ell=1,2,..N_\omega)$$

Through the assumption that the peak ordinates of the Fourier amplitude spectrum are almost equal to the undamped spectral velocity divided by T_d, one obtains an equation system for the unknown amplitudes a_ℓ of the component waves[1)]

$$\left|\sum_{\ell=1}^{N_\omega} C_{k\ell} a_\ell\right| = \frac{Sv_k}{T_d}, \quad (k=1,2,..N_\omega) \tag{4}$$

where the absolute value sign means the magnitude of the complex quantity. The above equations can be transformed into a set of real equations by squaring and adding the real and imaginary parts

$$\left[\sum Re(C_{k\ell})a_\ell\right]^2 + \left[\sum Im(C_{k\ell})a_\ell\right]^2 = \left[\frac{Sv_k}{T_d}\right]^2, \quad (k=1,2,..N_\omega) \tag{5}$$

Since the squares of the unknown a_ℓ's appear in these equations, this is a non-linear equation system. However, the most significant coefficient in the k-th equation is C_{kk}, namely the diagonal coefficient.

If the following quantities are defined as

$$\left.\begin{array}{l} s_r = \sum_{\ell=1}^{N_\omega} \mathrm{Re}(C_{k\ell})a_\ell \\ s_i = \sum_{\ell=1}^{N_\omega} \mathrm{Im}(C_{k\ell})a_\ell \end{array}\right\} (\ell \neq k) \qquad \left.\begin{array}{l} g_r = \mathrm{Re}(C_{kk}) \\ g_i = \mathrm{Im}(C_{kk}) \end{array}\right\} \tag{6}$$

then a_k becomes the root of the following quadratic equation

$$(g_r^2 + g_i^2)a_k^2 + 2(g_r s_r + g_i s_i)a_k + s_r^2 + s_i^2 - \left[\frac{Sv_k}{T_d}\right]^2 = 0 \tag{7}$$

The above property makes the application of a procedure very similar to the Gauss-Seidel Iteration Method possible. Starting with zero initial values for the unknown a_k's, the first equation of this non-linear equation system gives a non-zero value for a_1. Using this value of a_1 and zero for the others, the second equation yields a_2. Selection of the smaller of the two roots may be more convenient. In case the discriminant of the k-th equation becomes negative, that equation can simply be omitted.

In this way all the unknowns a_k's are obtained. These values are once again substituted into the equation system to obtain better values for a_k's. The iteration is repeated until the relative errors of the unknowns become less than a prescribed value.

For practical purposes, exact solution of the non-linear equation system is not necessary. An approximate solution to this equation system also constitute a good initial wave form. In this case, only the calculation of the diagonal coefficients is required. An approximate value for a_k becomes

$$a_k = \pm \frac{Sv_k}{|C_{kk}|\ T_d} \tag{8}$$

The $\pm$ sign at the right-hand side of Eq.8 can be taken as $(-1)^{k+1}$. After the a_k's are determined, the initial wave form can be computed from the superposition of all the component waves as

$$x(t) = \sum_{\ell=1}^{N_\omega} \sum_{m=m_1}^{m_2} a_\ell \sin\left[(m-m_1)\omega_\ell \Delta t\right] \tag{9}$$

IMPROVEMENT OF THE SELECTED WAVE FORM

The initial wave form, determined in the foregoing paragraph, has a Fourier amplitude spectrum, oscillating about the target velocity response spectrum. For zero damping, the computed velocity response spectrum generally coincides very well with the Fourier amplitude spectrum at the selected frequencies. Hence, this is a convenient initial wave form. For non-zero damping, the ordinates of the velocity response spectrum are naturally smaller than the Fourier amplitude spectrum values. However, numerical applications indicate that this does not create any inconvenience in the subsequent computations.

The selected wave form may be improved for all frequencies between j_{min} and j_{max}. This is done in the frequency domain by multiplying, for example, k-th transform value by the correction coefficient

$$b_k = \frac{Sv_k}{Svc_k} \quad (k=j_{min}, \dots j_{max}) \tag{10}$$

where

Sv_k = Target spectrum value for the k-th frequency,
Svc_k= Computed spectrum value for the k-th frequency.

The same correction should be made on N+2-k th transform value: which is the complex conjugate of the k-th value.

After the modification of the transform values in the frequency domain, a new wave form is obtained in the time domain by the inverse Fourier transform. Since N is selected as power of two, the fast Fourier Transform can be used in both directions. Thus, the computation time can be minimized.

An iteration is necessary to obtain a good agreement between the target velocity response spectrum and the computed velocity response spectrum.

NUMERICAL EXAMPLE

To illustrate the method, the generation of two artificial accelerograms is presented.

Input parameters are:

$$N = 256, \quad N_\omega = 20, \quad j_{min} = 3, \quad j_{max} = 100, \quad \Delta t = 0.05 \text{ sn}$$
$$N_1 = 28, \quad N_2 = 52, \quad N_3 = 88, \quad N_4 = 112$$

The initial wave form is the superposition of 20 component waves with amplitudes determined by the approximate procedure. This wave is shown in Fig.1-a. The target spectrum, the Fourier amplitude spectrum and the computed velocity response spectrum of this wave are as shown in Fig.1-b. Since all the figures are based on line printer outputs, the ordinate of the plots is not precise.

The velocity response spectrum is improved in successive iterations and finally takes the form of Fig.1-j at the 10 th step. At this stage, the final wave form. Accelerogram A, is shown in Fig.1-ℓ.

The sample points on Fig.1-b, d, f, h, and j correspond to equally spaced frequencies. They are modified during the iteration.

Fig.1-c, e, g, i, and k compare the calculated velocity response spectrum values with the target spectrum values for a different set of frequencies. In this case, the sample points correspond to equally spaced period values. These sample points can not be controlled during the iteration.

Fig.2 presents exactly the same information for 5 % critical damping. The final wave form, Accelerogram B, is given in Fig.2-ℓ.

CONCLUSIONS

In this paper, the author presents a simple method for the generation of artificial accelerograms to be used in time history analyses.

The main conslusions of the study are:

1- The selection of the initial wave form is very important in the generation of artificial accelerograms, since a good initial time history increases the speed of the convergence and reduces the computational effort significantly. In the above procedure, a good agreement is obtained between the target spectrum and the computed spectrum in ten iterations.

2- By using an initial time history composed of a limited number of sinusoidal waves which start and terminate at randomly determined instants, we obtain a very convenient starting wave form. This approach eliminates the use of an envelope function. The final wave form strongly resembles natural earhquakes and possesses weak and strong portions.

3- In the generation of artificial accelerograms, a major part of the computation time is spent for the determination of response spectra. Therefore, whatever the method is, the fast and accurate determination of response spectra is always of primary importance. In another paper[2)], the author discusses a useful method for the calculation of response spectra. This method is successfully utilized in this study.

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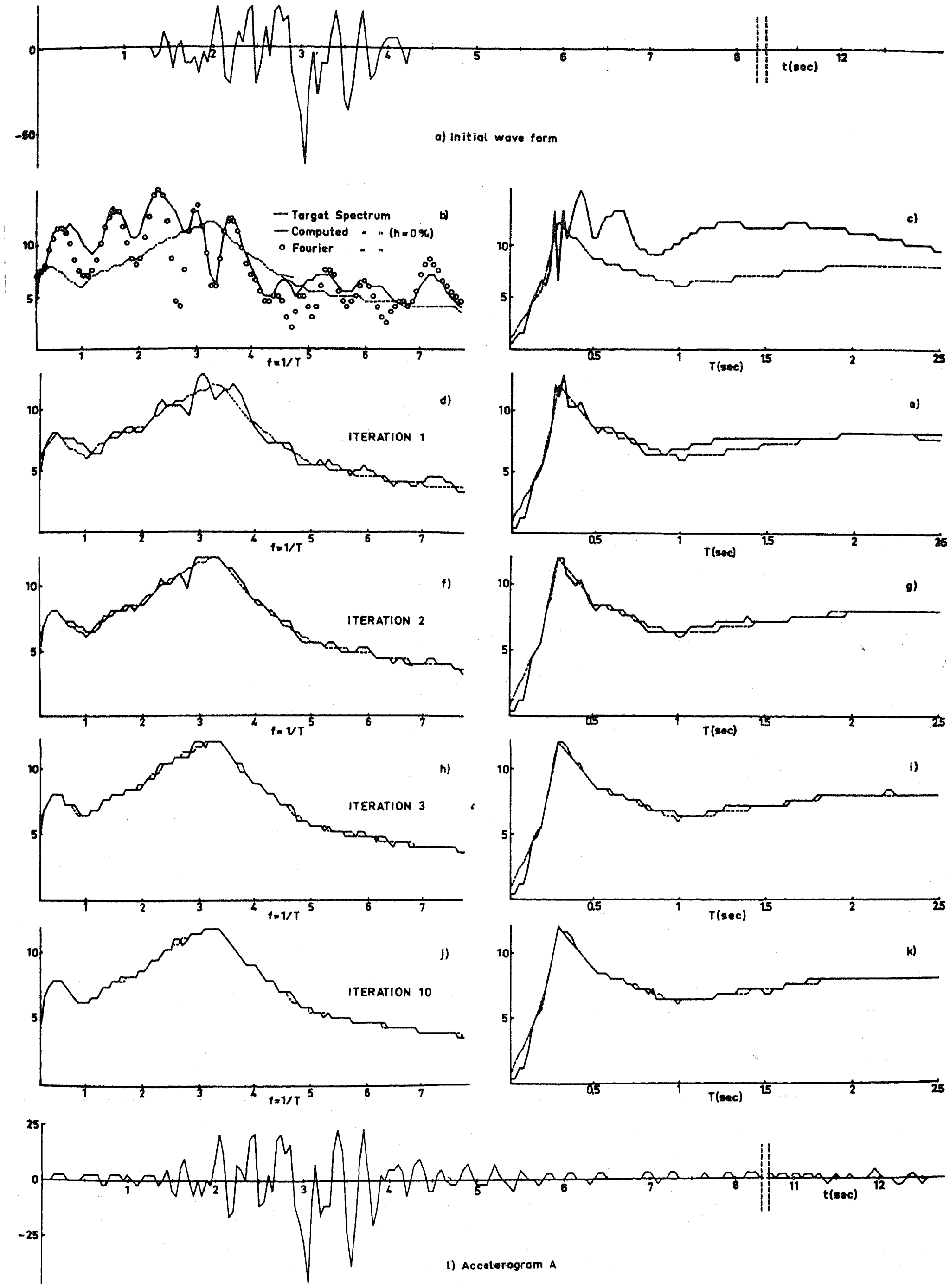
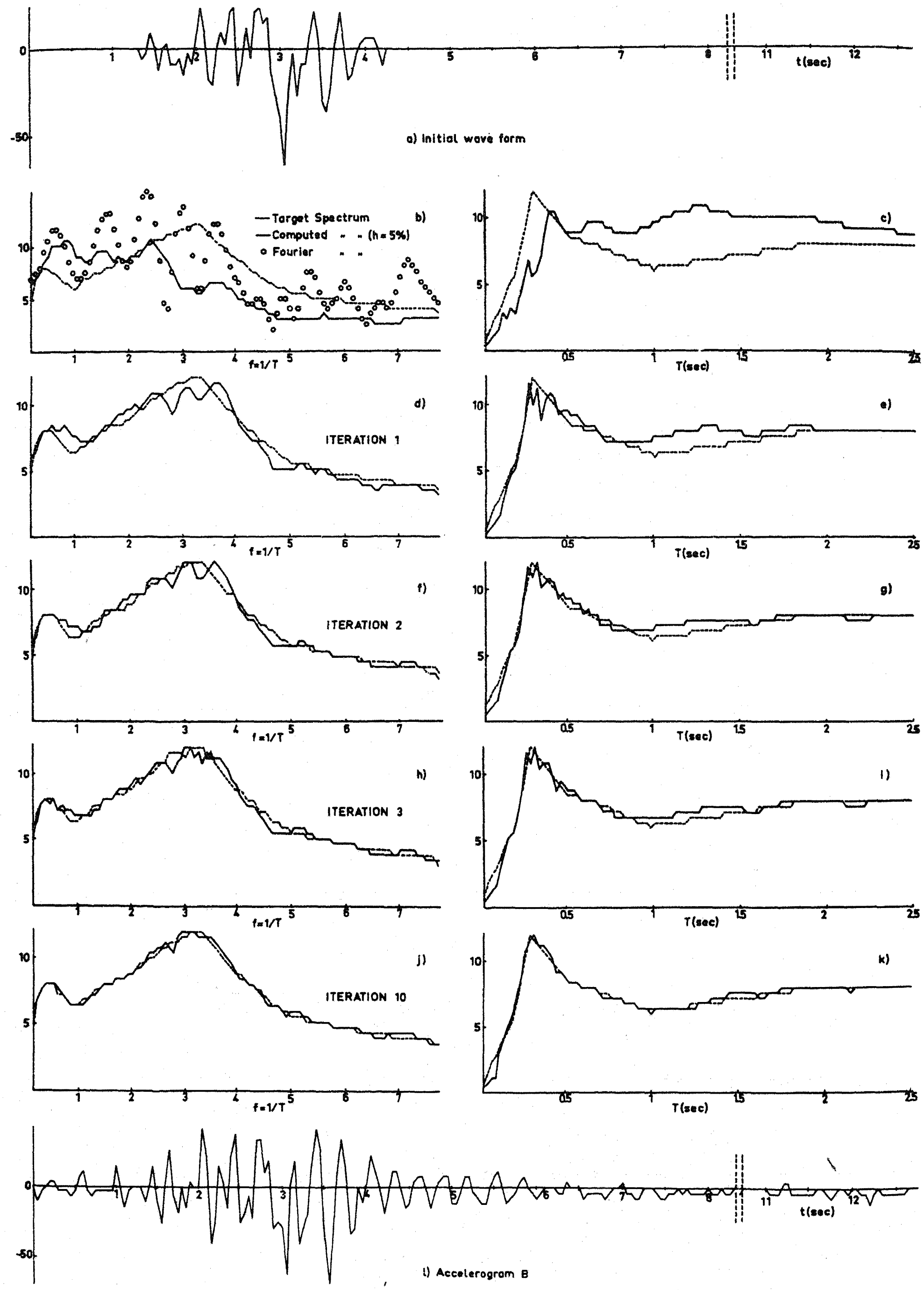


a) Initial wave form
t(sec)
Target Spectrum
Computed " " (h=0%)
Fourier " "
b)
c)
f=1/T
T(sec)
d)
e)
ITERATION 1
f)
g)
ITERATION 2
h)
i)
ITERATION 3
j)
k)
ITERATION 10
l) Accelerogram A

Figure 1



a) Initial wave form
t(sec)
— Target Spectrum
— Computed " " (h=5%)
o Fourier " "
b)
c)
f=1/T
T(sec)
d)
e)
ITERATION 1
f)
g)
ITERATION 2
h)
i)
ITERATION 3
j)
k)
ITERATION 10
l) Accelerogram B

Figure 2