

## RISK OPTIMIZATION CONSIDERING VIBRATIONAL BEHAVIOUR OF SUBSOIL

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### SUMMARY

According to theory of forced, damped vibration underlined by performance and damage observed in more than 2,300 buildings in Guatemala, it is advisable to consider probable subsoil periods in addition to building periods and soil-structure interaction. Vibrational data of 30 cases of soft soil show a relation between depth of soft layers and period which may be approximated by  $\underline{T} = a \exp \ln d^b$  or, perhaps for the time being more sensible by  $T = a \sqrt{d}$ .

### INTRODUCTION

Approximately periodic shaking has been noted by many observers with direct earthquake experience in particular if one has not been in the epicentral region. This impression is supported by theoretical physical considerations. Such site effects are caused by physical features of soft layers like their depth, consistency and configuration, and the picture is further complicated by factors like epicentral distance (non-linear transmission of earthquake energy), magnitude, source mechanism, etc.

Such "site effects" are occasionally mentioned (cf., e.g. (1)-(3)) and sometimes considered in special designs of buildings. Also some mechanical and electrical earthquake design provisions consider amplification due to interaction of force and item to be designed (4). However, earthquake building codes are generally mute in this respect. This is not only regrettable because an important damage causing and increasing factor is neglected but also because such buildings constitute the absolute majority of structures in an average earthquake zone and house most of the people.

### CONSIDERATIONS AND OBSERVATIONS

It is important to realize that the mass of shaking soft layers, on which many densely populated towns stand, is several to many hundred times larger than that of the buildings. This shaking soil mass represents the force and neglecting characteristic features of this force, and concentrating exclusively on building periods or, sometimes, on soil-structure interaction, reminds in the second case of an (unlikely) physicist who tries to analyse Barton's pendulum disregarding the force and horrifies any mechanical engineer who has been brought up with the tenet to avoid resonance between forces and pieces of machinery he designs.

The dramatic influence of characteristics of forces on amplitudes of distortion and therefore on damage to buildings may be seen when investigating forced oscillations. The amplitude of forced vibration is

$$D(\omega) = \frac{a}{\sqrt{(q - \omega^2)^2 + p^2 \omega^2}} \quad \text{Eq. 1}$$

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For  $p = 0$  (no resistance) the frequency of free oscillations of the system will be equal to the natural frequency. If  $q = \beta^2$ , and introducing the ratio of the frequency of the force to the frequency of free oscillations of the system ( $\omega/\beta = \lambda$ ), in order to be able to develop the resonance curve for forced, damped oscillations, the new equation will be

$$D(\lambda) = \frac{a}{\beta^2 \sqrt{(1 - \lambda^2)^2 + \gamma^2 \lambda^2}} \quad \text{Eq. 2}$$

if  $\gamma = p/\beta$ .

The maximum of that function will be reached for that value of  $\lambda$  for which the square of the denominator has a minimum and the minimum of  $\sqrt{(1 - \lambda^2)^2 + \gamma^2 \lambda^2}$  is reached when  $\lambda = \sqrt{1 - \gamma^2/2}$  and when this is equivalent to  $\gamma \sqrt{1 - \gamma^2/4}$ . Therefore the maximum amplitude will be

$$D_{\max} = \frac{a}{\beta^2 \gamma \sqrt{1 - \gamma^2/4}} \quad \text{Eq. 3}$$

It is now possible to construct the above mentioned graph of the function  $D(\lambda)$  in a general way, taking  $a$  and  $\beta$  as unity for the sake of definiteness. The curves obtained this way are resonance curves (Fig. 1). It is seen that for  $\gamma = 0.1$  the amplification factor for resonance is about 10 and still about 5 for  $\gamma = 0.2$ .

Although this represents only an approximation because, e.g. the system is neither linear particularly in the range of heavy to excessive loads, nor are buildings single-degree-of-freedom systems, transients, variation in damping, and changes in building period are not considered, it is still true that correspondence of predominant subsoil and building periods will lead to substantial amplification enhancing damage to these buildings considerably.

The author had an opportunity to assess the periods of more than 60 buildings damaged in Guatemala City by the M 7.5 earthquake on February 4, 1976. They were founded on comparatively similar soft subsoil and at a rather uniform distance of about 160 to 175 km from the epicenter. The periods of these buildings ranged from about 0.1 to 1.2 seconds. Plotting the number of damaged buildings against periods, a crowding in what may be called resonance bands was noted with about twice the number of cases per band as if performance would have been random. Also damage to buildings in these bands was heavier. A further support came from statistical evaluation of about 2,300 buildings which showed much heavier structural damage in storey groups coinciding with these resonance bands.

In view of this it is felt that site characteristics should receive more attention than so far, in particular as it appears feasible to produce guidance similar to formulas included in codes as a guideline for fundamental periods of buildings.

As a first attempt, it was tried to collect data available so far in order to see whether some systematic behaviour could be deduced (Fig. 2). Although the data is beset with many uncertainties and also definitions like depth of soft layer(s) may be debated, it appears that some useful generalizations could be possible, if not now then later after more and

better defined data has been collected. Data shown in Fig. 2 stems from various kinds of observations (made almost exclusively in Japan), from theory and models. It is therefore not surprising that scatter is considerable, but this is as well the rule for building periods (cf., e.g. (5)-(8)), and one could argue that factors causing deviation of actual building periods from those indicated by formulas in codes are not less numerous than those influencing subsoil periods.

One may try to represent the apparent relation between depth of soft subsoil and period by

$$T = a \exp \ln d^b \quad \text{Eq. 4}$$

or, by a simpler and for the time being more sensible relation like

$$T = a \sqrt{d} \quad \text{Eq. 5}$$

In Eq. 4  $a$  would be about 0.04 and  $b$  about 0.8 and this graph would rise somewhat steeper than the one produced by Eq.5 if  $a$  is taken as abt. 0.1, up to a depth of about 5,000 m.  $T$  would be the natural period in sec. and  $d$  is the depth to hard strata in meters.

#### CONCLUSIONS

As shown, not only deductions from theory of forced, damped vibrations but actual damage statistics demonstrate the adverse effect of site characteristics which lead to amplification of shaking in buildings and, consequently, to heavier damage. It would, therefore, be wise not to wait for further earthquakes to yield suitable information but to pursue the pertinent questions actively and without undue delay. This is particularly advisable in areas where population densities are high or where considerable values are concentrated.

#### REFERENCES

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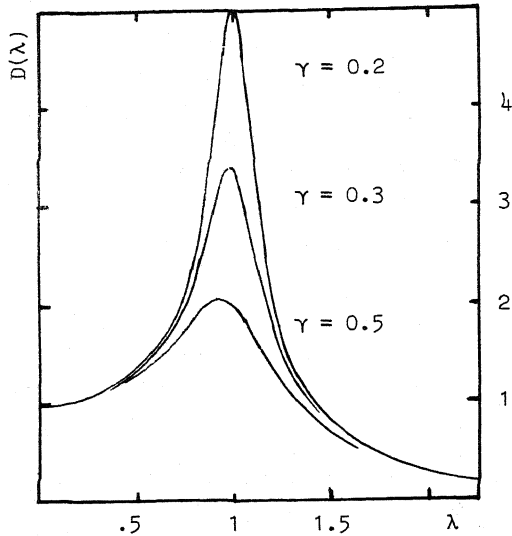


Fig. 1. Amplification as function of the frequency of the force to the frequency of free oscillations of the system ( $\omega/\beta = \lambda$ ) and depending on damping ( $\gamma$ ). It is known that damping in buildings is assumed to be smaller than shown here. As damping is reduced (e.g. due to incipient damage) the maximum shifts to higher values of  $\lambda$  and amplification increases.

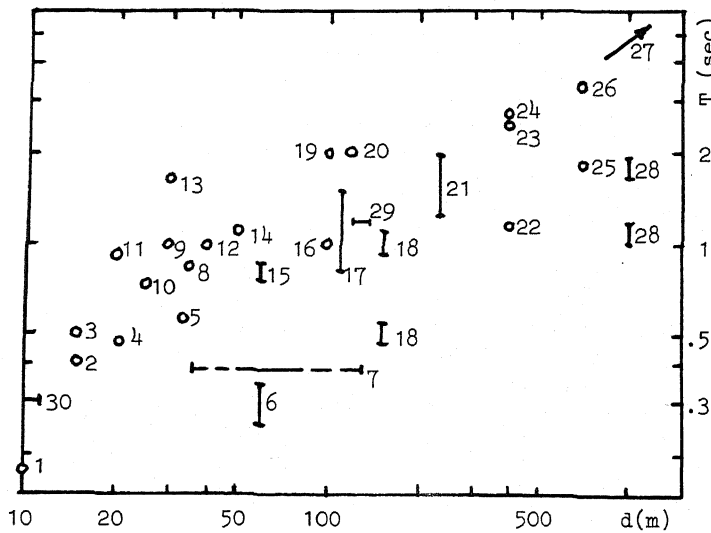


Fig. 2. A rather heterogeneous sample of periods of maximum amplifications, accelerations, velocities from observations and models is shown here against depth of soft subsoil. In spite of this small and mixed sample, some correlation is visible which warrants further investigations, in particular, when considering that resonance avoidance in a statistical sense would greatly improve building performance.

To economize on space, references are reduced to the minimum.  
 1 = 6th WCWW, 2-183, 228, 230, 231; 2 = *ibid.*; 3 = 6th WCEE, 2-26; 4 = 6th WCEE, 2-168; 5 = *ibid.*; 6 = 6th WCEE, 6-232; 7 = Bull. Seism. Soc. Am., 67, 1430 & own data; 8 = *cf.* 4; 9 = 5th WCEE, 43; 10 = 6th WCEE, 2-76; 11 = *ibid.*; 12 = *ibid.*; 13 = *cf.* 4; 14 = *cf.* 4; 15 = 6th WCEE, 2-66; 16 = 6th WCEE, 2-499; 17 = 6th WCEE, 2-115; 18 = Earthqu. Eng. & Struct. Dyn., 6, 401, 408; 19 = *cf.* 9; 20 = 5th WCEE, 332; 21 = *cf.* 17; 22 = *cf.* 1; 23 = *cf.* 1; 24 = *cf.* 3; 25 = *cf.* 1; 26 = *cf.* 3; 27 = *cf.* 16; 28 = *cf.* 18; 29 = Earthqu. Eng. & Struct. Dyn., 5, 157-179; 30 = 6th WCEE, 1-44, 1-47.