A SINGLE-DEGREE-OF-FREEDOM MODEL FOR NON-LINEAR SOIL AMPLIFICATION

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INTRODUCTION

For proper understanding of soil behavior during earthquakes and assessment of a realistic surface motion, studies of the large-strain dynamic response of non-linear hysteretic soil systems are indispensable. Most of the presently available studies are based on the assumption that the response of a soil deposit is mainly due to the upward propagation of horizontally polarized shear waves from the underlying berdock. Equivalent-linear procedures, currently in common use in non-linear soil response analysis, provide a simple approach and have been favorably compared with the actual recorded motions in some particular cases. In this regard, Idriss and Seed (1968), Seed and Idriss (1969), schnabel et.al. (1972) should be mentioned strain compatibilty in these equivalent-linear approaches is maintained by selecting values of shear moduli and damping ratios in accordance with the average soil strains, in an iterative manner. Truly non-linear constitutive models with complete strain compatibility have also been employed. (Constantopoulos, 1973; Papadakis, 1973; Faccioli et.al., 1973; Streeter et.al, 1974 Joyner and Chen, 1975).

Although these studies are complete in their methods of analysis, they inevitably provide applications pertaining only to a few specific soil systems and do not lead to general conclusions about the seismic behavior of soil deposits.

This paper attempts to provide a general picture of the soil response through the use of a single-degrees-of-freedom non-linear-hysteretic model. Although the investigation is based on a specific type of non-linearity and a set of dynamic soil properties, the method described does not limit itself to these assumptions and is equally applicable to other types of nonlinearity and soil parameters.

DESCRIPTION OF THE SOIL LAYER AND PROPERTIES

This paper considers the responses associated with the vertical propagation of SH waves through a non-linear hysteretic soil layer as shown in Fig.1. The layer rests on a rigid bedrock half-space and the soil layer-bedrock system is assumed to extend to infinity in the horizontal directions.

Dynamic properties of soil deposits have been investigated for various soil types (e.g. Hardin and Black, 1969; Ladd and Edgers, 1972; Hardin and Drnevich, 1972 a-b). These studies indicate that the Masing hypothesis is satisfied and, by an appropriate choice of parameters, the Ramberg-Osgood constitutive model shown in Fig.2 can approximate the dynamic behavior of real soils. The key parameters of this model are the maximum shear stress τ_m, and the low strain shear modulus G_m. On the basis of these parameters the initial loading curve can be given by.

$$\frac{\tau}{\tau_m} = \frac{\frac{\gamma}{\gamma_r}}{\frac{\gamma}{\gamma_r} + 1} \qquad (1)$$

where $\gamma_r = \tau_m/G_m$ is called the reference strain, (Hardin and Drnevich, 1972b). For normally consolidated, uniform cohesive soil and deposits the maximum shear stress can be assumed to vary linearly with the depth,

$$\tau_m = K_1 z, \tag{2}$$

and the low-strain shear modulus can be assumed to very linearly with the square root of depth

$$G_m = K_2 \sqrt{z} \tag{3}$$

Where z is the depth measured from the surface of the deposit and, K_1 and K_2 are constants based on physical properties of the medium.

METHOD OF ANALYSIS

An analogous single-degree-of freedom (SDF) model representing the first mode characteristics of the actual multi-degree-of-freedom soil deposit can be derived by setting the natural frequency of the SDF system equal to the first modal frequency of the soil layer and by insuring that for all strain levels the amount of energy dissipated per cycle by the SDF model is equal to the amount dissipated by the soil layer responding in its first mode. For nonlinear systems responding in small but non-zero strain levels the term frequency applies to the apparent frequency of vibration. For non-linearities of the type considered the apparent frequency if vibration in the small but non-zero ranges of strain can be satisfactorily approximated by employing the secant modulus of shear, G, corresponding to these strain ranges instead of the initial tangent modulus, G_m.

The equation of shear vibration of a soil column of unit cross sectional area can be given by

$$\frac{\partial \left(G_m \frac{\partial u}{\partial z}\right)}{\partial z} = \rho \frac{\partial^2 u}{\partial t^2} \tag{4}$$

Where u denotes the lateral shearing displacement, z is the depth measured along the vertical axis, and ρ is the mass density (Fig.1). For a soil deposit vibrating in its r^{th} mode, with a circular frequency of vibration, ω_r, the shearing displacements are given by:

$$u(z,t) = u_r(z) \sin \omega_r t \tag{5}$$

where

$$u_r(z) = \omega_r^2 \int_0^z \frac{1}{G_m(\beta)} \int_0^\beta \rho(\beta) u_r(\beta) d\beta \, d\beta - \int_0^H \frac{1}{G_m(\beta)} \int_0^\beta \rho(\beta) u_r(\beta) d\beta \, d\beta \tag{6}$$

conforming to the boundary conditions of

$$\frac{du_r}{dz} = 0 \quad \text{at} \quad z = 0 \tag{7}$$

$$u_r(H) = 0 \tag{8}$$

In these equations β is a dummy variable and H denotes the total depth of the soil deposit. Replacing G_m by the secant modulus of rigidity, G, as given below :

$$G = \frac{\tau}{\gamma} = \frac{G_m \tau_m}{\tau_m + \gamma G_m} \tag{9}$$

and through the use of an iterative scheme with an initial trial function of the form :

$$u_1(z) = u_o \;\{1 - (\frac{z}{H})^2\} \tag{10}$$

Eq. 6 yields the following first mode shape

$$u_1(z) = \frac{u_o}{1+\frac{35}{36}(\frac{u_o}{U_r})}\;\{1 - \frac{7}{6}(\frac{z}{H})^{3/2} + \frac{1}{6}(\frac{z}{H})^{7/2} + (\frac{u_o}{U_r})\;\frac{35}{36} - \frac{7}{6}(\frac{z}{H}) + \frac{7}{36}(\frac{z}{H})^4\} \tag{11}$$

and the following first mode frequency of vibration

$$\omega^2_{1a} = \omega^2_1 \;\{1/\;1 + 1.07\;(\frac{u_o}{U_r})\} \tag{12}$$

after a single iteration step.

In Eq. 10, 11 and 12 u_o stands for the surface displacement in the first mode of vibration, U_r will be termed the reference displacement" (analogous to the reference strain, γ_r) and is given by following equation:

$$U_r = \int_0^H \gamma_r \; dz \tag{13}$$

In Eq. 12 ω_{1a} is first mode apparent (strain dependent) frequency of vibration and ω_1 is the small strain first mode frequency of vibration.

Energy Dissipated Per Hysteresis Cycle The area bounded by the unloading and reloading paths in Fig. 2 is equal to the energy dissipated, e, by an infinitesimal soil element going through one complete hysteresis cycle with a normalized strain amplitude, ϵ , and is given by :

$$e = 2\tau_m\gamma_r\{\int_0^2 \frac{2\eta d\eta}{\eta+2} - \frac{1}{2}(2\varepsilon)\;\frac{2\;(2\varepsilon)}{2\varepsilon + 2}\} \tag{14}$$

where η is a dummy variable. Carrying out the integration the following equation is obtained.

$$e = \tau_m\gamma_r\;\{8\varepsilon \;-\; 8\ell n(\varepsilon\;1) \;-\; \frac{4\varepsilon^2}{\varepsilon\;1}\} \tag{15}$$

The normalized strain, ε, is given by $\varepsilon = \gamma/\gamma_r$. Replacing γ by,

$$\gamma = \frac{du_1(z)}{dz} \tag{16}$$

where $u_1(z)$ is given by Eq. 11, and expressing γ_r in terms of U_r and z/H, ε can be obtained as:

$$\varepsilon = \frac{\frac{3}{2}\;(u_o/U_r)}{1 + \frac{35}{36}(u_o/U_r)}\;\{-\frac{7}{4} + \frac{7}{12}(\frac{z}{H})^2 + (\frac{u_o}{U_r}) - \frac{7}{3}(\frac{z}{H})^{1/2} + \frac{7}{9}(\frac{z}{H})^{5/2}\} \tag{17}$$

The total energy lostby the soil deposit goin through a complete cycle of oscillation, E, is equal to the integral of Eq. 14 throughout the depth of the deposit (Erdik, 1979),

$$E_{soil} = U_r\;\tau_m \int_0^1 3(\frac{z}{H})^{3/2}\;(8\;\varepsilon - 8\ell n\;(\varepsilon + 1)\; - \frac{4\varepsilon^2}{\varepsilon+1}\cdot d(\frac{z}{H}) \tag{18}$$

where ε is given by Eq.17 and τ_m stands for:

$$\bar{\tau}_m = \frac{1}{H} \int_0^H \tau_m \, dz \qquad (19)$$

For given u_o/U_r the above equation can be numerically integrated to obtain $E_{soil}/(U_r \cdot \bar{\tau}_m)$. Integration results are shown in Table I for representative values of u_o/U_r.

TABLE I

u_o/U_r	$E_{soil}/U_r \bar{\tau}_m)$	$E_{SDF}D(U_r \bar{\tau}_m)$
0.5	0.105	0.115
1.0	0.556	0.575
2.0	2.374	2.378
5.0	11.602	10.899
10.0	31.579	29.240

<u>Analogous Single Degree of Freedom Model</u> Although the actual determination of the analogous SDF model involved some trial-and-error procedures, for purposes of presentation the model will be dricetly proposed and then tested to see how closely it resembles the first modal characteristics of the soil layer.

Let y denote the mass displacement and F denote the spring force of a simple SDF system consisting of a mass, M, and a nonlinear hysteretic spring. The constitutive characteristics of the spring satisfy the Masing hypothesis and are given by the following set of equations.

Initial loading path (Virgin Curve) $$\frac{F}{F_r} = \frac{(y/U_r)}{1+1.07\,(y/U_r)} \qquad (20)$$

Unloading or reloading path $$\frac{F}{F_r} = \frac{2(y/U_r)}{2+1.07(y/U_r)} \qquad (21)$$

where the origin of the coordinate system is transferred to the respective points of unloading and reloading.
In Eq. 20 and 21 U_r is the same reference displacement used in mathematical modelling of the soil deposit, and F_r is some reference spring force.

The energy lost by hysteresis in one complete oscillation cycle of the analogous SDF system, E_{SDF}, can be given as follows (Erdik, 1979) :

$$E_{SDF} = U_r \bar{\tau}_m \left\{ \frac{10(y/U_r)}{1.07} - \frac{10\ell n\,\{1.07(y/U_r)+1\}}{(1.07)^2} - \frac{5(y/U_r)^2}{1+1.07(y/U_r)} \right\} \qquad (22)$$

Values of $E_{SDF}/(U_r\,\bar{\tau}_m)$ computed on the basis of Eq. 22 are given in Table I for representative values of y/U_r.

Inspection of Table I reveals that if y, the SDF mass displacement, is taken equal to u_o, the soil surface displacement, the respective energies dissipated by the soil and SDF systems in one hysteresis cycle will be approximately equal to each other for all ranges of deformation.

The apparent, amplitude dependent, frequency of vibration to the small amplitude frequency of vibration of the analogous SDF system can be given by the following equation (Erdik, 1979).

$$\frac{(\omega_{SDF})a^2}{(\omega_{SDF})^2} = \frac{1}{1 + 1.07(y/U_r)} \tag{23}$$

Which can be seen to be identical to Eq. 12 if y is taken analogous to u_o.

In summary, the SDF system described by the constitutive relations given by Eq. 20 and 21 satisfies the first modal energy dissipation and frequency characteristics of the soil deposit, and the mass displacement of the SDF system is analogous to the surface displacement of the soil deposit, provided that the reference displacement and the small amplitude frequency of the analogous SDF system are set equal to those of the soil deposit, respectively.

For a given soil deposit U_r can simply be computed through the use of Eq. 13. To compute ω_{SDF} for any soil deposit, use of discrete Stodola Vianello procedure is recommended (Crandall, 1956). A good study of computational procedures for ω_{SDF} can be found in Dobry, et.al.(1976).

Response of the analogous nonlinear SDF model due to an earthquake excitation can be obtained through conventional numerical integration procedures.

ASSESMENT OF THE ACCURACY

The approximation provided by this analogy is tested for the following two cases. The first case involves the response of a 200 m deep soil deposit to the N21E component of 7.21.1952 Taft record scaled to a peak acceleration of 0.7g. The surface velocity as given by Joyner and Chen (1975), and as obtained on the basis of the analogous SDF model, with f_1 = 0.5Hz. and U_r = 22.7 cm, are provided in Fig.3. The second test case involves the response of the 50ft. deep soil deposit to the SOOE component of El-Centro record scaled to a peak acceleration of 0.1g. The surface velocity as given by Martin and Seed (1978), and as obtained on the basis of the analogous SDF model, with f_1 = 3.1Hz.and U_r = 1.1 cm, are provided in Fig. 4. Both of these comparisons indicate the excellent approximation provided by the analogous SDF model.

PARAMETRIC STUDIES AND CONCLUSIONS

Parametric studies conducted using different accelerograms as the bedrock motion (Erdik, 1979) have resulted in the iso-amplification contour envelopes given in Fig. 5 for different peak bedrock velocity and accelerations. The ranges of the reference displacement in the vertical axis and the fundamental frequency of vibration in the horizontal axis covers a wide group of soil deposits of importance in earthquake engineering. It can be assessed that : (1) for a given soil deposit, the greater the intensity o of bedrock motion the smaller is the acceleration, (2) for a given soil deposit and bedrock motion the velocity amplification is greater than that of acceleration, (3) for peak bedrock velocities greater than 20 cm/sec. the amplification is always less than two, (4) for peak bedrock accelerations greater than 0.2g the amplification is always less than 1.5 and the bedrock accelerations with peaks above 0.5g are de-amplified. Note that, in general the ATC-3 stipulations for site effects conform with these assessments.

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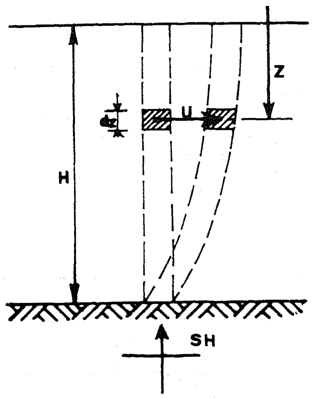


FIG.1 SOIL DEPOSIT HALF-SPACE

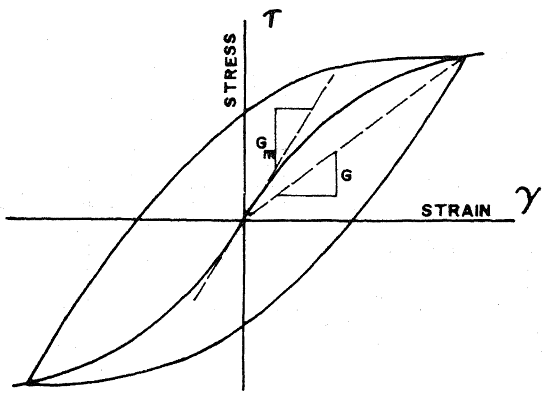


FIG.2 RAMBERG-OSGOOD CONSTITUTIVE MODEL

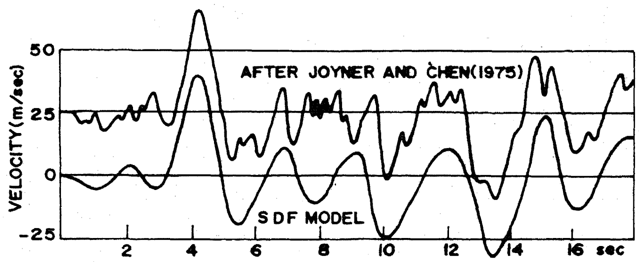
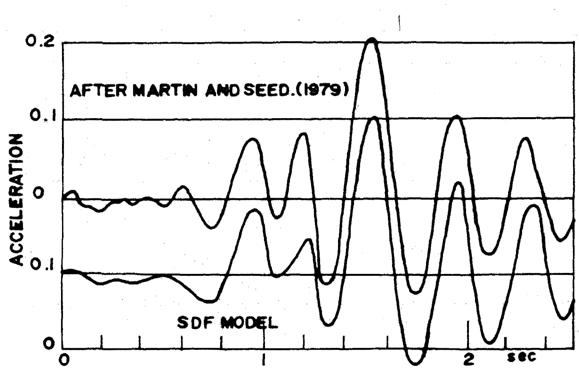


FIG.3 FIRST TEST CASE



FIG, 4, SECOND TEST CASE

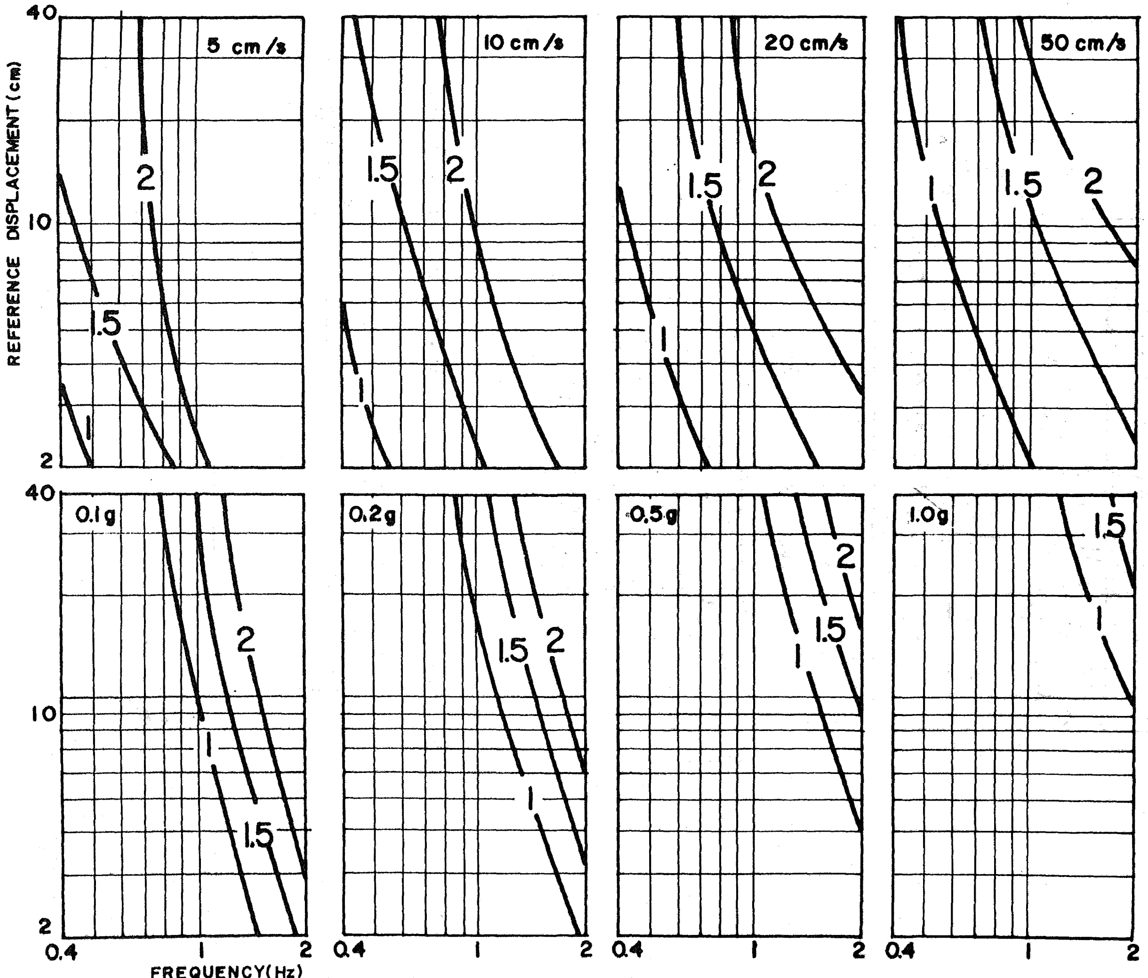


FIG.5 ISO-VELOCITY AND-ACCELERATION AMPLIFICATION CONTOURS