# A METHOD FOR NUMERICAL CALCULATION OF DYNAMIC-CHARACTERISTICS-CONVERSION OF SEISMOMETERS

by

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## SUMMARY

This paper presents a method for converting the numerical output waveform of a actual (real) seismometer into the output waveform of an imaginary (virtual) seismometer of desirable characteristics. The major drawback of conventional methods used for this purpose is their incapability of real-time processing. The proposed method for conversion of dynamic characteristics of seismometers is simple in nature and is capable of real-time processing which allows use of seismometers based on easy-to-handle accelerometers for measurement of velocity and displacements as well, bringing in a considerable saving in labor in measurement and maintenance.

## INTRODUCTION

In earthquake engeneering, velocity and displacement waveforms are often obtained by integration of acceleration waveforms. If the characteristics of a seismometer used for vibration measurement is unsuitable, the vibration waveform observed is found to be very much distorted, necessiating a correction in order to infer the real vibration waveform. In other cases where vibration waveforms obtained by various measurements are compared, waveform processing to unify the instrument characteristics ("instrument characteristics conversion processing")is required for the output waveform of each instrument.

Calculation for correction of waveform distorsion can be performed by "instrument characteristics conversion processing". Further, "instrument characteristics conversion processing" also enables differentiation and integration of waveforms, which is understandable by the fact that the acceleration of earthquake motion is measured by short-period seismometers and its displacement by long-period seismometers. In this case, long-period and short-period components are filtered out in integration and differentiation, respectively, allowing automatic reduction of components liable to disturb differentiation and integration operations.

Conventional methods[1] et al. related to diffentiation, integration and distortion-correction of waveform are not capable of real-time processing due to limited number of waveform data that can be processed. However, if such calculations can be performed on real-time basis with high accuracy being maintained, not only the use of an accelerometer, easier to be handled and installed than a displacement meter, will be allowed for continuous monitoring of displacement, but also the installation of only one seismometer will be required for obtaining acceleration, velocity and displacement on real time.

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This paper describes with numerical example the seismometer-characteristics-conversion method capable of real-time processing in which differentiation, integration and calculations for distortion-correction of waveforms are replaced by calculations for conversion of instrument characteristics.

## CALCULATION METHOD2)

Suppose that the equation of motion of the 'real' seismometer having natural circular frequency and damping factor of $\omega_1$ and $h_1$ , respectively, is expressed by the following equation:

$$\ddot{x}_1 + 2h_1\omega_1\dot{x} + \omega_1^2 x = m\ddot{y} \qquad (1)$$

where $\ddot{y}$ is the real accelaration waveform, $x_1$ is the relative displacement of the 'real' seismometer pendulum to the ground, and $\dot{x}_1$ and $\ddot{x}_1$ first and second derivatives of $x_1$ with respect to time, respectively.

In general, the value obtained as an output of a seismometer is $x_1$ or $\dot{x}_1$. The coefficient m is a constant depending on the type of a seismometer as shown in Table 1. To convert the instrument characteristics is to obtain the output of a 'virtual' seismometer in the form of natural circular frequency $\omega_2$ and damping factor $h_2$ based on the output $x_1$ or $\dot{x}_1$ of the 'real' seismometer.

Case 1 : 'Real' seismometer output is relative displacement (x-output).

First, response is obtained when a value n-times the output of the 'real' seismometer is input to the 'virtual' seismometer.

$$\ddot{x}_2 + 2h_2\omega_2\dot{x}_2 + \omega_2^2 x_2 = nx_1 \qquad (2)$$

The coefficient n is determined with respect to combination of the types of 'real' and 'virtual' seismometers (described later ). Eq.2 is solved by numerical calculations, where all of $\ddot{x}_2$, $\dot{x}_2$ and $x_2$ are calculated at each time step. From these values, the value X defined by the following equation is calculated:

$$X = \ddot{x}_2 + 2h_1\omega_1\dot{x}_2 + \omega_1^2 x_2 \qquad (3)$$

Calculation for conversion of instrument characteristics consists of determination of the value of X as detailed below and determination of value of coefficient n on the right hand side of Eq.2.

By eliminating $x_1$ , $x_2$ and their derivatives from Eqs.1, 2 and 3, the following equation is obtained:

$$\ddot{X} + 2h_2\omega_2\dot{X} + \omega_2^2 X = nm\ddot{y} \qquad (4)$$

Comparing Eq.4 with Eq.1, it is seen that X is equal to the output of the 'virtual' seismometer (x-output type) for the input of real acceleration $\ddot{y}$. The characteristics of the 'real' seismometer is not included in the converted output X any more.

The coefficient nm on the right hand side of Eq.4 is a constant depending on the type of a 'virtual' seismometer as shown in Table 1. The value of the coefficient n can be determined from this value. Table 2 shows the value of n for all combinations of the types of the 'real' and 'virtual' seismometers.

Case 2 : 'Real' seismometer output is relative velocity ($\dot{x}$-output). Proceeding in the same manner as in Case 1, the response of the 'virtual' seismometer is obtained first, when a value n'-times the output of the 'real' seismometer is input to it.

$$\ddot{x}_2 + 2h_2\omega_2\dot{x}_2 + \omega_2^2 x_2 = n'\dot{x}_1 \qquad (5)$$

The value of coefficient n' is determined with respect to the combination of the type of the 'real' and 'virtual' seismometers. The value Y defined by the following equation is calculated from the values of $\dot{x}_2$ and $x_2$ obtained at each time step, and the value of $\int x_2 dt$ (See Appendix A) computed from $\dot{x}_2$, $x_2$ and $n'\dot{x}_1$ :

$$Y = \dot{x}_2 + 2h_1\omega_1 x_2 + \omega_1^2\int x_2 dt \qquad (6)$$

In fact, finding the value of Y completes the calculation of conversion. As in Case 1, the following equation is obtained by eliminating $x_1$, $x_2$ and their derivatives from Eqs.1 , 5 and 6.

$$\ddot{Y} + 2h_2\omega_2\dot{Y} + \omega_2^2 Y = n'm\ddot{y} \qquad (7)$$

Therefore, Y is equal to the output of the 'virtual' seismometer (x-output type). The value of n'm is given in Table 1, with respect to the types of 'virtual' seismometers, from which the value of n' is determined depending on the combination of the types of 'real' and 'virtual' seismometers as shown in Table 2 .

To sum up, the entire calculations for characteristics conversion boils down to determine simply the value of X or Y. The calculations of Eqs.2 and 3 or Eqs.5 and 6 to determin this value are mainly response calculations of one-degree-of-freedom system. Various methods[3)&4)] are available for analyzing the response of such a system. The calculation is not restricted by the number of sampling data or sampling time interval, and can be performed with a small-capacity computer. All the calculations in next section are done with a micro computer with 6K byte RAM.

## NUMERICAL EXAMPLE

Input Waveform. Instead of an actually observed earthquake motion, the waveforms shown in Fig.1 are adopted as a model ground motion in order to verify the method described above. The numerical formulas of the model ground motion are given in Table 3 . Fig. 2 represents Fourier Spectra of the model ground motion.

Examples of Characteristics Conversion Calculation. For this model ground motion, the characteristics of the 'real' seismometer (x-output type) are supposed to be those given in Table 4 . In the characteristics of each seismometer, the damping factor differs from the one generally recommended. Fig. 3 shows the results when the waveforms measured by the 'real' seismometers were simulated by calculating the input waveform at every 1/100 sec.. Fig.4 shows the examples of 'virtual' seismometer outputs calculated from the simulated outputs of each 'real' seismometer when the characteristics of the 'virtual' seimometers were supposed to be those given in Table 4. In this figure, the directly observed waveforms by the seismomemeters having the characteristics of 'virtual' seismometers are shown in dotted lines. However, the calculated waveform coincides with the directly observed waveform to such an extent that the dotted line cannot be discriminated in each case.

A numerical example for a 'real' seismometer of relative velocity output type is shown next. Suppose that this 'real' seismometer is an accelerometer of natural frequency $f_n=0.25$ Hz and damping factor $h = 3$. Fig. 5(a) shows the simulation result of the 'real' seismometer. Fig.5(b),5(c) and 5(d) representing acceleration, velocity and displacement, respectively, are the outputs of the 'virtual' seismometers in Table 4 calculated on the basis of the results in Fig. 5(a). The correct answers are also shown in dotted lines in these figures, but they cannot be clearly discriminated due to good coincidence with the converted waveforms.

As seen from the example given above, this calculation method allows easy conversion of seismometer characteristics.

## CONCLUSION

A numerical calculation method has been described that realizes differentiation, integration and distortion correction calculations by converting the characteristics of seismometers. If the accuracy of AD conversion of waveform is sufficiently high, the proposed method will yield accurate acceleration, velocity and displacement of ground motion as is expected from theory.

## ACKNOWLEDGEMENT

The auther would like to express his gratitude to Mr. Akio Saito, member of Structure Lab., Railway Technical Research Inst.,J.N.R., his drawing and data arrangement.

## REFERENCE

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## APPENDIX A: Calculation of $\int x\,dt$.

Equation of motion: $\ddot{x} + 2h\omega\dot{x} + \omega^2 x = a$ (A-1)

If sampling time interval is $\Delta t$, the equation of motion between the step i+1 is approximately expressed as follows according to the theorem of medium value:

$$\frac{\dot{x}_{i+1}-\dot{x}_i}{\Delta t} + 2h\omega\frac{x_{i+1}-x_i}{\Delta t} + \omega^2\frac{\int_0^{t_{i+1}} x\,dt - \int_0^{t_i} x\,dt}{\Delta t} = \frac{a_{i+1}+a_i}{2}$$

Therefore,

$$\int_0^{t_{i+1}} x\,dt = \int_0^{t_i} x\,dt - \frac{2h(x_{i+1}-x_i)}{\omega} - \frac{\dot{x}_{i+1}-\dot{x}_i}{\omega^2} + \frac{(a_{i+1}+a_i)\Delta t}{2\omega^2} \quad \text{(A-2)}$$

If the response of Eq.(A-1) is to be calculated on the assumption that values vary linearly within every $\Delta t$ interval, exact results can be obtained from Eq.(A-2). In general, if the response calculation is completed up to

Step (i+1), all the turm on the right hand side of Eq.(A-2) are known. Hence calculation of $\int^{t} x\,dt$ at Step (i+1) can be easily done by the use of Eq.(A-2).

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Table 1. Coefficient m

| Type | x-output | $\dot{x}$-output |
|---|---|---|
| A | $\omega^2$ | $2h\omega$ |
| V | $2h\omega$ | 1 |
| D | 1 | * |

notice) A,V and show Accelerometer, Velocity meter and Displacement meter, respectively.

Table 2. Coefficient n and n'

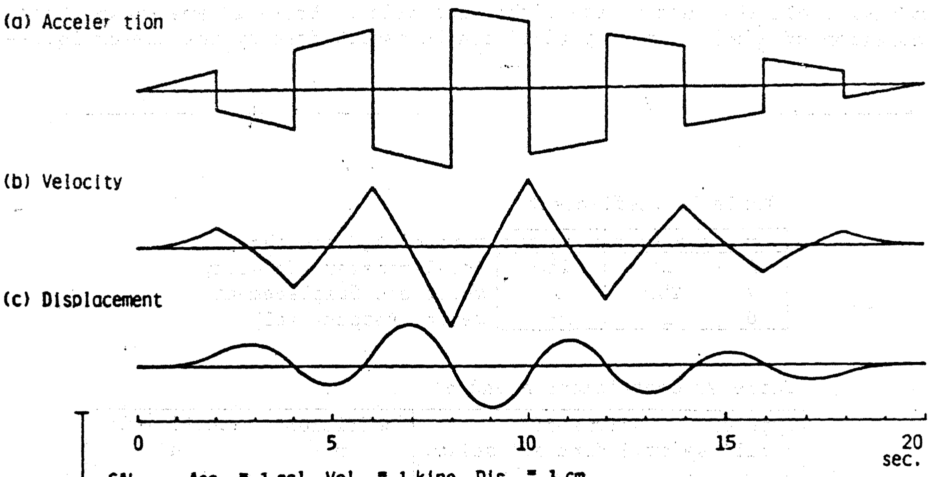
| Type of 'Real' seism. $\omega_1$, $h_1$ | Type of 'Virtual' seism. $\omega_2$, $h_2$ | Coefficient n | Coefficient n' |
|---|---|---|---|
| A | A | $\omega_2^2/\omega_1^2$ | $\omega_2^2/2h_1\omega_1$ |
| A | V | $2h_2\omega_2/\omega_1^2$ | $2h_2\omega_2/2h_1\omega_1$ |
| A | D | $1/\omega_1^2$ | $1/2h_1\omega_1$ |
| V | A | $\omega_2^2/2h_1\omega_1$ | $\omega_2^2$ |
| V | V | $2h_2\omega_2/2h_1\omega_1$ | $2h_2\omega_2$ |
| V | D | $1/2h_1\omega_1$ | 1 |
| D | A | $\omega_2^2$ | * |
| D | V | $2h_2\omega_2$ | * |
| D | D | 1 | * |

Table 3. Numerical Formulas of Model Ground Motion

| Time t | Acceleration | Velocity | Displacement |
|---|---|---|---|
| 0- 2 | $t/8$ | $t^2/16$ | $t^3/48$ |
| 2- 4 | $-t/8$ | $1/2 - t^2/16$ | $-2/3 + t/2 - t^3/48$ |
| 4- 6 | $t/8$ | $-3/2 + t^2/16$ | $14/3 - 3t/2 + t^3/48$ |
| 6- 8 | $-t/8$ | $3 - t^2/16$ | $-40/3 + 3t - t^3/48$ |
| 8-10 | $5/3 - t/12$ | $5 - (t-20)^2/24$ | $-64 + 5t - (t-20)^3/72$ |
| 10-12 | $-5/3 + t/12$ | $-10/3 + (t-20)^2/24$ | $424/9 - 10t/3 + (t-20)^3/72$ |
| 12-14 | $5/3 - t/12$ | $2 - (t-20)^2/24$ | $-280/9 + 2t + (t-20)^3/72$ |
| 14-16 | $-5/3 + t/12$ | $-1 + (t-20)^2/24$ | $152/9 - t + (t-20)^3/72$ |
| 16-18 | $5/3 - t/12$ | $1/3 - (t-20)^2/24$ | $-56/9 + t/3 - (t-20)^3/72$ |
| 18-20 | $-5/3 + t/12$ | $(t-20)^2/24$ | $(t-20)^3/72$ |

Table 4. Characteristics of Seismometer

| Type | 'Real' seism. | | 'Virtual' seism. | |
|---|---|---|---|---|
| | $f_n$ (Hz) | h | $f_n$ (Hz) | h |
| A | 1 | 0.3 | 10 | 0.7 |
| V | 0.5 | 3 | 0.25 | 20 |
| D | 0.1 | 0.7 | 0.01 | 0.55 |



CAL. : Acc. = 1 gal, Vel. = 1 kine, Dis. = 1 cm

Fig. 1. Model Waveforms of Ground Motion

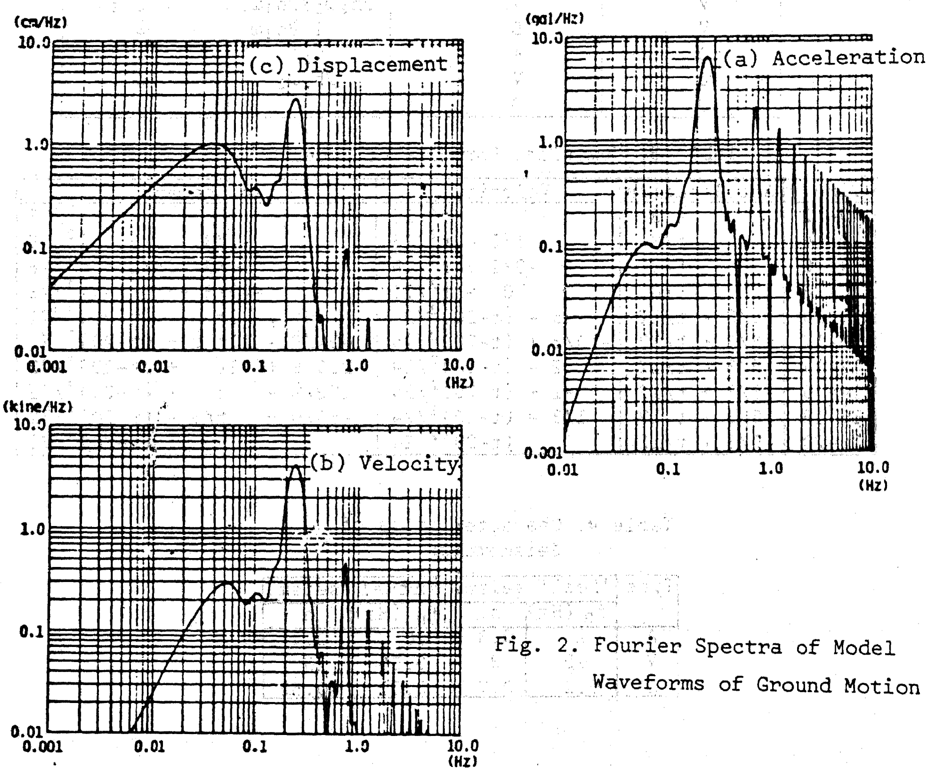


Fig. 2. Fourier Spectra of Model Waveforms of Ground Motion

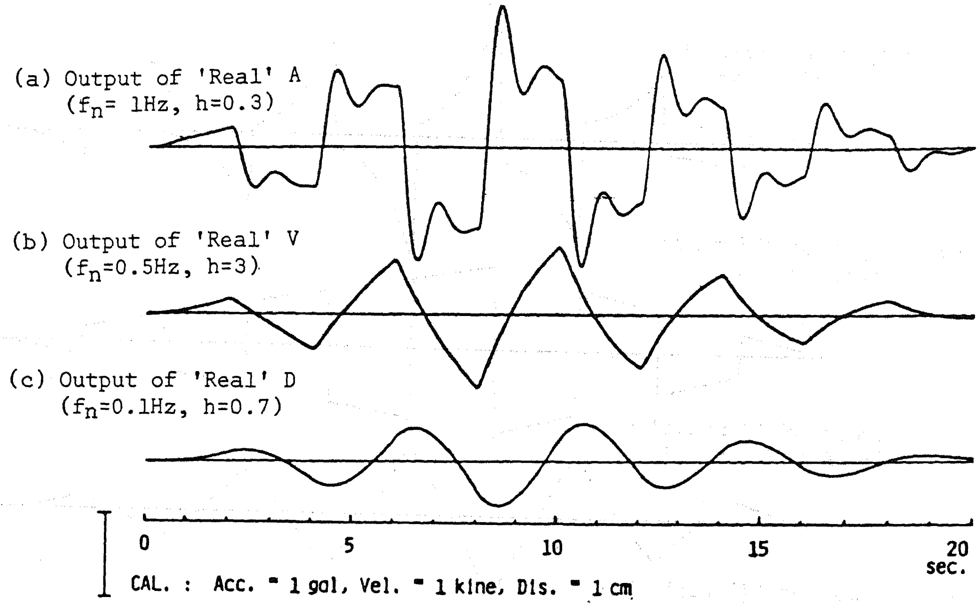


Fig. 3. Simulated Output Waveforms of 'Real' Seismometers

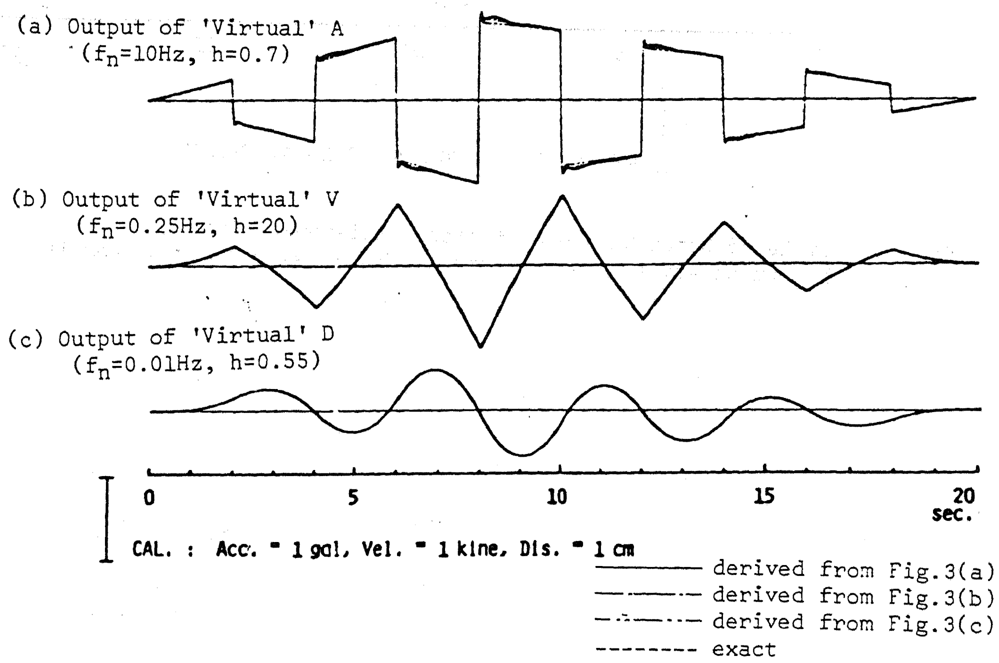


Fig. 4. Output Waveforms of 'Virtual' Seismometers derived from Outputs of 'Real' Seismometers in Fig. 3

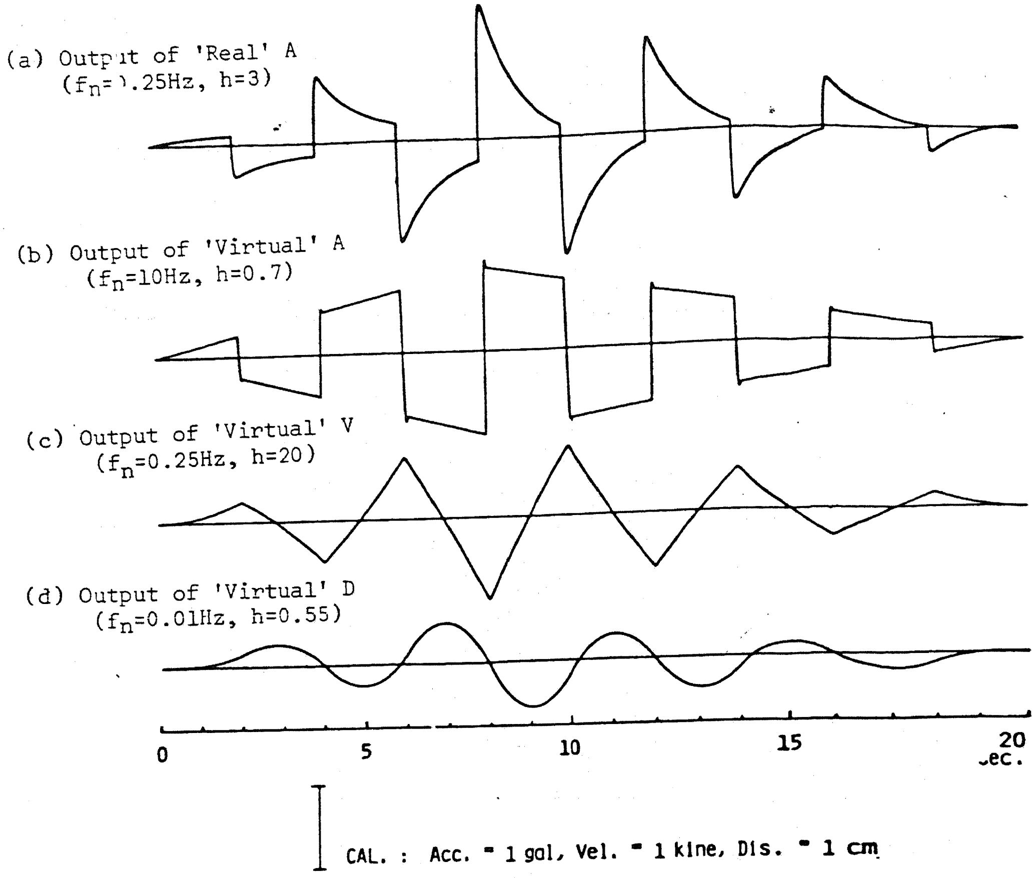


Fig. 5. Simulated Outpt Waveform of 'Real' Seismometer and the Output Waveforms of 'Virtual' Seismometer derived from it