EFFECTS OF CANYON TOPOGRAPHY ON DINAMIC SOIL-BRIDGE INTERACTION FOR INCIDENT PLANE SH WAVES

by

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SUMMARY

The influence of a semi-circular cylindrical canyon on dynamic soil-bridge interaction is analyzed for the case of incident harmonic plane SH waves. Bridge abutments are assumed rigid and support a shear beam on the edges of the canyon. The problem is formulated as one of scattering and diffraction of seismic waves. The scattered and the free-field solutions are expanded in three coordinate systems in terms of Hankel and Bessel functions, respectively. Comparisons of the response are provided for the present solution and the one obtained without canyon.

INTRODUCTION

Field measurements during earthquakes (6, 7) emphasize the important effect of the spatial variation of the ground motion on the response of long structures. Reports on damage of long structures with many supports are extensive (9, 10). In some cases the causes of the failure are unclear and they could be explained considering soil-structure interaction. On the other hand, it is well known that irregular topographies and soft deposits can induce large amplifications or decrements of ground motion at nearby sites (4, 11-13). The phenomenon involves scattering and diffraction of seismic waves.

Simple elastic models have been used to study the response of long structures with many supports when subjected to horizontal and vertical motions (8). Using a probabilistic approach, a design criterion has been proposed for long-multi-supported structures (5). The phase differences in the support motion were taken into account. With a finite element analysis the three-dimensional response of an arbitrary structure supported on rigid footings has been partially solved (14). Harmonic Rayleigh and plane body waves were considered.

For incident harmonic SH waves a recent work has been presented to study the dynamic soil-bridge interaction of a single span bridge (1). The bridge was modeled as a shear beam supported by two rigid abutments with a semi-circular portion embedded on the surface of an elastic half-space. The driving forces and the impedance matrix were obtained considering the problem of scattering. The influence of various parameters on the response was studied. Parameters included are: mass, stiffness and geometric ratios, and the frequency and angle of incidence of the incoming wave field.

In this paper an extension is presented of Abdel-Ghaffar and Trifunac's work (1). The main difference is that the present analysis assumes a semi-circular cylindrical canyon between the abutments of the bridge. The problem is solved in two steps; firstly the driving forces and the impedance

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-stiffness- matrix for the foundations are obtained. Secondly, the motion of the bridge itself is considered. From boundary conditions, an infinite set of simultaneous equations arises. The system is truncated in order to obtain the numerical solution. The influence of the canyon on the response is shown for different canyon-soil-bridge configurations and SH wave fields. Comparisons are made with the solution obtained without the canyon.

FORMULATION OF THE PROBLEM

Let us consider a homogeneous, isotropic, linear elastic half-space with a semi-circular cylindrical cavity on the surface. Near the edges of the canyon there are two rigid bodies with a semi-circular part embedded in the half-space. Such bodies are the abutments of a shear beam. This two-dimensional model of a canyon-bridge system is shown in Fig. 1. Three systems of polar coordinates are defined in Fig. 2. In the propagation of harmonic SH waves the antiplane displacement u must satisfy the reduced wave equation, which in each system j = 0, 1, 2 is

$$\frac{\partial^2 u}{\partial r_j^2} + \frac{1}{r_j}\frac{\partial u}{\partial r_j} + \frac{1}{r_j^2}\frac{\partial^2 u}{\partial \phi_j^2} + k^2 u = 0 \qquad (1)$$

where $k = \omega/\beta$, ω = circular frequency, $\beta = \sqrt{\mu/\rho}$ = shear wave velocity, μ = shear modulus and ρ = density of the medium.

The problem is solved by using the superposition principle; the displacement field wich satisfies Eq. 1 is given by

$$u = u_I + u_{II}\,\Delta_I + u_{III}\,\Delta_2 \qquad (2)$$

where u_I, u_{II} and u_{III} are the displacements due, respectively, to an incident SH wave with the footings fixed, a unitary displacement exp $(i\omega t)$ in the left abutment with the right abutment fixed, and a unitary displacement in the right abutment with the other fixed. The displacements of the abutments are given by Δ_1, and Δ_2. Throughout the paper the time factor, exp $(i\omega t)$, is understood. Each of the described displacement fields must satisfy the free-boundary condition $\partial u/\partial n = 0$ in the canyon border and the half-space surface.

Writing the field u_I in terms of the *free-field* solution, $u^{(o)}$, and the fields diffracted by the cavity and the abutments, the relationship is

$$u_I = u^{(o)} + u_o^{(d)} + u_1^{(d)} + u_2^{(d)} \quad , \qquad (3)$$

where

$$u^{(o)} = \sum_{m=o}^{\infty} A_m^{(o)}\, J_m\,(kr_o)\,\cos m\phi_o \quad ,$$

$$A_m^{(o)} = 2(-i)^m\,\varepsilon_m \cos m\theta \quad , \varepsilon_o = 1,\ \varepsilon_m = 2 \text{ if } m > 0 \quad ,$$

θ = angle of incidence, $i = \sqrt{-1}$,

$J_m(\cdot)$ = Bessel function of the first kind and order m,

r_o, ϕ_o = polar coordinates in the cavity

$$u_o^{(d)} = \sum_{m=o}^{\infty} A_m H_m^{(2)}(kr_o) \cos m\phi_o \quad ,$$

$$u_1^{(d)} = \sum_{m=o}^{\infty} B_m H_m^{(2)}(kr_1) \cos m\phi_1 \quad ,$$

$$u_2^{(d)} = \sum_{m=o}^{\infty} C_m H_m^{(2)}(kr_2) \cos m\phi_2 \quad ,$$

and where A_m, B_m, C_m = complex constants to be determined from boundary conditions, $H_m^{(2)}(\cdot)$ = Hankel function of the second kind and order m, and r_1, ϕ_1 and r_2, ϕ_2 are the coordinates associated with the abutments.

In order to satisfy boundary conditions in each system it is necessary to refer the terms in Eq. 3 to the system of coordinates considered. For this purpose Graf's addition theorem (2) is used. The displacement field u_{II} and u_{III} are very similar to u_I, the main difference being that $u^{(o)} \equiv 0$. The excitation is given by the prescribed motion of the supports as was pointed out before.

Boundary conditions, $\partial u/\partial r_o = 0$ at the canyon border and those of the abutment displacements, yield an *infinite set of simultaneous equations for* u_I, u_{II}, and u_{III}, respectively. As the coefficient matrix is the same, the three independent vectors can be considered toghether. In order to obtain the numerical solution the system is truncated to one of finite dimension. The size of the system required to obtain convergent solutions depends strongly on the frequency and on the relative dimensions of the canyon and abutments. In matrix notation the system of equations can be written as

$$A\,X = B \quad , \tag{4}$$

where A = coefficient matrix of order M x M, X = unknown matrix of order M x 3, and B = matrix of independent terms of the same order.

DRIVING FORCES AND IMPEDANCE MATRIX

Once the system in Eq. 4 is solved, the total forces F_1 and F_2 exherted by the soil on the abutments 1 and 2, respectively, can be obtained readily by integration of the stresses in the contact areas. These forces, in matrix notation, can be written as

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \begin{Bmatrix} F_1^{(o)} \\ F_2^{(o)} \end{Bmatrix} + \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{Bmatrix} \Delta_1 \\ \Delta_2 \end{Bmatrix} \tag{5}$$

or

$$F = F^{(o)} + K\,\Delta \quad , \tag{6}$$

where $F^{(o)}$ = vector of the driving forces, K = impedance matrix and Δ = displacement vector. It is clear that $F^{(o)}$ is obtained from u_I since it depends on the excitation and the geometric and mechanical properties of the system. The impedance matrix K is obtained from u_{II} and u_{III}.

RESPONSE OF THE BRIDGE

The bridge has been modeled as a shear beam and therefore the displacements u_b of the beam must satisfy the equation

$$\frac{\partial^2 u_b}{\partial x^2} + k_b^2 u_b = 0 \quad , \tag{7}$$

where $k_b = \omega/\beta_b$, $\beta_b = \sqrt{\mu_b/\rho_b}$, μ_b = shear modulus and ρ_b = mass density. The solution of Eq. 7, satisfying the boundary conditions $u_b(-D_1)=\Delta_1$ and $u_b(D_2) = \Delta_2$, is given by

$$u_b(x) = \frac{1}{\sin \nu} \left[\Delta_1 \sin k_p(D_2-x) + \Delta_2 \sin k_p(D_1 + x)\right] \quad , \tag{8}$$

where $\nu = k_p L$ and $L = D_1 + D_2$. The beam forces $F_b^{(1)}$ and $F_b^{(2)}$ acting on the abutments can be written as

$$\begin{Bmatrix} F_b^{(1)} \\ F_b^{(2)} \end{Bmatrix} = c \begin{pmatrix} -\cot \nu & \csc \nu \\ \csc \nu & -\cot \nu \end{pmatrix} \begin{Bmatrix} \Delta_1 \\ \Delta_2 \end{Bmatrix} \tag{9}$$

or

$$F_b = \omega^2 M_b K_b \Delta \quad , \tag{10}$$

where $c = M_b\omega^2/\nu$, $M_b = \rho_b hL$ = beam mass per unit length and h = thickness of the beam.

From the balance of dynamic forces on each abutment, the displacements become

$$\Delta = \left[\frac{1}{2} k\hat{a} M_s^{-1}(M - M_b K_b) - \hat{K}\right]^{-1} \hat{F}^{(o)} \tag{11}$$

where $\hat{a}$ = diagonal matrix with the radii of the abutments, *diag* $\hat{a} = (a_1, a_2)$, $M_s = \rho\pi\hat{a}^2/2$ = mass matrix of the soil removed by the footings, $M = \rho_f\pi\hat{a}^2/2$ = mass matrix of the footings with density ρ_f, $K = \mu k\pi\hat{a}\hat{K}$ and $F^{(o)} = \mu k\pi\hat{a}\hat{F}^{(o)}$.

RESULTS

The amplitudes of the displacements of the abutments for different bridges are given in Figs. 3 and 4. These are drawn *vs* normalized frequency as given by $\eta = k a_o = 2\pi a_o/\lambda$, where a_o = radius of the canyon, and λ = incident wave-length. Results are presented for three incidence angles ($\theta = 0$, 45 and 90 degrees) and $L/a_1 = 5$, 10 corresponding, respectively, with $a_1/a_o=2/3$, 1/3. The dimensions and properties of the abutments are the same for each bridge, thus modeling symmetric canyon-bridge systems. The mass of the elements are given in terms of the mass of the soil replaced by abutment one, that is by means of M_b/M_{S1} and M_1/M_{S1}, where M_b = mass of the shear beam, M_1 = mass of the rigid abutment, and M_{S1} = mass of the replaced soil. As previously stated $M_1 = M_2$ and $M_{S1} = M_{S2}$. The relative stiffness between the soil and the bridge is given in terms of $\varepsilon = k_b L/(k\, a_1)$ for which values of 2 and 3 were chosen.

To show the effects of the canyon on the bridge response, the displacements are compared with those calculated without the canyon, that is, using the previously described solution (1). The maximum response is reached for frequencies wich are smaller than those of the bridge without canyon, except for the large-span bridge. For vertical incidence the displacements are equal and in phase because of symmetry; they are also smaller than those obtained for other incidence angles. The differences between the two cases (canyon and no-canyon) are small for vertical incidence for which the barrier effect of the canyon is not significant. These differences grow for large values of L/a_1. The maximum values of Δ_1 and Δ_2 increase when $M_b > M_1$.

CONCLUSIONS

The influence of the canyon on the response is significant for incidences which differ from the vertical. In some cases the maximum abutment displacements are greater than those of the bridge without canyon. For long bridges the influence of one abutment on the motion of the other is small; this could simplify the impedance matrix since terms K_{12} and K_{21} could be neglected for certain frequencies. It should be expected that for bridges of greater fexibility than those treated here, the barrier effect would be of importance. Although a more complete parametric study is being developed (3), the results presented show that the effects of canyon topography on soil-bridge interaction can be significant.

ACKNOWLEDGEMENTS

Thanks are given to L. Esteva and J.I. Ojinaga for their comments and suggestions.

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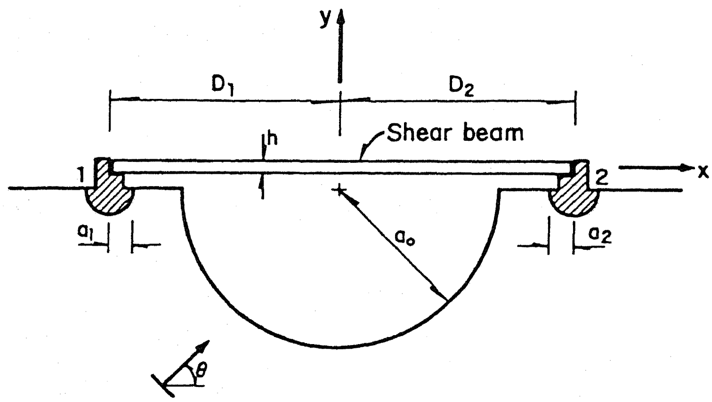

Fig 1. Two-dimensional model of a canyon-bridge system and incident plane SH wave

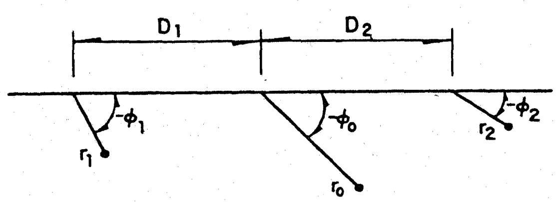

Fig 2. Systems of polar coordinates

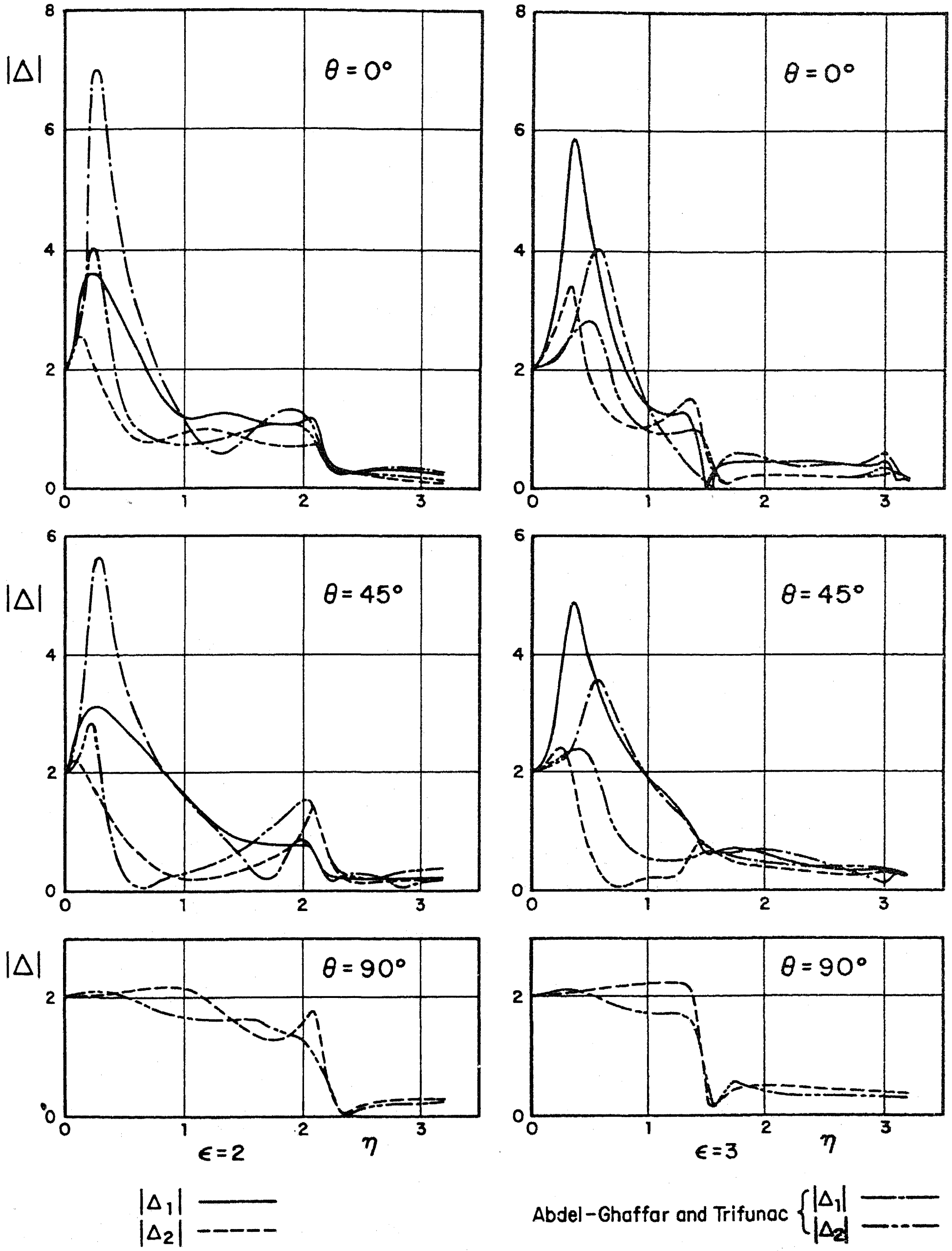

Fig 3. Displacement amplitudes of the two fundations for $M_b/M_{s_1}=2, M_1/M_{s_1}=4, L/a_1=5, a_1/a_0=2/3$ and $\epsilon=2,3$. Incidence angles $\theta=0°, 45°, 90°$

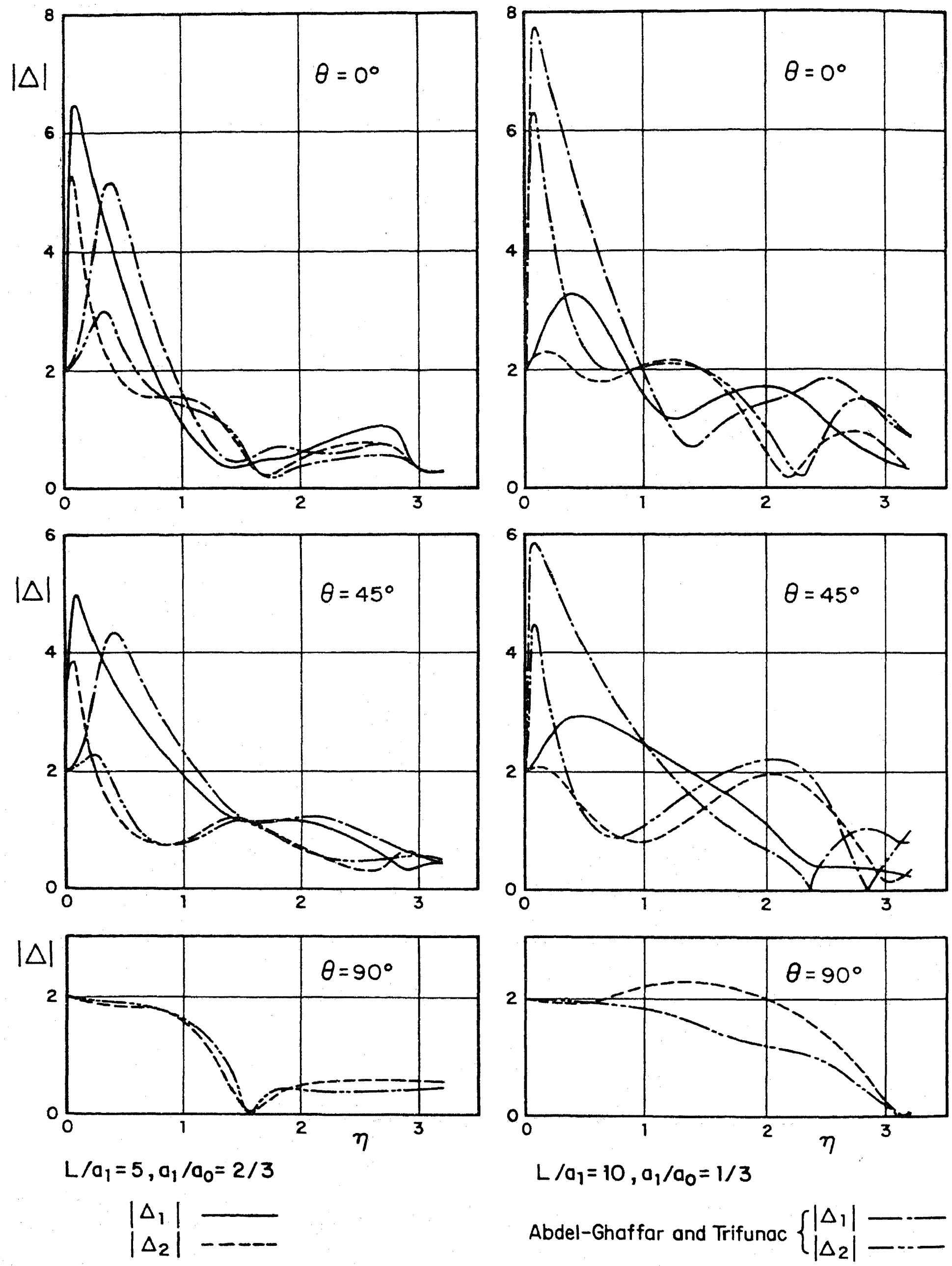

Fig 4. Displacement amplitudes of the two fundations for $M_b/M_{s_1} = 4, M_1/M_{s_1} = 2, L/a_1 = 5, 10, a_1/a_0 = 2/3, 1/3$ and $\epsilon = 3$. Incidence angles $\theta = 0°, 45°, 90°$